Image Registration Algorithm Based on Copula Distribution Estimation

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ABSTRACT: In order to solve the problem that the measure function is easy to fall into the local extremum because of much local extremes in the mutual information registration method, this paper designs an optimization algorithm applied to image registration. One measure function based on mutual information and gradient similarity is constructed combining with evolutional copula estimation of distribution algorithm (ECEDA). New algorithm fully pays attention to the correlation between multidimensional variables joining with the linear weighted based on nonparametric estimation method to overcome the randomness of nonparametric estimation method. Experimental results show that comparing with the traditional copula estimation of distribution algorithm and other existing registration algorithms, the proposed algorithm has higher accurate rate and robustness.

Keywords: image registration; copula estimation of distribution algorithm; mutual information; nonparametric estimation

1 INTRODUCTION

Image registration is a key technology of image processing in the military, medical and other practical application with important significance. The purpose of image registration is to coordinate transformation to map the pixels of an image (floating images) to another image (original image). Using optimization algorithm to find the maximum extremum of registration measure function and the corresponding registration parameters [1], its essence is a multi-parameter optimization problem.

At present, the two main methods of image registration are the method based on region and method based on the characteristics of the images. The method based on region includes mutual information method, the cross-correlation method and the phase correlation method based on FFT. The most widely used method is mutual information method [2–3], in that the mutual information is only related to the gray scale. Due to the lack of spatial information, the registration results of mutual information method can’t meet the practical requirement. J.P.W. Pluim [4] combined mutual information with the spatial information of images to improve registration performance. Since then, many registration methods based on mutual information and spatial information have been proposed. Among them, registration method united mutual information with gradient similarity was put forward by Chen [5] in 2009. In 2012, Liu [6] proposed a kind of normalized mutual information measure function based on weighted gradient. The registration precision is further improved. Only having registration measure function is not enough. In order to solve the problem, some optimization algorithms are needed usefully. The commonly used optimization strategy has genetic algorithm, Powell, particle swarm optimization (PSO), firefly algorithm, etc. Wang [7] applied multi-parameter segmentation mutating method to image registration improving the accuracy of the search. Wei [8] proposed the modified PSO with a certain improvement effect. Zhu [9] combined cultural improved particle swarm optimization with maximum mutual information to improve the calculation speed. Du [10] put forward an improved firefly algorithm for medical image registration, to some extent, improving the precision of registration.
Based on the literature [5], this paper lets the mutual information measure combining with gradient similarity as the registration measure function and proposes the Copula Estimation of Distribution Algorithm to solve the image registration problem. The improved algorithm fully pays attention to the correlation among multidimensional variables joining with the linear weighted based on nonparametric estimation method to overcome the randomness of traditional multidimensional nonparametric estimation method. Experiments show that this method overcomes the local extremum, and improves the precision of registration and registration rate.

2 PRELIMINARIES

Image registration aims to make the images from the same (or different) image detector on the corresponding point on the image space, using computer technology to seek for one or a series of spatial transformation. The modeling process of image registration is actually to establish a similarity measure function to measure the level of similarity of two images under the three-dimensional coordinate transformation depending on optimization algorithm to search for an optimal geometric transformation.

2.1 MI combining with gradient similarity

Mutual information (MI) is an important concept in the information statistics, used to describe the statistical correlation between the two systems. Generally entropy expresses the mutual information. The system A’s entropy is

\[ H(A) = -\sum_x p_A(x) \log (p_A(x)) \]  

(1)

MI considers the information contributed to the overlapping region by each image being registered together with the joint information. MI of the two images can be defined as following:

\[ I(A,B) = H(A) + H(B) - H(A,B) \]  

(2)

Where \( H(A) \) and \( H(B) \) denote the separate entropy values of A and B. \( H(A,B) \) means the joint entropy.

