Research on Comprehensive Decision Model Based on Analytic Hierarchy Process and Entropy Method

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Abstract. Many evaluation problems in product scheme design belong to multi-person, multi-level and multi-objective comprehensive evaluation problems. At present, the analytic hierarchy process (AHP) is widely used in the evaluation of complex product design scheme. In the establishment of judgment matrix, experts compare the value of each evaluation index and give them scores, with strong subjective factors. And another commonly used method of determining the weight -- the entropy weight method is to determine the objective weight according to the size of the index variance. According to the characteristics of complex product scheme evaluation and the actual needs, this paper will establish the decision-making model with the AHP method and entropy method, realizing the comprehensive evaluation of the scheme of complex products.

The Establishment and Standardization of the Evaluation Index System

The design scheme of complex product has different evaluation indexes at different angles and different design levels. It generally presents a multi objective and multi hierarchy structure, which is shown in Figure 1.

Assuming that decision theory domain $U = \{u_1, u_2, \ldots, u_m\}$ is a collection of product design, $V = \{v_1, v_2, \ldots, v_n\}$ is an evaluation index set, then by the $U$ and $V$ of any pair of elements $(u_i, v_j)$ constitute the $N$ evaluation criteria for the product solution of the Descartes product set. Record $f_{ij}$ as the elements in $(u_i, v_j)$, then $m \times n$ $f_{ij}$ constitute the evaluation index matrix of design scheme $F$, and $F = (f_{ij})_{m \times n}$. 

![Evaluation target tree.](Image)
The Establishment of a Comprehensive Decision-Making Model

AHP to Determine the Subjective Weights of Indicators

AHP is a decision-making method proposed by Satty, T.L in 70s, because the method has the advantages of simple, practical and effective, so it has been widely used, and has established a complete application mode, has a solid theoretical foundation of mathematics. The main steps of general analytic hierarchy process are as follows: (1) establishment of a hierarchical model of multi-objective decision-making problems (Figure 2); (2) the criterion layer from the same layer of a layer of factors belonging to the factors, establish pairwise comparison matrix; (3) to calculate the weight vector, and for consistency analysis.

AHP is the factors contained in the product design and their relationship, the problem can be decomposed into different elements, and these elements belong to different levels, so as to form a hierarchical structure, according to a certain rule in each level, the level of elements for each comparison judgment matrix is established. Through the orthogonal feature vector maximum the characteristics of calculated values and the corresponding judgment matrix, the weights of the criteria for layer elements, calculate the level factor weights for the upper level and on the basis of.

Invite relevant experts in the field of each scale are relatively objective scoring, assuming that the W layer for this level of the single sort process of the weights of the factors, there are the following steps:

Firstly, we calculate the product of each line of the judgment matrix

\[ G_i = f_{i1} \times f_{i2} \times \cdots \times f_{in} \]  

Then calculate:

\[ O_i = \sqrt[n]{G_i} \]  

And then the vector:

\[ \mathbf{O} = (O_1, O_2, \ldots, O_n) \]  

is normalized like this:

\[ W_i = O_i / (O_1 + O_2 + \cdots + O_n) \]

The calculation of the Wi that is the weight of the various factors in the ranking order.

Entropy Weight Method to Determine the Objective Weight of Index

The weight of AHP is determined by the expert's knowledge and decision intention and preference based on, despite the consistency inspection makes the ranking is quite reasonable, but still unable to overcome the subjectivity and randomness of the defect, and the entropy method, the index of the
product design evaluation index system in a sample the data of observation value, the greater the difference between the effect of the index of the system is bigger, is said to contain and transfer the index information of the greater weight will be given a higher. The calculation of entropy weight theory is based on the observation of the sample, which is the object of the study in the perspective of objective data. Can combine the two to make the weight of the results more reasonable. The specific operation steps are as follows:

Assuming that $e_i$ is the first $j$ evaluation index, according to the calculation formula of entropy:

$$e_i = -\frac{1}{\ln n} \sum_{i=1}^{n} f_{ij} \ln f_{ij}, \quad e_j > 0$$  \hfill (5)

$$f_{ij} = \frac{x_{ij}}{\sum_{i=1}^{n} x_{ij}}$$  \hfill (6)

$f_{ij}$ is the characteristic proportion of the $j$th index in the $i$th system; $x_{ij}$ is the characteristic proportion of the first $i$th index in the $j$th system. And $\sum_{i=1}^{n} x_{ij}$ are the sum of all the system data of the $J$ index.

Assuming that $w_j$ is the entropy weight of the evaluation index of $J$, the formula is as follows:

$$W_j = \frac{1-e_j}{n=\sum_{i=1}^{n} e_i}$$  \hfill (7)

So the entropy weight is calculated as the weight of the index.

