A Comprehensive Model of Evaluating Management Cadres in Universities and Colleges Based on Principal Component Analysis and Fuzzy Mathematics

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Abstract. Applying fuzzy comprehensive evaluation theory, a comprehensive evaluation model can be established by determining its relevant factors and their weight. The comprehensive evaluation for the management cadres can be achieved by adopting the methods of Cluster Analysis and Principal Component Analysis. This model provides a comprehensive and scientific tool for the assessment, selection and appointment of cadres for the personnel department and organization department in colleges and universities.

Introduction
How fast colleges and universities should push ahead is related to the competence of their management cadres. One of the key issues of the personnel department is how to evaluate and select outstanding cadres. The work of cadre assessments is the foundation of selecting and appointing qualified applicants, with evaluating factors play different roles in the procedures. In addition, the subjects evaluating and objects evaluated are not only overlapped, but also equipped with the characteristic of fuzziness. This paper tries to find a comprehensive model to evaluate management cadres in higher schools using the Principal Component Analysis and Fuzzy Mathematics theory.

In the process of comprehensive evaluation, the commonly used methods, such as Fuzzy Information and Analytic Hierarchy Process (AHP), focus on transforming the subjective evaluation indices into objective ones which are quantifiable and visualized. However, the mentioned methods themselves have some certain subjectivity in determining the various indices, which may lead to the shortage of objectivity. Considering it from another angle, the authors try to explore a scientific and feasible method based on the Fuzzy Mathematics theory. Such method focuses on objective evaluation and assesses the comprehensive quality of the management cadres in higher schools by using the technique of Cluster Analysis and the Principal Component Analysis.

The Establishment of a Comprehensive Evaluation Index System
According to the overall planning for the national reform of the cadre and personnel management system and the basic requirements of the management cadres in higher schools and in accordance with the scientific concept of development, talent and the correct accomplishments’ view, the colleges and universities should establish a fair, just, open, scientific and reasonable index system and mechanism for evaluating talents. Such mechanism should be oriented by morality, competence, diligence and performance, in which work performance and the masses’ recognition should be the key point. In line with the principle of choosing and appointing cadres with competence and morality, the evaluation index system can be divided into four aspects: morality, competence, diligence, performance [1-3], and each aspect can be decomposed into multiple secondary indices, as shown in table 1.
Table 1. The comprehensive evaluation index system for management cadres in universities and colleges.

<table>
<thead>
<tr>
<th>First-level Indices</th>
<th>Weight</th>
<th>Variable</th>
<th>Second-level Indices</th>
<th>Weight</th>
</tr>
</thead>
<tbody>
<tr>
<td>Morality</td>
<td>0.2</td>
<td>$X_1$</td>
<td>Ideological Level</td>
<td>0.3</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$X_2$</td>
<td>Understanding of Policy</td>
<td>0.3</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$X_3$</td>
<td>Team Spirit</td>
<td>0.15</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$X_4$</td>
<td>Working Style</td>
<td>0.25</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$X_5$</td>
<td>Professional Ability</td>
<td>0.35</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$X_6$</td>
<td>Management Ability</td>
<td>0.2</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$X_7$</td>
<td>Coordinating Ability</td>
<td>0.25</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$X_8$</td>
<td>Knowledge Update</td>
<td>0.2</td>
</tr>
<tr>
<td>Competence</td>
<td>0.3</td>
<td>$X_9$</td>
<td>Mental Attitude</td>
<td>0.2</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$X_{10}$</td>
<td>Working Efficiency</td>
<td>0.35</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$X_{11}$</td>
<td>Working Discipline</td>
<td>0.2</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$X_{12}$</td>
<td>Forging Ahead</td>
<td>0.25</td>
</tr>
<tr>
<td>Diligence</td>
<td>0.2</td>
<td>$X_{13}$</td>
<td>Target Achievement</td>
<td>0.2</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$X_{14}$</td>
<td>Target Achieved</td>
<td>0.3</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$X_{15}$</td>
<td>Working Effect</td>
<td>0.3</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$X_{16}$</td>
<td>Masses’ Recognition</td>
<td>0.2</td>
</tr>
</tbody>
</table>

Comprehensive Evaluation Model

The Ordered Clustering Method

Clustering method is used to distinguish the level and classification of all the evaluated objects, which, basing on the indices of observed samples, first achieve the statistical magnitude by analyzing the similarity between the probable samples or indices, and then classify the statistics samples. The authors will adopt the ordered clustering method to conduct analysis on the evaluation index system of management cadres [4].