Due to the mutual information was not involved in the calculation of space information, different imaging techniques can lead to the same organization with different gray level to multimode images. Because of the gradient vector of the corresponding points would point to the same or the opposite direction, introduce the direction similarity measure. If the gradient vector modulus value of a set of corresponding points is resemble, the points have the similar gradient vector measured in the modulus value similarity. The combination of direction similarity measure and modulus value similarity as gradient similarity coefficient is multiplied by mutual information as the final registration measure [5] (GSNMI).

Assume point \( x \)'s gradient vector is \( \hat{x} \), and point \( y \)'s gradient vector is \( \hat{y} \). The included angle is

\[ \alpha_{x,y} = \arccos \frac{\hat{x} \cdot \hat{y}}{||\hat{x}|| \cdot ||\hat{y}||} \]  

(3)

The direction similarity of the corresponding points is as followed:

\[ \omega(\alpha_{x,y}) = \frac{\cos(2\alpha_{x,y}) + 1}{2} \]  

(4)

The module similarity of the corresponding points is as followed:

\[ g_{x,y} = \begin{cases} \min(\|x\|, \|y\|); & \max(\|x\|, \|y\|) \neq 0 \\ 1; & \max(\|x\|, \|y\|) = 0 \end{cases} \]  

(5)

The gradient similarity is shown below:

\[ GS(A,B) = \sum_{(x,y) \in (A \times B)} \omega(\alpha_{x,y}) g_{x,y}/2 \]  

(6)

Therefore, the final registration measure (GSNMI) can be expressed as followed:

\[ GSNMI = GS(A,B) \cdot NMI(A,B) \]  

(7)

2.2 Copula estimation of distribution algorithm

Estimation of Distribution Algorithm (EDA) is a novel evolutionary computation, which mainly depends on learning and sampling mechanisms to manipulate the evolutionary search, and has been proved a potential technique for complex problems. 1996, EDA [11] is put forward for the first time, used to improve the Genetic Algorithm. Instead of crossover and mutation operation, on the basis of the probability distribution model, EDA estimates the advantage group and gets the samples for population evolution. Due to the proposed Sklar theorem [12], copula theory was used in EDA [13] (CEDA).

Sklar's theorem in n-dimensions:

Let \( H \) be n-dimensional distribution function with margins \( F_1,F_2,...,F_n \). Then there exists an n-copula \( C \) such that for all \( x \) in \( \mathbb{R}_n \),

\[ H(x_1,...,x_n) = C(F_1(x_1),\cdots,F_n(x_n)) \]  

(8)

If \( F_1,F_2,...,F_n \) are all continuous, then \( C \) is unique;
otherwise, $C$ is uniquely determined on $RanF_1 \times \ldots \times RanF_n$. Conversely, if $C$ is an $n$-copula and $F_1, F_2, \ldots, F_n$ are distribution functions, then the function $H$ defined by (8) is an $n$-dimensional distribution function with margins $F_1, F_2, \ldots, F_n$.

Sklar’s theorem is the foundation of copula theory. It elucidates the role that copulas play in the relationship between multivariate distribution functions and their univariate margins [14].

3 EVOLUTIONAL COPULA ESTIMATION OF DISTRIBUTION ALGORITHM

When image $A$ and image $B$ are completely matched, similar to the maximum mutual information method, GSNMI contains the largest information. Establish a mathematical model based on the GSNMI to measure the relationship between the two images. The model can be defined as:

$$GSNMI_0 = \arg \left( \max_T \left[ GSNMI \left( A, B \left[ T \left( x', y', \theta \right) \right] \right) \right] \right)$$

(9)

Where $T$ is the rigid transformation function ($t_x$, $t_y$, for $x$ and $y$ translation and rotational $\theta$), $B(T)$ is the transformed image.

3.1 Parameter Estimation

According to Sklar’s theorem, the joint distribution of a random vector could be constructed by the one-dimensional distribution of each random variable through a copula. Therefore, the hard work to estimate the joint distribution is simplified to estimate the marginals and to produce a copula. This article assumes that the marginal functions obey normal distribution, and use the Clayton copula as copula function.

