Contract Design and Supplier Quality Management under Asymmetric Information

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Keywords: Contract design, Game Theory, contract model, supplier's quality Management, asymmetric information.

Abstract. This paper investigated how to design contract models under asymmetric information to manage supplier quality and achieve supply chain coordination. We employed game theory to design contract models: optimal contracts in centralized control situation, profit margin contract and profit sharing contract in decentralized control situation. Under asymmetric information, the manufacturer designed optimal contract to manage supplier quality and to achieve coordination. The numerical analyses show that profit sharing contract is an optimal decision and coordination mechanism in decentralized control situation.

Introduction

Supplier quality management (SQM) is considered as a proactive approach in the buyers’ perspective to seek for continuous supply quality improvement and collaborative ongoing alliance between buyers and suppliers. Studies of successful Supplier Quality Management (SQM) frameworks consistently showed that the orchestration of supplier alliances, supplier development and supplier monitoring are the three foundational elements to a globally scalable, secure supplier network. (I-Ki, Chin, 2004)[1].

Agrawal and Muthulingam (2015) investigated how the depreciation of organizational knowledge (organizational forgetting) affects quality performance. They analyze information on 2,732 quality improvement initiatives implemented by 295 vendors of a car manufacturer and find that organizational forgetting affects quality gains obtained from both learning-by-doing (autonomous learning) and quality improvement initiatives (induced learning); more than 16% of quality gains from autonomous learning and 13% of quality gains from induced learning depreciate every year. Their results highlight the ubiquity of organizational forgetting and suggest the need for continued attention to sustain and enhance quality performance in supply chains [2].

Supply chain contracts are considered as a useful tool to structure the costs and rewards of all of its members so as to achieve coordination in a decentralized situation. Yang et al. (2012) studied a buyer’s strategic use of a dual-sourcing option when facing suppliers possessing private information about their disruption likelihood and solved for the buyer’s optimal procurement contract. Their research showed that the optimal contract can be interpreted as the buyer choosing between diversification and competition benefits. Better information increased diversification benefits and decreases competition benefits [3].
Chaturvedi and Martínez-de (2011) considered a buyer facing multiple potential suppliers, each having an associated (exogenous) reliability that quantifies its risk of supply failure and designed optimal mechanisms that depend on the buyer’s level of information regarding the suppliers’ cost of production and reliability. Their specific contribution was to provide guidelines for designing optimal procurement mechanisms when there is a risk of supplier failure [4].

Liu and Wang (2015) developed a quality control game model for logistics service supply chain and analyzed the impact of different risk attitude of a logistics service integrator and a functional logistics service provider [5].

In related works on supply chain coordination under asymmetry information research literature, Gümü and Gurnani (2012) modeled a supply chain consisting of a single buyer and two suppliers, both of which compete for the buyer’s order and face risk of supply disruption. One supplier is comparatively more reliable but also more expensive, whereas the other one is less reliable but cheaper and faces higher risk of disruption. They analytically characterize the equilibrium contracts for the two suppliers and the buyer’s optimal procurement strategy [6].

Li et al. (2012) analyzed two supply chain inventory models (the vendor has complete information about the buyer’s cost structure, and the buyer possesses private cost information), they designed the coordination mechanism by using principal agent model to induce the buyer to report his true cost structure [7].

**Contract Design in Centralized Control Situation under Asymmetric Information**

We consider the standard setting with a single supplier and single buyer who sells the supplier’s product to the final market. Manufacturer orders from supplier according to market demand $Q$. The situation is described as follows. The market demand function is given by linear demand function in quality and price: $q = Q(p, x) = \alpha + \varepsilon x - \beta p$, where $\alpha > 0$, $\beta > 0$ and $\varepsilon > 0$ are known parameters, $x$ denotes the supplier quality, and the retail price selling to the customer is $p$. The supplier’s production cost is given by $s = S(q, x) = (\lambda + \delta x)q + f + \phi x^2$, the supplier’s variable cost is $\lambda + \delta x$, $\lambda$ denotes the variable production cost, the unit variable cost increases (decreases) by $|\delta x|$. The supplier’s fixed production cost is $f + \phi x^2$ (similar to Banker 1998) [8].

The buyer’s internal variable costs are $m$, the wholesale price is $w$. The buyer’s order quantity is $Q$, without loss of generality, we assume that the supplier follows a lot-for-lot policy, i.e., the supplier’s production lot size is equal to the lot size shipped to the buyer. Let $\pi$ and $\Pi$ denote the profit and the expected profit respectively.