The optimal classification can be established by the optimal partition method. A partition should be found first, which leads to the largest index difference between segments with minimal difference between the internal index values. Set index values as $x_1, x_2, \cdots, x_n$ and each is the P dimensional vector, the optimal segmentation steps are as follows:

1. Define the Diameter of Cluster

   Let $G_{ij}$ is $\{v_j, x_{j+1}, \cdots, x_{j+l}\}$, $j \geq i$, and the mean vector can be expressed as:

   $$\bar{x}_i = \frac{1}{j-i+1} \sum_{j=i}^{j+l} x_j$$

   (1)

   Then, the diameter of the cluster $G_{ij}$ is defined as the sum total of differences between the various internal indices, which can be expressed as the sum of squares of deviations. Make $D(i,j)$ be the diameter of $G_{ij}$, then

   $$D(i,j) = \sum_{i \neq j}^l (x_j - \bar{x}_i)^2$$

   (2)

2. Define the Objective Function

   N ordered index values can be divided into k classes. Set a certain splitting method as

   $$p(n,k) = \{i_1, \cdots, i_k - 1\} \cup \{i_2, \cdots, i_2 - 1\} \cup \cdots \cup \{i_k, \cdots, n\}$$
where the point of division \(1=i_1<i_2<\cdots<i_k<i_{k+1}=n\) (define the loss function or objective function in this classification as the sum of squares of deviations within a class), then
\[
e[P(n,k)] = \sum_{j=0}^{k} D(i_j, i_{j+1} - 1)
\] (3)

when \(n, k\) are fixed, the smaller the value of \([e/P(n, k)]\) is, the smaller the sum of squares of deviations within each class is, and the more reasonable the classification is. Therefore, the authors need to find a splitting method to minimize the value of the objective function \(P(n, k)\).

(3) Gaining Optimal Classification

Assume \(P^*(n,k)\) as the minimal classification gained by \(e[P(n,k)]\), as \(n\) ordered data are divided into \(k\) classes, namely they are first divided into two parts \([1,2,\cdots,j-1],[j,j+1,\cdots,n]\) and then \([1,2,\cdots,j-1]\) will be divided into \(k-1\) types, while \([j,j+1,\cdots,n]\) is sorted out into an individual class, that is \(p(n,k):P(j-1,k-1),(j,j+1,\cdots,n)\). So
\[
e[P^*(n,k)] = \min_{e[P]*} \left\{e[P^*(j-1,k-1)] + D(j,n)\right\}
\] (4)

When we divide \(n\) ordered data into \(k\) classes, \(j_k\) first satisfy:
\[
e[P^*(n,k)] = e[P^*(j_k-1,k-1)] + D(j_k,n)
\] (5)

First get the \(k\)th class \(G_k = (j_k,j_k+1,\cdots)\), then find \(j_{k-1}\) to get the \(k-1\)th class. Then one by one, all of the classes can be obtained.
\[
P^*(n,k) = (G_1,G_2,\cdots,G_k)
\] (6)

Based on matrix \([e[P(n,k)]]_{n\times n}\), image of \(k\) can be drawn on \(e[P(n,k)]\) and the optimal partition class number can be obtained, namely the obvious \(k\) at the turning point as shown in figure 2:

![Figure 1. Image of k on e[P(n,k)].](image)

