Research on Interval Multiple Attribute Decision-Making Method Based on Gray Correlation Degree and TOPSIS

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Abstract. This paper investigates an interval multiple attribute decision making problem. In view of the fact that Euclidean distance and gray correlation degree can fully and faithfully reflect the proximity of the selected scheme to the ideal scheme from the distance and the different shape. So we propose a new decision making method based on grey correlation degree and TOPSIS. It accurately evaluates interval numbers multi-attribute index scheme. Finally, an example of a multiple attribute decision making problem with intervals is given to illustrate the feasibility and validity of the proposed method.

1. Introduction

Technique for Order Preference by Similarity to Ideal Solution (TOPSIS) is an effective method for multiple attribute decision making. The solutions are arranged by the comparison of the distance between every choice and the positive or negative ideal solution. If the object is closest to the optimal solution and which is also the farthest from the inferior solution, it is best. Otherwise it’s the worst. While each index of the positive solution achieves the optimal value of each evaluation index, each index of the negative solution achieves the worst value of each evaluation index.

This thesis presents a new method which integrates the theory of gray correlation degree with the basic idea of TOPSIS. It provides new algorithm for multiple attribute decision making problem. But nowadays, the decision-making information we obtain is usually grey Numbers in the field of the economy, finance, military and engineering technology and so on. Therefore, using interval numbers to express decision information is quite common. This decision making based on clear number is extended to deal with interval numbers. Research on this issue has both theoretical significance and also practical value.

2. The priority method of interval numbers Based on Gray Correlation Degree and TOPSIS

2.1 Coherent concepts and operational rules about interval numbers

Definition 1 If \( a = [a^l, a^r] = \{x | a^l \leq x \leq a^r, a^l, a^r \in R\} \), \( a \) is an interval number. Particularly, when \( a^l = a^r \), \( a \) devolve into a real number.

Definition 2 If \( a = [a^l, a^r], b = [b^l, b^r] \), \( d_{(a,b)} = \sqrt{(a^r - b^l)^2 + (a^l - b^r)^2} \) which is called standard Euclidean of interval \( a \) and interval \( b \).

2.2 The priority method of interval numbers Based on Gray Correlation Degree and TOPSIS

Grey related analytical method is a mean that measures the relevance degree between factors. TOPSIS is a sort method which approaches ideal solution. This method in the paper integrates the advantage of TOPSIS and grey relation analysis on the curve position and the curve trend. We give
a new metric based on the TOPSIS method and the incidence degree. A simplified formula is given to calculate the relative proximity which measures how close the project approaches the ideal project. In order to realize its practicality and operability, we quantify the index. (T1 is efficiency index; T2 is cost index).

\[
y_{i,j}^{+} = \frac{x_{i,j} - \min_{i} x_{i,j}^{+}}{\max_{i} x_{i,j} - \min_{i} x_{i,j}^{+}}, \quad y_{i,j}^{-} = \frac{x_{i,j} - \min_{i} x_{i,j}^{-}}{\max_{i} x_{i,j} - \min_{i} x_{i,j}^{-}}, \quad i \in N, j \in T_{1}
\]

\[
y_{i,j}^{+} = \frac{x_{i,j}^{+} - \min_{i} x_{i,j}^{+}}{\max_{i} x_{i,j}^{+} - \min_{i} x_{i,j}^{+}}, \quad y_{i,j}^{-} = \frac{x_{i,j}^{+} - \min_{i} x_{i,j}^{-}}{\max_{i} x_{i,j}^{+} - \min_{i} x_{i,j}^{-}}, \quad i \in N, j \in T_{2}
\]

**Definition 3** If the weighted standardized vector is \( x_{i} = (x_{i1}, x_{i2}, \ldots, x_{in})^{T} \) \((i = 1, 2, \ldots, n)\), \( x_{ij} = [x_{ij}^{+}], x_{ij}^{-} \) is nonnegative interval grey number on \([0, 1]\).

\[
x_{ij}^{+} = \max_{i,j} x_{ij}, x_{ij}^{-} = l; \quad x_{ij}^{+} = 0, x_{ij}^{-} = \min_{i,j} x_{ij}; \quad x_{ij}^{+} = \max_{i,j} x_{ij}, x_{ij}^{-} = l; \quad (j \in T_{1})
\]

\[
x^{+} = \left[ x_{11}^{+}, x_{12}^{+}, \ldots, x_{ij}^{+}, \ldots, x_{n1}^{+}, x_{n2}^{+}, \ldots, x_{nj}^{+} \right]^{T}
\]

\[
x^{-} = \left[ x_{11}^{-}, x_{12}^{-}, \ldots, x_{ij}^{-}, \ldots, x_{n1}^{-}, x_{n2}^{-}, \ldots, x_{nj}^{-} \right]^{T}
\]

It's said to be positive ideal solution of schemes; \( x^{+} \) is ideal solution of schemes.