The Combined Weight

Assuming that $w_a$ is the final combined weight, $w_b$ is the AHP weight, and $w_c$ is the entropy weight. We establish a linear combination of the three weights, The formula for calculating the combined weights is as follows:

$$W_j^a = \gamma W_j^b + (1 - \gamma)W_j^c$$  \hfill (8)

Among them, the gamma is the weight tradeoff factor, the greater the $\gamma$, the AHP determines the weight of the overall weight of the impact of the greater; conversely, the entropy weight method to determine the weight of the impact of the comprehensive weight. Through the change of the $\gamma$ to meet the needs of different evaluation occasions, so that the AHP-entropy method has a very good adaptability.

Example Analysis

Introduction to the Basic Situation

With the continuous development of cities, traffic congestion has been a terrible problem in China. Viaduct, Subway networks optimization measures did not help to improve the traffic capacity of road network congestion problems, different structures will naturally be different, on the traffic capacity of road network structure which is stronger, which plays a vital role in improving the traffic congestion.

Using the Evaluation Model to Evaluate

The Selection of Evaluation Index

The selection of evaluation index is summarized after combing the relevant literature, selection of the following principles, the first is the high frequency index, high frequency index appeared several
times in a typical writer in index to focus on, if these indicators for the current environment is still applicable, should be considered into the evaluation system. Followed by a strong representative, pointing to a clear indicator, should also be included in the evaluation system. Based on the above principles, the paper puts forward the following three indicators of first level and ten level two indexes: traffic condition index B1 (Accident rate C1, Road occupancy rate C2, Road flow rate C3, Capacity of road network C4), Index of road network structure B2 (Density of road network C5, Load balancing degree of road network C6, Accessibility of road network C7), Road traffic quality index B3 (travel delay rate C8, Link flow C9, Average running speed C10).

In the construction of evaluation system, we invite relevant experts to participate in the evaluation system, and provide practical and feasible suggestions for the construction of the evaluation system.

Using the AHP Method and Entropy Weight Method to Calculate the index Weight Respectively

Using Excel and MATLAB tools, take the scores from experts on the indexes, AHP, entropy and entropy weight of each index is calculated as shown in table 1.

Table 1. AHP, entropy and entropy weight of evaluation indexes.

<table>
<thead>
<tr>
<th>index</th>
<th>entropy value</th>
<th>entropy weight</th>
<th>AHP weight</th>
</tr>
</thead>
<tbody>
<tr>
<td>C1</td>
<td>0.92</td>
<td>0.005</td>
<td>0.19</td>
</tr>
<tr>
<td>C2</td>
<td>0.812</td>
<td>0.164</td>
<td>0.07</td>
</tr>
<tr>
<td>C3</td>
<td>0.901</td>
<td>0.025</td>
<td>0.052</td>
</tr>
<tr>
<td>C4</td>
<td>0.968</td>
<td>0.002</td>
<td>0.119</td>
</tr>
<tr>
<td>C5</td>
<td>0.558</td>
<td>0.273</td>
<td>0.331</td>
</tr>
<tr>
<td>C6</td>
<td>0.74</td>
<td>0.186</td>
<td>0.089</td>
</tr>
<tr>
<td>C7</td>
<td>0.899</td>
<td>0.099</td>
<td>0.003</td>
</tr>
<tr>
<td>C8</td>
<td>0.853</td>
<td>0.125</td>
<td>0.052</td>
</tr>
<tr>
<td>C9</td>
<td>0.913</td>
<td>0.007</td>
<td>0.023</td>
</tr>
<tr>
<td>C10</td>
<td>0.881</td>
<td>0.114</td>
<td>0.071</td>
</tr>
</tbody>
</table>

Calculating the Comprehensive Weight

In this situation, the comprehensive weight should not be too subjective or objective, to make the sum of squares of deviations between the subjective weight, objective weight and combination weight is the least, we set up the objective function:

\[
\min Z = \sum_{j=1}^{m} [(W_j^a - W_j^b)^2 + (W_j^a - W_j^c)^2]
\]  

(9)

Combined formula (8) and (9), we finally know that the value of \( \gamma \) is 0.5, so,

\[
W_j^a = 0.5W_j^b + 0.5W_j^c
\]

(10)

Taking data in table 1 into the formula (10), we can obtain the comprehensive weight coefficient matrix of influencing factors of road traffic capacity of road network:

\[
W^a = (0.0975, 0.117, 0.0385, 0.0605, 0.302, 0.1375, 0.051, 0.0885, 0.015, 0.0925)
\]
Conclusion
In order to eliminate the influence of the subjectivity in the analytic hierarchy process and the deficiency of the single method, this paper establishes a comprehensive evaluation model based on analytic hierarchy process and entropy weight method. Finally, by analyzing examples, we use the established evaluation model to calculate the strength of the impact of various index on road traffic capacity. So the validity of the method is verified, which provides a certain reference value for the evaluation of road traffic capacity.

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References