Define quality as the supplier’s conforming rate, that is, the proportion of units that satisfy product specifications, the widely used quality measure. Given the many production uncertainties, supplier’s conforming rate of the produced units is a random variable (Yan, Zhao and Tang 2015) [9].

In general, the buyer doesn’t know the supplier’s quality $x$; we assume the buyer holds a prior distribution function $H(x)$ with continuous density function $h(x)$, and mean value of $\mu$, the Variance is $\sigma^2$. $H(x)$ is different, strictly increasing and is defined on $[x, \bar{x}]$, where $x \in [0, \infty)$. Let $H(0) = 0$ and $\bar{H}(x) = 1 - H(x)$. All parameters except $x$ are common knowledge.
For the purpose of benchmark, we first investigate the centralized situation where one central decision maker seeks to maximize total system profits. Then the supply chain system profit can be written as

\[ \Pi(p) = \int_{x} [(\alpha + \epsilon x - \beta p)(p - \lambda - \delta x - m) - f - \phi x^2]h(x)dx. \]

From the first optimal condition \( \partial \Pi(p)/\partial p = 0 \), we obtain the optimal retail price is

\[ p^c = \frac{\alpha + \beta(\lambda + m) + (\epsilon + \beta \delta) \mu}{2\beta} \]

The optimal order quantity is

\[ Q^c = \frac{\alpha - \beta(\lambda + m) + (\epsilon - \beta \delta) \mu}{2} \]

Therefore, the maximum supply chain expected profit is given by

\[ \Pi^c = \left[ \frac{\alpha - \beta(\lambda + m) + (\epsilon - \beta \delta) \mu}{4\beta} \right]^2 - f - \phi(\sigma^2 + \mu^2) \]

**Contract Design in Decentralized Control Situation under Asymmetric Information**

**Profit Margin Contract**

Just as a dominated supplier will declare a unit wholesale price \( w \), the economics and marketing literature has long recognized that a dominated buyer can declare a required profit margin. The buyer’s profit function can be written as: \( \pi_M = Q(p - w - m) \). After the dominant manufacturer declares his required profit margin \( M \), i.e., \( M = \pi_M/Q \), the supplier knows that, for whatever she quotes, the unit retailer price will be \( p = w + m + M \). Hence, the supplier’s profit function is

\[ \pi_s(w) = \left[ \alpha - \beta(w + m + M) + \epsilon x \right](w - \lambda - \delta x) - f - \phi x^2. \]

From the first optimal condition \( \partial \pi_s(w)/\partial w = 0 \), we obtain the optimal wholesale price \( w(M) = \left[ \alpha + \beta(\lambda - m - M) + (\epsilon + \beta \delta) x \right]/(2\beta) \).

Substituting \( w(M) \) into the buyer’s expected profit function \( \Pi_M(M) = \int_{x} \pi_M(M)h(x)dx \), solving \( \partial \Pi_M(M)/\partial M = 0 \), we obtain optimal profit margin \( M^\star \). By calculating the above function, we can derive Lemma 1.

**Lemma 1.** The profit margin contract in decentralized control setting under asymmetric supplier quality information is following.

The optimal profit margin is

\[ M^{\text{pm}} = \frac{\alpha - \beta(\lambda + m) + (\epsilon - \beta \delta) \mu}{2\beta} \]

The optimal wholesale price is

\[ w^{\text{pm}} = \frac{\alpha + \beta(3\lambda - m) + (\epsilon + 3\beta \delta) \mu}{4\beta} \]

The optimal retail price is:

\[ p^{\text{pm}} = \frac{3\alpha + \beta(\lambda + m) + (3\epsilon + \beta \delta) \mu}{4\beta} \]

The optimal order quantity is:

\[ Q^{\text{pm}} = \frac{\alpha - \beta(\lambda + m) + (\epsilon - \beta \delta) \mu}{4} \]
The buyer’s expected profit is:
$$\Pi_m^P = \frac{\left[\alpha - \beta (\lambda + m) + (\epsilon - \beta \delta) \mu \right]^2}{8\beta} \quad (8)$$

The supplier’s expected profit is:
$$\Pi_s^P = \frac{\left[\alpha - \beta (\lambda + m) + (\epsilon - \beta \delta) \mu \right]^2}{16\beta} - f - \phi(\sigma^2 + \mu^2) \quad (9)$$

The supply chain system’s expected profit is:
$$\Pi_m^C = \frac{\left[\alpha - \beta (\lambda + m) + (\epsilon - \beta \delta) \mu \right]^2}{16\beta} - f - \phi(\sigma^2 + \mu^2) \quad (10)$$

By calculating, we can obtain $\Pi_m^P < \Pi_m^C$, so supply chain don’t achieve coordination in profit margin contract.