**Definition 4** (Gray correlation degree) The correlation coefficient between each weighted and standardized schemes and the positive or negative ideal solution of schemes are calculated separately. The formula follows (Among them \( \lambda = 0.5 \)).

\[
r_{i,j}^{+} = \frac{1}{2} \left( \frac{\min_{i,j} \left| x_{ij}^{+} - x_{ij}^{-} \right| + \lambda \max_{i,j} \left| x_{ij}^{+} - x_{ij}^{-} \right|}{\max_{i,j} x_{ij}^{+} - \min_{i,j} x_{ij}^{-}} + \frac{\min_{i,j} \left| x_{ij}^{-} - x_{ij}^{+} \right| + \lambda \max_{i,j} \left| x_{ij}^{-} - x_{ij}^{+} \right|}{\max_{i,j} x_{ij}^{-} - \min_{i,j} x_{ij}^{+}} \right)
\]

\[
r_{i,j}^{-} = \frac{1}{2} \left( \frac{\min_{i,j} \left| x_{ij}^{+} - x_{ij}^{-} \right| + \lambda \max_{i,j} \left| x_{ij}^{+} - x_{ij}^{-} \right|}{\max_{i,j} x_{ij}^{+} - \min_{i,j} x_{ij}^{-}} + \frac{\min_{i,j} \left| x_{ij}^{-} - x_{ij}^{+} \right| + \lambda \max_{i,j} \left| x_{ij}^{-} - x_{ij}^{+} \right|}{\max_{i,j} x_{ij}^{-} - \min_{i,j} x_{ij}^{+}} \right)
\]

The resulting correlation coefficient matrix between the decision alternative and positive ideal solution of schemes is R+. While, the resulting correlation coefficient matrix between the decision alternative and negative ideal solution of schemes is R-.

**Definition 5** The correlation coefficients of the mean between the \( m \) indexes and the positive or negative ideal solution sequence of schemes are calculated separately to reveal relationships between the decision alternative and positive or negative ideal solutions sequence. We call it the correlation \( (r^{+}, r^{-}) \).

\[
r_{i}^{+} = \frac{1}{m} \sum_{j=1}^{m} r_{i,j}^{+}, \quad r_{i}^{-} = \frac{1}{m} \sum_{j=1}^{m} r_{i,j}^{-} \quad i = 1, 2, 3, \ldots, n
\]

**Definition 6** The Euclidean distance between each weighted and standardized schemes and positive or negative ideal solution sequence of schemes are calculated separately. The formula follows:

\[
D_{i}^{+} = \sqrt{\sum_{j=1}^{n} \left( x_{i,j} - x_{ij}^{+} \right)^{2}}, \quad D_{i}^{-} = \sqrt{\sum_{j=1}^{n} \left( x_{i,j} - x_{ij}^{-} \right)^{2}} \quad i = 1, 2, 3, \ldots, n
\]

3. **The Decision-Making Method Based on Gray Correlation Degree and TOPSIS**

(1) The normalization is processed to the origin data and the standardized matrix is gotten \( Y \).
(2) Using the method of entropy values determines the combination weight of the evaluation indexes W.
(3) Calculate the weighted and standardized sequence U.
(4) Define the positive and negative ideal solution.
(5) Calculate the Euclidean distance from each schemes to positive or negative ideal solution \( D_i^+ \) and \( D_i^- \).
(6) Calculate gray correlation degree between each schemes and positive or negative ideal solution \( r_i^+ \) and \( r_i^- \).
(7) Make the distance and the correlation degree being dimensionless.
\[
M_{\text{new}} = M_i / \max(M_i) \quad (M_i \text{ which represent } D_i^+, D_i^-, r_i^+, r_i^-)
\]
(8) The larger the number (\( D_i^- \) and \( r_i^+ \)) is, the closer the schemes will get to the positive ideal solution; the larger the number (\( D_i^+ \) and \( r_i^- \)) is, the farther away the schemes are from the positive ideal solution. You can associate non-dimensionalized distance with the group. Then the following formula can be used.
\[
S_i^+ = a_1 D_i^- + a_2 r_i^+, \quad S_i^- = a_3 D_i^+ + a_4 r_i^- \quad i=1,2,3,\ldots,n
\]
\( a_1 \) and \( a_2 \) reflect the decision-maker's preference over position and shape. And it satisfy the condition that \( a_1 + a_2 = 1 \). \( S_i^+ \) reflects the distance between every choice and the positive or negative ideal solution. The larger the number is, the better the value of scheme is. Meanwhile the larger the number of \( S_i^- \) is, the worse the value of scheme is.
(9) Computing the relative closeness degree of scheme.
\[
C_i^+ = S_i^+ / (S_i^+ + S_i^-) \quad i=1,2,3,\ldots,n
\]
The new closeness degree is based on Euclidean distance and gray correlation degree. It integrates the advantage of TOPSIS and grey relation analysis on the curve position and the curve trend. The physical significance is clear.
(10) The alternatives are ranked by the relative closeness degree. The larger the closeness degree is, the better the value of scheme is. The smaller the closeness degree is, the worse the value of scheme is.