Generally speaking, Archimedean copula function can represent the relationship of variables of the optimization problems. It simplifies the work to estimate the distribution model of the promising population. At present, there are lots of parameter estimation methods. For instance, the nonparametric method proposed by Genest and Rivest [15], maximum likelihood method (ML), marginal inference method (IFM) and kernel density estimation method. Among them, the parameter estimation is the simplest way. Considered the symmetry of Archimedean copula function, Du [16] proposed an improved nonparametric method closely expressing the actual situation. But this method ignores the correlation of any two dimensional variables. In order to solve the problem, propose an improved nonparametric estimation method based on weight theory for multi-dimensional variables. We experimentally demonstrated that the proposed method is feasible and effective. The method is given as:

Suppose that $n$-dimensional Archimedean copula function is $C(F_1(x), \ldots, F_n(x))$, and the 2-dimensional joint is $C(F_1(x), F_2(x)), q \neq r$. In $n$-dimensional ($n>2$) Copula Estimation of Distribution Algorithm, the selected advantage groups $(X_1, \ldots, X_n)$’s copula is $C_{\theta}$. Value $r_{n}$ can be expressed as:

$$\tau_{\theta} = \left( \sum_{1 \leq p < q \leq n} S_{pq} \right) / \left( \sum_{1 \leq p < q \leq n} S_{pq} \right)^2$$

(10)

$$S_{pq} = \sum_{1 \leq i \leq J, 1 \leq j \leq s} \text{sign} \left( \left( x_{ip} - x_{jp} \right) \left( x_{iq} - x_{jq} \right) \right)$$

+ \sum_{1 \leq i \leq J, 1 \leq j \leq s} \text{sign} \left( \left( x_{ip} - x_{ip} \right) \left( x_{iq} - x_{iq} \right) \right)

According to value $r_{n}$, estimate value $\theta$. When the function is Clayton copula, $\theta$ is $2\tau_{\theta}/(1 - \tau_{\theta})$.

3.2 Algorithm Steps

Let iterations $I=100$ when population number $P=1000$. Let selectivity $s=0.2$ when the mutation rate $r=0.25$. The specific steps of the improved estimation of distribution algorithms (ECEDA) are as follow:

- a) Input images, and determine the transformation space.
- b) Initialize the population.
- c) Transform the floating images.
- d) Interpolant the transformed floating images using the bilinear interpolation.
- e) Calculate the GSNMI between the original image and the corresponding floating images.
- f) If iterations $i < I$, continue to perform g). Otherwise, output the optimization results.
- g) Select the best $s \times P$ individuals from the current population as the promising population.
- h) Suppose the dominant population obey normal distribution.
- i) Estimate the Copula parameter $\theta$.
- j) Sample from the copula $C_{\theta}$. Generate $l$ vectors who obey the joint $C_{\theta}$.
- k) Compose the next generation by the following 3 parts. 1) Reserve the best individuals of the current generation to the next generation. 2) Get the new individuals. 3) Generate some random individuals in the search space depending on certain mutation rate. Let the next generation return to c).

4 EXPERIMENTAL ANALYSIS

4.1 The Data Source

In this paper, the image data derive from the computer vision group [17] of Universidad de Granada and The Whole Brain Atlas [18]. Select 118 2D gray level images (256*256) from computer vision group and dif-
Different brain fault MR-T1, MR-T2 2D images from The Whole Brain Atlas to do the experiment.

Figure 1. MR-T1 (a). Figure 2. MR-T2 (a).

Figure 3. MR-T1 (b). Figure 4. MR-T2 (b).

Figure 5. MR-T1 (c). Figure 6. MR-T2 (c).