**Basic Principles of Fuzzy Evaluation[5-6]**

(1) Determine the Evaluation Factors Set

First evaluation factors of objects to be assessed should be chosen, and then the evaluation index system determined, which is composed of many specific-combined and interrelated indices. Set \(U\) as evaluation factors set:
\[
U = \{u_1, u_2, \cdots, u_i, \cdots, u_n\}
\]
where \(u_i\) (\(i=1,2,\cdots,n\)) are first-level factors, with each consist of multiple second-level factors:
\[
U_1 = \{u_{11}, u_{12}, \cdots, u_{1k_1}\};
U_2 = \{u_{21}, u_{22}, \cdots, u_{2k_2}\};
\cdots \quad \cdots \\
U_n = \{u_{n1}, u_{n2}, \cdots, u_{nk_n}\}
\]
(2) Determine the Remarks Set
While evaluating the objects, suitable remarks should be selected. Set \( V = \{v_1, v_2, \cdots, v_m\} \).

According to the remarks set gained by cluster analysis introduced in section 3.2, the paper, through experimental analysis, obtains \( V=\) (excellent, good, passed and unqualified).

(3) Determine the Evaluation Factors’ Weight
In line with the degree of importance of various factors, their quantitative weight should be determined respectively and form the weight set expressed by \( A: A = \{a_1, a_2, \cdots, a_n\} \). Then, it can be gotten \( A = \{0.2, 0.3, 0.3, 0.2\} \) from Table 1.

Similarly, its second-level weight set will be \( A_1, A_2, \cdots, A_n \) namely:
\[
A_1 = \{a_{11}, a_{12}, \cdots, a_{1k}\}; \\
A_2 = \{a_{21}, a_{22}, \cdots, a_{2k}\}; \\
\cdots \cdots \cdots \\
A_n = \{a_{n1}, a_{n2}, \cdots, a_{nk}\}
\]

Several methods calculating weight value are commonly used, such as Analytic Hierarchy Process (AHP), Expert Weighting Method, Subjective Weighting Method, Delphic Method, etc. This paper, calculating all the weighted values by Expert Weighting Method, obtains:
\[
A_1 = \{0.3, 0.3, 0.15, 0.25\}; \\
A_2 = \{0.35, 0.2, 0.25, 0.2\}; \\
A_3 = \{0.2, 0.35, 0.2, 0.25\}; \\
A_4 = \{0.2, 0.3, 0.3, 0.2\}
\]

**Principal Component Analysis (PCA)**

The focus of PCA, used by this paper, is how to translate the multiple indices into fewer comprehensive indices. Comprehensive indices refer to a linear combination of multiple original indices.

(1) Data Standardization
Each cadre to be evaluated is regarded as a sample. Suppose there are \( n \) samples and 18 indices \( X_i(i=1, 2, \cdots, 18) \), \( x_j(i=1, 2, \cdots, 18; j=1, 2, \cdots, n) \), where \( x_j \) represents the score for the \( j \)th management cadre at the \( i \)th index.

According to the matrix technique[7]:
\[
x'_{ij} = \frac{x_{ij} - \bar{x}_j}{\sqrt{\text{Var}(X_j)}} (i=1, 2, \cdots, 18; j=1, 2, \cdots, n),
\]
where
\[
\bar{x}_j = \frac{1}{n} \sum_{i=1}^{18} x_{ij}; \text{Var}(X_j) = \frac{1}{n-1} \sum_{i=1}^{18} (x_{ij} - \bar{x}_j)^2 (j=1, 2, \cdots, n).
\]

(2) Calculate correlation matrix \( R \) of the sample
\[
R = \begin{bmatrix}
    r_{11} & r_{12} & \cdots & r_{1,18} \\
    r_{21} & r_{22} & \cdots & r_{2,18} \\
    \vdots & \vdots & \ddots & \vdots \\
    r_{18,1} & r_{18,2} & \cdots & r_{18,18}
\end{bmatrix}
\]

where \( r_{ii} = 1(i=1, 2, \cdots, 18) \), \( r_{ij} = r_{ji} (i \neq j) \)

