Game Analysis of Military Equipment Demands Reporting Based on Principal-agent Theory
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Abstract. This paper makes an incentive game analysis based on the principal-agent theory on the work of military equipment demands reporting, tries to find an optimization to satisfy both side of army and assistant, and give a rational solution to the moral hazard between clints and agents. Provides a new research direction to adapt to the new equipment management situation after the reform of army.

Introduction
The equipments work of military are mainly composed of three parts: raising, storing and supplying, and the three of them are connected by the time. However, one thing that easily ignored by managers is the report work of the needs of equipments. Because the demand reporting is the forerunner of raising, storing and supplying, an unsatisfactory equipment demand plan lays the foreshadowing of the following series of problems at the source.

The tenure cycle of an assistant in most units of army is about 3-4 years. So in many cases, the departments have to rely on inexperienced assistants give full play to the subjective initiative to complete this task which affects the subsequent work related to all the equipments. Therefore, it is some problem about how to guide the assistants that lack of work experience to make proper demand plans.

Generally speaking, the following aspects should be considered in a proper demand plan:
1. The demand plans in the past years, especially the ones with high equipment satisfaction rate and less backlog in the year-end feedback. Then, compared the tasks of the current year based on the task volumes at that time.
2. The tast volume of the current year. And the equipments model and state and other technical indicators of the unit should be taken account into.
3. A rough demand plan derived from the equipment management system based on the outline and the corresponding formula.
4. The knowledge about the regular pattern of the equipments consuming.

Problem Description
Based on the principal-agent theory, this paper makes an incentive game analysis on the work of equipment demand reporting. The paper takes army as the principal and assistant as agent. The army hopes the assistant to make a satisfactory demand plan according to the comprehensive benefit of the military, while it cannot evaluate assistant selection plan immediately, but only observe the backlog or shortage of equipment in the process of supply and consumption. (We assume that the equipment supply satisfaction rate is 100%) Therefor the final question is that how to reward and punish agents according to the observed information to achieve the purpose of motivation.

Modeling of Incentive Game
Assignments of Symbols and Assumptions
\( A \): All optional demand combinations of assistants.
\( a \): A specific demand plan. \( a \in A \)

\( \theta \): Exogenous random variable out of the assistant’s control. The paper set \( \theta \) as the stability of consumption of equipment \( a \) (Can be expressed by the standard deviation or variance).

\( \rho \): Range of values \( \theta \) (the higher, the more accurate the consumption forecast is).

\( G(\theta), g(\theta) \): The distribution function and probability density function of \( \theta \) on \( \rho \).

\( x(a, \theta) \): An observable result codetermined by both \( a \) and \( \theta \), such as stock backlog, shortage of goods, etc.

\( \pi(a, \theta) \): Comprehensive benefit codetermined by both \( a \) and \( \theta \), and the direct beneficiaries is the client.

\( s(x), s(\pi) \): The implementation cost of the incentive system \( s \) which conclude by the result \( x \) designed by army for the assistant; the profit gained by assistant from the total income \( \pi \) under the incentive system \( s \).

\( c(a) \): The cost of \( a \), \( c' > 0, c'' > 0 \).

\( v(\pi - s(x)) \): The expected effectiveness of the army, \( v' > 0, v'' \leq 0 \), \( \mu(s(\pi) - c) \): The expected effectiveness of the assistant, \( \mu' > 0, \mu'' \leq 0 \)

Assumption 1: \( \pi \) is a strictly increasing and incrementally reduced function of \( a \). If the value of \( \theta \) is certain, the better work assistant have done, the higher the effectiveness of the output, but the Unit added value of output will reduce with the increase of \( a \). \( a \) and \( \theta \) cannot be observed.

Assumption 2: \( \pi \) is a strictly increasing function of \( \theta \). The more stable the equipment consumption, the more conducive to the precision of the demand and perfect meets the needs.

Assumption 3: Both army and assistant are risk aversion or neutral type.

Assumption 4: Function \( G, x, \pi, \mu, v, c \) are army and assistant’s Common knowledge.

Model Establishment

Function \( \pi \) increases with \( a \). Therefore, army hopes to assistant to work hard, to give full play to the advantages of information mastery about the consumption of the equipments. But \( c' > 0 \), which means assistant hope to pay less effort. That’s why army should give sufficient incentives to their assistants.

The expected effectiveness of the army:

\[
\int v(\pi(a, \theta) - s(x(a, \theta)))g(\theta)d\theta
\] (1)

The work of army is to select one of the satisfaction of the assistant effort level \( a \) and to design an incentive \( s(x) \), maximize formula(1).