4.2 Registration algorithms

This paper implements several registration algorithms, for instance, the method combined GSNMI with Genetic Algorithm(GA), the method combined GSNMI with Self-adaptive Genetic Algorithm(SGA), the proposed method [9] based on MI and Cultural Improved Particle Swarm Optimization(CIPSO), the method [10] based on MI and Firefly Algorithm (FA), the method combined GSNMI with Copula Estimation of Distribution Algorithm(CEDA) and the method combined GSNMI with Evolutional Estimation of Distribution Algorithm(ECEDA) for this experiment. Compare the optimization parameters with the standard parameters, and calculate the Mean Absolute Error (MAE). According to the MAE, measure the optimization results.

Rigid transformation involves three variables. In GA, SGA, CEDA and ECEDA, the hunting zone of $\theta$, $t_x$, $t_y$ respectively is $[-180,180]$, $[-128,128]$ and $[-128,128]$. In FA and CIPSO, the hunting zone of $\theta$, $t_x$, $t_y$ respectively is $[-30,30]$, $[-30,30]$ and $[-30,30]$.

In the process of image transformation bilinear interpolation is used as interpolation algorithm.

4.3 Data analysis

Table 1. Registration rate of 79 images (256 x 256) of miscellaneous set.

<table>
<thead>
<tr>
<th>Algorithms</th>
<th>GA</th>
<th>SGA</th>
<th>CIPSO([9])</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number</td>
<td>62</td>
<td>69</td>
<td>70</td>
</tr>
<tr>
<td>Rate</td>
<td>78.48%</td>
<td>87.34%</td>
<td>88.61%</td>
</tr>
<tr>
<td>Algorithms</td>
<td>FA([10])</td>
<td>CEDA([13])</td>
<td>ECEDA</td>
</tr>
<tr>
<td>Number</td>
<td>71</td>
<td>65</td>
<td>74</td>
</tr>
<tr>
<td>Rate</td>
<td>89.87%</td>
<td>82.28%</td>
<td>93.67%</td>
</tr>
</tbody>
</table>

Table 1 shows the registration rate of existing registration methods and our proposed registration algorithm. Compared with the existing methods, the rate of our method is the highest, 93.67%, showing our method fully considering the correlation among variables is better than the existing methods and the traditional CEDA method.

Table 2. Registration rate of 39 images (256 x 256) of biomedical set.

<table>
<thead>
<tr>
<th>Algorithms</th>
<th>GA</th>
<th>SGA</th>
<th>CIPSO([9])</th>
</tr>
</thead>
<tbody>
<tr>
<td>Miscellaneous</td>
<td>8/12</td>
<td>9/12</td>
<td>10/12</td>
</tr>
<tr>
<td>Nematodes</td>
<td>0/11</td>
<td>3/11</td>
<td>7/11</td>
</tr>
<tr>
<td>MR</td>
<td>14/16</td>
<td>16/16</td>
<td>15/16</td>
</tr>
<tr>
<td>Rate</td>
<td>56.41%</td>
<td>71.79%</td>
<td>82.05%</td>
</tr>
<tr>
<td>Algorithms</td>
<td>FA([10])</td>
<td>CEDA([13])</td>
<td>ECEDA</td>
</tr>
<tr>
<td>Miscellaneous</td>
<td>11/12</td>
<td>9/12</td>
<td>11/12</td>
</tr>
<tr>
<td>Nematodes</td>
<td>8/11</td>
<td>2/11</td>
<td>4/11</td>
</tr>
<tr>
<td>MR</td>
<td>15/16</td>
<td>16/16</td>
<td>16/16</td>
</tr>
<tr>
<td>Rate</td>
<td>87.18%</td>
<td>69.23%</td>
<td>79.49%</td>
</tr>
</tbody>
</table>

Table 2 shows the registration rate of our proposed registration algorithm is not higher than FA and CIPSO’s. The reason for the decline is that the search area of ECEDA is greater than FA and CIPSO. The rotation angle impacts on the registration measure. In the process of optimization, the algorithm falls into the local maximum values leading to this method can’t identify the correct rotation angle. For instance, the relationship between the measure GSNMI of image nemacb4 and rotation angle (ANG) can be shown in the figure below:

Figure 7. nemacb4.pgm. Figure 8. ANG-GSNMI.
In experiment, MR-T1 (a) and MR-T2 (a) were obtained from the Whole Brain Atlas. The data are shown in Table 3:

<table>
<thead>
<tr>
<th>Algorithms</th>
<th>GA</th>
<th>SGA</th>
<th>CIPSO [9]</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\theta / (\circ)$</td>
<td>5</td>
<td>2.8099</td>
<td>4.0486</td>
</tr>
<tr>
<td>$X / (\text{pixel})$</td>
<td>10</td>
<td>10.5595</td>
<td>10.2806</td>
</tr>
<tr>
<td>$Y / (\text{pixel})$</td>
<td>10</td>
<td>7.6025</td>
<td>9.0685</td>
</tr>
<tr>
<td>GSNMI / ($E+03$)</td>
<td>6.1435</td>
<td>6.1696</td>
<td>6.1034</td>
</tr>
<tr>
<td>MAE</td>
<td>1.7155</td>
<td>0.7212</td>
<td>0.4758</td>
</tr>
</tbody>
</table>

In experiment, MR-T1 (b) and MR-T2 (b) were obtained from the Whole Brain Atlas. The data are shown in Table 4:

<table>
<thead>
<tr>
<th>Algorithms</th>
<th>GA</th>
<th>SGA</th>
<th>CIPSO [9]</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\theta / (\circ)$</td>
<td>5</td>
<td>5.1278</td>
<td>4.8929</td>
</tr>
<tr>
<td>$X / (\text{pixel})$</td>
<td>10</td>
<td>10.9107</td>
<td>10.8207</td>
</tr>
<tr>
<td>$Y / (\text{pixel})$</td>
<td>10</td>
<td>10.4532</td>
<td>10.6004</td>
</tr>
<tr>
<td>GSNMI / ($E+03$)</td>
<td>6.1192</td>
<td>6.2282</td>
<td>6.1934</td>
</tr>
<tr>
<td>MAE</td>
<td>0.4972</td>
<td>0.5071</td>
<td>0.3763</td>
</tr>
</tbody>
</table>

In experiment, MR-T1 (c) and MR-T2 (c) were obtained from the Whole Brain Atlas. The data are shown in Table 5:

<table>
<thead>
<tr>
<th>Algorithms</th>
<th>GA</th>
<th>SGA</th>
<th>CIPSO [9]</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\theta / (\circ)$</td>
<td>5</td>
<td>-0.2314</td>
<td>4.7893</td>
</tr>
<tr>
<td>$X / (\text{pixel})$</td>
<td>10</td>
<td>11.0316</td>
<td>11.9715</td>
</tr>
<tr>
<td>$Y / (\text{pixel})$</td>
<td>10</td>
<td>7.9225</td>
<td>10.5691</td>
</tr>
<tr>
<td>GSNMI / ($E+03$)</td>
<td>8.2356</td>
<td>8.4771</td>
<td>8.5436</td>
</tr>
<tr>
<td>MAE</td>
<td>2.7802</td>
<td>0.9171</td>
<td>0.6570</td>
</tr>
</tbody>
</table>

In multi-modal image registration, when GSNMI achieves maximum value, calculate the $\text{MAE}$ of the corresponding registration parameters. The dates in Table 3, Table 4 and Table 5 show that the registration results $\text{MAE}$ of ECEDA is the minimum than GA, SGA, CIPSO, FA and CEDA suggesting new algorithm veritably reflects the correlation among the multi-dimensional variables together with excellent global search ability.

5 CONCLUSION

An improved multidimensional non-parametric estimation method which can better reflect the multidimensional variable correlation has been proposed. The image registration method based on traditional mutual information and gradient similarity is discussed and the influence of image rotation to the registration performance is analyzed. Applying Copula Estimation of Distribution Algorithm to the image registration problem, the experimental results show that compared with the existing methods, the proposed method has higher precision and robustness.

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