(3) Determine Principal Component
The eigenvalue of \( R \) and its corresponding unit characteristic vector are:
With the data standardized, the correlation matrix features \( \sum_{i=1}^{18} \sigma_i^2 = \sum_{i=1}^{18} |\lambda_i| \) (\( \sigma_i \) is the standard deviation of \( X_i \)), while \( \eta = \frac{\sum_{i=1}^{m} \lambda_i}{\sum_{i=1}^{18} \lambda_i} \) is the accumulative variance contribution rate of \( F_1, F_2, \cdots, F_m (m \leq 18) \), which will be appropriate to take \( \eta \geq 85\% \) in practice. When \( m < 18 \), \( F_1, F_2, \cdots, F_m \) simplifies the original index system \( X_1, X_2, \cdots, X_{18} \), and can also reflect more than 85% variance of each one of \( X_1, X_2, \cdots, X_{18} \).

### Load of Variance on the Principal Component

To illustrate the load of variance for \( X_i \) on the principal component, the load matrix can be calculated first, which is

\[
A = \begin{bmatrix}
  u_{11} & u_{12} & \cdots & u_{1,18} \\
  u_{21} & u_{22} & \cdots & u_{2,18} \\
  \vdots & \vdots & \ddots & \vdots \\
  u_{18,1} & u_{18,2} & \cdots & u_{18,18}
\end{bmatrix} \begin{bmatrix}
  \sqrt{\lambda_1} \\
  \sqrt{\lambda_2} \\
  \vdots \\
  \sqrt{\lambda_{18}}
\end{bmatrix}
\]

(8)

Since the variance of each \( x'_i \) is 1, the percentage of variance of \( X_1, X_2, \cdots, X_m \) carried by each of \( F_1, F_2, \cdots, F_m \) can be calculated. When the percentage of the variance \( X_i (i = 1, 2, \cdots, 18) \) carried by \( F_1, F_2, \cdots, F_m \) together is greater than 85\%, \( F_1, F_2, \cdots, F_m \) can reflect not only the index information, but also the load of \( X'_i \) on \( F_1, F_2, \cdots, F_m \).

### (5) Comprehensive Evaluation

Principal component \( F_1, F_2, \cdots, F_m \) can reflect the management cadre’s quality from different aspects. In order to assess the strength of the comprehensive ability synthesized by principal component \( F_1, F_2, \cdots, F_m \) on the original information, it needs to calculate the comprehensive quality of the principal component \( F_1, F_2, \cdots, F_m \) for each evaluated object and its comprehensive evaluation branch, working as the basis of evaluation.

\[
F = \sum_{i=1}^{m} \omega_i F_i, \quad \omega_i \text{ is the weight, with } \omega_i = \frac{\lambda_i}{\sum_{j=1}^{18} \lambda_j}, (i = 1, 2, \cdots, m).
\]

### Examples in Practice

The comprehensive evaluation for each management cadre in the universities and colleges come from their corresponding superiors, peers, subordinates, and themselves. In addition to self-evaluation, each of the other three evaluation teams is made up of 5 people, with weight distribution as \( P = \{0.35, 0.3, 0.25, 0.1\} \) on binary logic (yes or no), where 1 means “yes”, 0 “no”.

#### Comprehensive Evaluation from Superiors

(1) Fuzzy evaluation matrix established based on the first-level indices of Morality \( R_1 \) :
The evaluation result of Morality $B_1$:

$$B_1 = A_1R_1 = (0.2917, 0.4333, 0.1750, 0)$$

In accordance with maximum membership principle, the evaluation result of the cadre’s “morality” is “good”.

(2) Fuzzy evaluation matrix established based on the first-level indicators of Competence $R_2$:

$$R_2 = \begin{bmatrix}
\frac{2}{6} & \frac{3}{6} & \frac{1}{6} & 0 \\
\frac{1}{6} & \frac{3}{6} & \frac{2}{6} & 0 \\
\frac{2}{6} & \frac{4}{6} & 0 & 0 \\
\frac{2}{6} & \frac{3}{6} & 0 & \frac{1}{6}
\end{bmatrix}$$

The evaluation result of Competence $B_2$:

$$B_2 = A_2R_2 = (0.3000, 0.5417, 0.1250, 0.0333)$$

In accordance with maximum membership principle, the evaluation result of the cadre’s “competence” is “good”.