The army finds itself facing two constraints from assistant:

For the one, the purpose of motivation is to win a win-win situation. The effectiveness when assistant participate this incentive should better than the one assistant don’t participate. That’s what we called participation constraint:

\[
\int u(s(x(a, \theta)))g(\theta)d\theta - c(a) \geq U
\] (2)

Another, army has no idea about \( a \) and \( \theta \) when it begin to design incentive scheme, so after the introduction of the incentive system, assistant always choose the action \( \bar{\pi} \) to maximize their effectiveness. If army want assistant to choose action \( a \) which satisfy it properly, the incentive must let the assistant believe that they will get better effectiveness if they choose action \( a \), not \( a' \).

That means:

\[
\int u(s(x(a, \theta)))g(\theta)d\theta - c(a) \geq \\
\int u(s(x(a', \theta)))g(\theta)d\theta - c(a'), \forall a' \in A
\] (3)

In summary, the problem of army is to choose \( a \) and \( s(x) \) which maximize its profit:
Analysis of Incentive Game

Assume an assistant has two options to choose, plan H and plan L, representing the demand plan by hard work and lazy work, respectively. Assume the minimum possible value of $\pi$ is $\underline{\pi}$, and the maximum possible value of $\pi$ is $\overline{\pi}$. If $a = H$, the distribution function and density function of $\pi$ are $F_H(\pi)$ and $f_H(\pi)$, respectively; If $a = L$, the distribution function and density function of $\pi$ are $F_L(\pi)$ and $f_L(\pi)$. Because $\pi$ is an increasing function of $a$, then $\pi \in [\underline{\pi}, \overline{\pi}]$, $F_L(\pi) \geq f_H(\pi)$. The probability of getting high returns is larger if the assistant works harder.

From the army’s perspective, they would like the assistant to choose $a = H$, while from the assistant’s perspective, $c(H) > c(L)$. So the problem of the army is to design $s(x)$:

$$\max_{\pi, s(x)} \int v(\pi - s(x)) f_H(\pi) d\pi$$

s.t. $\int u(s(x)) f_H(\pi) d\pi - c(a) \geq U$

$\int u(s(x)) f_L(\pi) d\pi - c(a) \geq$

$\int u(s(x')) f_L(\pi) d\pi - c(a')$, $\forall a' \in A$

(4)

The Lagrange multiplier of $\lambda$ and $\mu$ as constraints A and B, respectively,

$-v' f_H(\pi) + \lambda u' f_H(\pi) + \mu u' f_L(\pi) - \mu u' f_L(\pi) = 0$

we can get:

$$\frac{v'(\pi - s(\pi))}{u'(s(\pi))} = \lambda + \mu(1 - \frac{f_L}{f_H})$$

(6)

We come to the conclusion that assistant revenue changes with $f_L / f_H \cdot f_H / f_H$. Shows how much $\pi$ comes from distribution of $f_L$, not $f_H$.

In another way, army can modify the probability of assistant’s hard work from the inventory in end of the year which can be easily observed. According to the Bayes rule, $\gamma = \prob(H)$ is the priori probability when army think assistant choose $H$, $\tilde{\gamma} = \prob(H / \pi)$ is the posteriori probability when army think assistant choose $H$ after army observe $\pi$:

$$\tilde{\gamma}(\pi) = \frac{f_H \gamma}{f_H \gamma + f_L (1 - \gamma)}$$

And:

$$\frac{f_L}{f_H} = \frac{\gamma - \gamma * \tilde{\gamma}(\pi)}{\tilde{\gamma}(\pi) (1 - \gamma)}$$

Put this formula into (6):
\[
\frac{v'(\pi - s(\pi))}{u'(s(\pi))} = \lambda + \mu \left( \frac{\tilde{y}(\pi) - \gamma}{\tilde{y}(\pi)} (1 - \gamma) \right)
\] (7)

Therefore, if army have to make a downward correction about the probability of assistant choosing H after \( \pi \) is observed, that means \( \tilde{y}(\pi) < \gamma \), then assistant should get punishment; in contrast, if \( \tilde{y}(\pi) > \gamma \), assistant should get reward.

**Summary**

This paper establishes a basic model about the game of equipment demand reporting based on principal-agent theory; lead a way to solve this problem. But the following aspects should be get further consideration before it been applied to pratical work.

1. Analyse the influence auxiliary analysis of equipment management information system make on the report work of equipment demand, mainly the influence on the function \( c(a) \). Because assistants will pay less time and efforts to work out the equipment demand reporting of same level of quality with the development of the equipment management information system. That means, the value of \( c(a) \) will reduce.

2. Analyse the how much the weight of the report work on equipment in assistant post assessment, influence \( s(\pi) \). There are more than equipment demands reporting on assistants working description. So the equipment demand reporting shoud be given a proper weight assistant post assessment, and the larger this weight is, the better effect the incentive will get, and the better work assistants will do.

3. Analyse the cycle of \( \pi(a, \theta) \). Our military equipment turnover cycle is quite long, about one year or so. And superior usually require 100% to meet the equipment demand plan of the unit. So it is impractical to perceive whether this kind of equipment is overstock or not.

**References**