4. Case study
The node location selection is a planning process that chooses a location to set up a logistics node in an economy zone with multi-demands. Different layouts can be wildly different depending on the operating costs of logistics system. In present conditions, therefore, it is crucial that how to set a center for logistics and reduce the logistics expenses of logistics system to the minimum. How to provide best service and make social benefit to be the sole criterion? Using cluster analysis method, main factors that influence the city center location are established. The following indexes are Construction costs \( S_1 \), Economic factors \( S_2 \), Natural environment \( S_3 \), Policy factors \( S_4 \). And here is the comprehensive assessment process of the three port city to be voted for.

Table 1. The quantitative index value of each logistics center location.

<table>
<thead>
<tr>
<th>Index values</th>
<th>( S_1 )</th>
<th>( S_2 )</th>
<th>( S_3 )</th>
<th>( S_4 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>[150, 165]</td>
<td>[70, 80]</td>
<td>[80, 90]</td>
<td>[65, 75]</td>
</tr>
<tr>
<td>B</td>
<td>[135, 155]</td>
<td>[80, 90]</td>
<td>[85, 95]</td>
<td>[75, 85]</td>
</tr>
<tr>
<td>C</td>
<td>[160, 170]</td>
<td>[75, 85]</td>
<td>[60, 70]</td>
<td>[70, 80]</td>
</tr>
</tbody>
</table>

(1) The normalization is processed to the origin data and the standardized matrix is gotten. As shown in table 2:
(2) Using the method of entropy values determines the combination weight of the evaluation indexes: $W_1 = 0.37$, $W_2 = 0.32$, $W_3 = 0.20$, $W_4 = 0.11$.

(3) Calculate the weighted and standardized sequence $U$.

$$
U = \begin{bmatrix}
0.052, 0.211 & 0.0, 160 & 0.114, 0.172 & 0.0, 0.055 \\
0.159, 0.370 & 0.160, 0.320 & 0.142, 0.200 & 0.086, 0.110 \\
0.0, 107 & 0.080, 0.240 & 0.058 & 0.028, 0.083
\end{bmatrix}
$$

(4) The positive and negative ideal solution.

$$
x^+ = ([0.0, 0.107], [0.161], [0.142, 1], [0.086, 1])
$$

$$
x^- = ([0.159, 1], [0.0, 0.16], [0.0, 0.058], [0.0, 0.055])
$$

(5) Calculate the Euclidean distance from each schemes to positive or negative ideal solution $D_i$.

$$
D^+ = (0.583, 0.497, 0.584); D^- = (0.167, 0.125, 0.209)
$$

(6) Calculate gray correlation degree between each schemes and positive or negative ideal solution $r_i^+$ and $r_i^-$. The resulting correlation coefficient matrix between the decision alternative and positive ideal solution of schemes is $R^+$. While, the resulting correlation coefficient matrix between the decision alternative and negative ideal solution of schemes is $R^-$. 

$$
R^+ = \begin{bmatrix}
0.713 & 0.346 & 0.552 & 0.408 \\
0.490 & 0.705 & 0.686 & 0.673 \\
1 & 0.442 & 0.347 & 0.460
\end{bmatrix}
$$

$$
R^- = \begin{bmatrix}
0.395 & 1 & 0.604 & 1 \\
0.707 & 0.535 & 0.560 & 0.686 \\
0.334 & 0.674 & 1 & 0.841
\end{bmatrix}
$$

$$
r^+ = (0.505, 0.638, 0.562); r^- = (0.750, 0.622, 0.712)
$$

(7) Make the distance and the correlation degree being dimensionless ($\alpha_1 = 1/2; \alpha_2 = 1/2$).

(8) Computing the relative closeness degree of scheme.

$$
C^* = (0.443, 0.487, 0.491)
$$

(9) The alternatives are ranked by the relative closeness degree.

$C > B > A$. The C city is the best solution for a logistics center.

5. Conclusion

Classical TOPSIS method can’t incarnate the comparability between the different projects. This paper proposes an improved TOPSIS method based on the grey relational coefficient. It integrates the advantage of TOPSIS and grey relation analysis on the curve position and the curve trend. The physical significance is clear. It makes analysis more comprehensive and objective. In the meantime, it has certain value to employ and extend.

References