(3) Fuzzy evaluation matrix established based on the first-level indices of Diligence $R_3$:

$$R_3 = \begin{bmatrix}
\frac{3}{6} & \frac{2}{6} & \frac{1}{6} & 0 \\
\frac{3}{6} & \frac{2}{6} & \frac{1}{6} & 0 \\
\frac{3}{6} & \frac{2}{6} & \frac{1}{6} & 0 \\
\frac{2}{6} & \frac{2}{6} & \frac{2}{6} & 0
\end{bmatrix}$$

The evaluation result of Diligence $B_3$:

$$B_3 = A_3R_3 = (0.4583, 0.3333, 0.2083, 0)$$

In accordance with maximum membership principle, the evaluation result of the cadre’s “competence” is “excellent”.

(4) Fuzzy evaluation matrix established based on the first-level indices of Performance $R_4$:

$$R_4 = \begin{bmatrix}
\frac{3}{6} & \frac{2}{6} & \frac{1}{6} & 0 \\
\frac{2}{6} & \frac{3}{6} & \frac{1}{6} & 0 \\
\frac{2}{6} & \frac{2}{6} & \frac{1}{6} & \frac{1}{6} \\
\frac{4}{6} & \frac{2}{6} & 0 & 0
\end{bmatrix}$$

The Evaluation result of Performance $B_4$:

$$B_4 = A_4R_4 = (0.4333, 0.3833, 0.1333, 0.0833)$$

In accordance with the principle of maximum membership, the evaluation results of the cadre’s performance is “excellent”.

According to the above-mentioned evaluation result, the fuzzy relation matrix of comprehensive evaluation of the superiors to the cadre is

$$R = \begin{bmatrix}
B_1 & B_2 & B_3 & B_4 \\
0.2917 & 0.4333 & 0.1750 & 0 \\
0.3000 & 0.5417 & 0.1250 & 0.0333 \\
0.4583 & 0.3333 & 0.2083 & 0 \\
0.4333 & 0.3833 & 0.1333 & 0.0833
\end{bmatrix}$$
The comprehensive evaluation result of the superior to the cadre is $C_1$.

$C_1 = AR = (0.3700, 0.4308, 0.1542, 0.0350)$

In accordance with the principle of maximum membership, the comprehensive evaluation result of the superior to the cadre is “good”.

**Comprehensive Evaluation of Each of the Other Teams to the Cadre**

According to the fuzzy relationship matrix, it can be calculated similarly each of the other teams’ comprehensive evaluation to the cadre. The evaluation result of peers:

$C_2 = (0.3769, 0.4362, 0.1669, 0)$

The evaluation result of subordinates:

$C_3 = (0.3789, 0.4639, 0.1372, 0)$

Self-evaluation result: $C_4 = (0.5050, 0.4150, 0.0600, 0)$

The final evaluation result of the University to the cadre is $C_{PD} = (0.3878, 0.4391, 0.1443, 0.0123)$

In accordance with the principle of maximum membership, the comprehensive evaluation result of the university to the cadre is “good”.

**Comprehensive Evaluation Scores**

Suppose the rank matrix $H = (90, 80, 70, 60)$,

$W = CH^T = (0.3878, 0.4391, 0.1443, 0.0123)(90, 80, 70, 60)^T = 80.8690$

The comprehensive evaluation score of the cadre is 80.8690.

**Conclusion**

The various indices evaluating morality, competence, diligence and performance of the management cadres in universities and colleges can be achieved by using the method of Cluster Analysis, Principal Component Analysis. The model provides a comprehensive and scientific evaluation tool for the assessment, selection and appointment of cadres for the personnel department and organization department, whose result can objectively reflect the real situation to ensure the authenticity and reliability of cadres’ comprehensive evaluation.

**References**