**Profit Sharing Contract (coordination situation)**

We now design a profit sharing contract to coordinate the supply chain. Under the profit sharing contract, let $\psi$ be the fraction of channel expected profit the retailer keeps, $\psi \in [0,1]$, so $1-\psi$ is the fraction the supplier earns. The buyer’s expected profit is $\Pi_m^P = \psi \Pi_m^PS$, and the supplier’s expected profit is $\Pi_s^P = (1-\psi) \Pi_m^PS$. With coordination, the system expected profit is $\Pi_m^PS = \Pi_m^C$, the optimal order quantity is $Q_m^PS = Q_m^C$, and the optimal retail price is $p_m^PS = p_m^C$.

The buyer’s expected profit function can be written as $\Pi_m^PS = E\left[Q \cdot M\right]$, it also can be written as $\Pi_m^PS = E\left[Q^C \cdot M\right] = \psi \Pi_m^C$. By calculating the above function, we can derive Lemma 2.

**Lemma2.** The profit sharing contract in decentralized control setting under asymmetric supplier quality information is following.

The optimal profit margin is
$$M_m^PS = \frac{\psi \left[\alpha - \beta (\lambda + m) + (\epsilon - \beta \delta) \mu \right]}{2\beta} \quad (11)$$

The optimal wholesale price is
$$w_m^PS = \frac{(2-\psi)(\alpha - \beta m) + (2+\psi)\beta \lambda + \left[(2-\psi)\epsilon + (2+\psi)\beta \delta \right] \mu}{4\beta} \quad (12)$$

The optimal retail price is:
$$p_m^PS = \frac{\alpha + \beta (\lambda + m) + (\epsilon + \beta \delta) \mu}{2\beta} \quad (13)$$

The optimal order quantity is:
$$Q_m^PS = \frac{\alpha - \beta (\lambda + m) + (\epsilon - \beta \delta) \mu}{2} \quad (14)$$

The buyer’s expected profit is:
$$\Pi_m^PS = \psi \left\{ \frac{\left[\alpha - \beta (\lambda + m) + (\epsilon - \beta \delta) \mu \right]^2}{4\beta} - f - \phi(\sigma^2 + \mu^2) \right\} \quad (15)$$

The supplier’s expected profit is:
$$\Pi_s^PS = (1-\psi) \left\{ \frac{\left[\alpha - \beta (\lambda + m) + (\epsilon - \beta \delta) \mu \right]^2}{4\beta} - f - \phi(\sigma^2 + \mu^2) \right\} \quad (16)$$

The supply chain system’s expected profit is:
$$\Pi^P = \frac{[\alpha - \beta(\lambda + m) + (\varepsilon - \beta \delta) \mu]^2}{4\beta} - f - \phi(\sigma^2 + \mu^2)$$  \hspace{1cm} (17)

Results

In this section, we give several numerical examples to analyze the effects of the supplier quality level on the order quantity and supply chain system’s expected profit. Let $\alpha = 30, \beta = 3, \varepsilon = 50, \lambda = 3, \delta = 1.5, m = 5, f = 50, \phi = 0.5, \sigma = 10$. We assume the quality conform rate of $\mu$ varies from 1 to 10.

Impact on Optimal Order Quantity

Fig.1 illustrates the impact of varying $\mu$ on the optimal order quantity. From this figure, we can see that optimal order quantity is a linearly increasing function of $\mu$.

Comparing the optimal order quantity in profit margin contract and profit sharing contract in decentralized control setting in Fig.1, the latter is more than the former. This implies that latter is a more optimal and adaptable instrument for channel coordination.

![Figure 1: $\mu$ versus optimal order quantity.](image)

Impact on System’s Expected Profit

Fig.2 illustrates the impact of varying $\mu$ on the supply chain system’s expected profit. From this figure, we can see that the system’s expected profit is a non-linearly increasing function of $\mu$.

Comparing the system’s expected profit in profit margin contract and profit sharing contract in decentralized control setting in Fig.2, the latter is more than the former. This implies that latter is a more optimal and adaptable instrument for channel coordination.

![Figure 2: $\mu$ versus system’s expected profit.](image)

Conclusion

In this paper, we have investigated contract design in a supplier-manufacturer supply chain in centralized and decentralized setting under asymmetric supplier quality information and studied how the manufacturer control supplier’s quality. From the analysis above, we can draw
a conclusion that the profit sharing contract was the optimal strategy and could achieve supply chain channel coordination in decentralized setting, besides, the order quantity and the expected profit are increasing functions of supplier quality level.

References


