Lag Pendulum Model and Its Application in Wind Resistance
Stability of the Structure

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Abstract. Air resistance in the study of architectural design and other sports is an important factor that cannot be ignored. On the basis of the lag pendulum model formed by motion resistance, from the aspects of geometric structure we analysis the wind resistance stability of cable-stayed bridge and Guangzhou tower. Results showed that the tilt design of sola cable can effectively improve the wind stability of the bridge; the outer barrel tilted inclined column and brace structure in Guangzhou tower can effectively resist strong wind resistance, outer barrel rigid node can withstand wind loads, resistance to bending moments. With the lag pendulum model, the paper validation sola bridge and Guangzhou tower, and the rationality of the design which make sure the safety of structure.

Introduction

As we know, wind drag has a great impact on our developing society. In fact, it may alter the condition of structures and movements, thus changing the physical state such as secure performance, speed or tracks [1, 2]. Wind resistance should be taken into consideration when it comes to the construction of large structures like bridges and towers [3, 4]. Cable-stayed bridge is the most popular and main design for large bridge, which has taken the place of beam bridge and suspension bridge, with higher classification of wind resistance [3]. Canton Tower is located in south China, where it may suffer from the typhoon frequently. What’s more, the tower has the longest height worldwide. All these factors require higher standard of construction materials, along with which we should also lay more importance on the wind resistance of geometrical structure [4-7]. Anyway, lag pendulum is a typical blocked hysteresis moving model, with which we would be able to calculate the stability of the Canton Tower and other cable-stayed bridges. By making comparisons, inclined designs are found out to work well in resistance against typhoon, and the upsetting moment it brings about also has a better performance in fighting against strong wind and earthquakes as well as staying stable.

Moving Model of the Lag Pendulum

A pendulum consists of a piece of tough string and a weight. When the suspension point begins to circle, the weight does the same thing. Anyway, it’s well known that the air drag causes the lagging phase position of the weight, relative to the suspension point, which is the moving model of the lagging pendulum. Apparently, the moving state of the lag pendulum is affected by the turning radius and speed of the suspension point, pendulum length and weight mass. The Figure1 shows the instant figure of a typical hysteresis moving. As is seen in the picture, R1 and R2 are respectively for the turning radius of the suspension point and the weight, similarly T for pull in the string, mg for the gravity of the weight and f for the air drag the weight faced with. Once again, w means the angular velocity and α stands for the lagging angle. The moving pattern is fairly different under
different initial parameters. But just bear it in mind that there is no fundamental difference.

The Application of the Lag Pendulum Theory

Chair Plane

The movement in the Figure 1 is fairly similar to the movement of chair plane, displayed in the Figure 2. Due to the air drag, the phase position of the chair plane is lagging behind the suspension point. Obviously the lagging angle is to be enlarged with the increasing of the pendulum length, the damping coefficient and the rotating speed. Faced with the air drag, the chair is inclined and lagging behind, pulled by the string, thus a component coming up, which gets the air drag balanced. In other words, the air drag could be counteracted in this way.

Cable-stayed Bridge

For an originally stationary hanging pendulum, it would be okay even encountered with strong wind, since it may counteract the wind drag with its inclined string. There are many good examples in large structures, clarifying the well performance of inclined design counteracting the wind resistance. In fact, cable-stayed bridges get bending moment declined while span-length increased, with the extending several inclined cables raising the elasticity load within one beam span. The whole process includes the force transmission from beam to the inclined cables and finally to the bridge tower. Nowadays cable-stayed bridge design is the most widely adapted with its obvious stability (shown in the Figure 3) in wind resistance [3].
Canton Tower

The height of the Canton Tower is 454m. Two columns consist of the tower body which has two larger ends, thus making the nickname “slim waistline” widely spread (shown in the Figure 4). The tower structure is sensitive to the wind load, with 24 columns and 46 circles as well as 46 struts to make up the outer canister. It may be surprising that when looked from above, the figure is an oval one. The whole pattern is made up of two ellipses turning several degrees. One of the ellipses is 10m underground while another is 450m on ground. Anyway the axis of the ellipse has a 45-degree clockwise rotation, with its center moving 7.07m towards northwest. Given its unique tower body, some design might not fit in the rules in general, for example the excellent wind load ability, far beyond the ordinary structures. These factors make the structure of Canton Tower worth lots of researches. The main outer canister acts like steel spatial grid, with 60% of the vertical load and 85% of the overturning moment laid in its foundation, showing its extraordinary ability to resist the lateral load [5-7]. Wind force often brings about the lateral load so that the unlimited wind resistance research becomes the hotspot in the research on Canton Tower. There are yet many winds engineering research on Canton Tower including Engineering Weather Analysis and Calculation, Landform and Wind Tunnel Test and The Overall Model Test, as well as the Project of assessment on Canton Tower’ wind resistance [4-7]. What is actually more significant, a good structure need not only state stability but also dynamic stability. In the following passage, the reason why Canton Tower is dynamic stable will be discussed with the lag pendulum compared with.

Figure 4. Canton Tower.

Canton Tower is located in the south China near the ocean where typhoons usually come from southeast. In order to reduce the windlag, oval structure is implemented in the construction of the outer canister. The axis of the ellipse is east 45 degrees to south (shown in the Figure 5), thus reducing the tower body’s windward area, which may decrease the overturning moment. What’s more, the turning axis may increase the sloping angle of the columns and the struts, which could leads to a greater ability to resist the lateral wind. To find out the characteristic of resistance to wind load, the oval structure needs to be simplified into a round one, with the columns transmitted into a main, stronger and tough one, thus forming the lag pendulum model. In this moving model, the top end is circling in a flat plane. As the circling speed is increasing, air drag the column encountered is getting bigger, too. This factor causes the enlarging phase difference, in other words, the other end is lagging behind the top one. In this process, the figure of the system gradual becomes a slim-waistline one (shown in the Figure 6) from its former style of being a circular frustum. A slim-waistline lagging system is a dynamic stable one, which is just natural to make the equilibrium of mechanical structure, so that it could be a reference to some real constructions.

The outer shape of the canister (shown in the Figure 4) of Canton Tower is fairly similar to a slim-waistline lag pendulum (shown in the Figure 6). When faced with the wind blowing, the inclined columns and struts may play a significant role in preventing the body from upsetting, with the force transmitted in the rigid structures and links. What’s more, since both of the column and the strut are just upsetting in different directions (shown in the Figure 4), both side of the tower body may resist the upsetting moment through the inclined structures. And if seen comparatively, the
equilibrium of mechanical structure of Canton Tower is fairly similar to the natural lag pendulum system. Thus we may conclude that the Canton Tower system is also a slim-waistline lagging system, which also has the natural dynamic stability. So Canton Tower is a good example where the dynamic model of physical movement is successfully used in practical [8, 9].

Figure 5. Bird’s-Eye View for Canton Tower’s Structure.

Figure 6. Slim-Waistline Lag Pendulum.

**Summary**

The lag pendulum is a physical moving model, where air drag can be counteracted and slim-waistline model may come into being under some circumstances. And the lagging pendulum is the key to keep the dynamic equilibrium, now widely applied in large building constructions, e.g. the cable-stayed bridge. The shape of Canton Tower is just similar to the lag pendulum’s dynamic stable model, which could be an ideal model to resist the strong wind. In the other hand, it is the lag pendulum model that enhances the beauty and the outstanding ability of Canton Tower to be non-upsetting.

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References


