The Scheduling Problem in Case of Movable Service Performing Sites

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Abstract. This paper identifies a new type of scheduling problems, in which both customer and service provider are movable. Therefore this type of problems has to decide both the sequence of service performing and the sites where service is performed. Two possible mathematical models are discussed and illustrated.

Introduction

In common customer-supplier model, the place for performing service is fixed as part of demand description. For example, in jobshop scheduling, the parts are processed at corresponding machines without option; in taxi scheduling, the taxi goes unquestionably to the place where a customer initiates taxi calling. The scheduling problem aims to satisfy customers’ demands and determine the sequence or timetable of performing services.

With today’s rapid development of technology and convenient communication facility, a new type of problems emerged, in which the sites for service performing are movable. For example, in mobile battery charging/swapping system for electric vehicles [1,2], electric vehicles and mobile battery charging/swapping facility vehicles are coordinated to meet at a site. Air refueling of planes in mission [3-5] requires selecting an intermediary refueling location for a pair of plane and tanker aircraft too. Medical teams in battlefield have similar requirement. In these problems, there are multiple choices of service performing sites. The solution to these problems has to give consideration to both movable customers and service suppliers. Usually a timely response to demand is required and customers require traversing less distance.

In general, the new type of scheduling problem has to determine both the sites of performing service and the sequence of performing services. The meeting site for a pair of customer and supplier can be different from the initial site at the time the demand is lodged. This paper discussed the mathematical model of the scheduling problem in case of movable service performing sites.

Problem Description

Suppose that there are I mobile resource suppliers for J moving customers. The initial locations (coordinates) of both mobile suppliers and customers are given. Each customer travels...
to his destination and needs a service during his trip. The problem is to decide at which location the service is performed and the sequence of the customers who accept service from a corresponding supplier under several constraints:

1) The customer and the mobile suppliers meet at the location within an acceptable time lag.
2) The customer will reach his trip destination under any circumstances.
3) Only one mobile supplier provides service for one customer.
4) Each demand is satisfied at one site.

To simplify the problem, we make following assumptions:

1) Number of movable suppliers \( I > 1 \)
2) Number of customers \( J > 1 \)
3) The time to travel from one location to another location can be estimated.
4) All suppliers have the same service providing capability and provide the same service.

The overall objective function is to optimize the whole scheduling process and minimize the overall travelling distance for the service suppliers and customers (One may alternatively consider cost, time, etc.).

![Diagram of Movable Service Performing](image)

(a) One way of Movable Service Performing

(b) Another Way of Movable Service Performing

Figure 1. Schematic of Movable Service Performing.

As a matter of fact, each customer has two needs to be satisfied: accepting service and arriving destination. Fig. 1 shows two ways of implementation. In Fig. 1(a), customers queue up at a site, where the service supplier is allocated. In Fig. 1(b), the service suppliers travel to some locations on the predetermined routes of customers to provide service. In both cases, there are multiple options for meeting sites. Nevertheless, the latter satisfies one-to-one customer needs,
at the same time customers need not change route. The following section discusses the mathematical models for the two implementations.

**Mathematical Model I**

In this model, customers queue up at the service performing place. The problem is solved with two stages. Firstly, decide the meeting site coordinates and each customer accepts which supplier’s service; secondly, solve the sequence problem. Assume at time \( t \), there are \( J \) service suppliers \((b_j, j=1, 2, \ldots, J)\) providing service for \( I \) customers \((a_i, i=1, 2, \ldots, I)\) \((I \geq J)\). The location of customer \( a_i \) at the time is known as \((x_i^a, y_i^a)\), and the location of supplier \( b_j \) at the same time is known as \((x_j^b, y_j^b)\). There are \( N \) possible sites for service performing, with location \((x_n^c, y_n^c)\) \((n=1, 2, \ldots, N)\). Decision variable \( z_{ijn} \) can be 0 or 1, 1 means that customer \( a_i \) accepts service from supplier \( b_j \) at site \( n \).

Objective function:

\[
\text{min} \sum_{n=1}^{N} \left[ \sum_{i=1}^{I} (d_{in} \times z_{ijn}) + \sum_{j=1}^{J} (d_{jn}' \times z_{ijn}) \right]
\]

s.t.

\[
\sum_{n=1}^{N} \sum_{j=1}^{J} z_{ijn} = 1 \quad i = 1, 2, \ldots, I \quad (2)
\]

\[
\sum_{n=1}^{N} \sum_{i=1}^{I} z_{ijn} \geq 1 \quad j = 1, 2, \ldots, J \quad (3)
\]

\[
\sum_{j=1}^{J} \sum_{i=1}^{I} z_{ijn} \leq J \quad (4)
\]

\[
z_{ijn} \in (0, 1) \quad \text{for all } i, j \quad (5)
\]

In Eq. (1), \( d_{in} \) denotes the distance between customer \( i \) and site \( n \), \( d_{jn}' \) denotes the distance between supplier \( j \) and site \( n \). Eq. (2) ensures that every demand must be satisfied, and only one supplier provide service at one site. Eq. (3) ensures that one supplier must provide service for at least one customer. Eq. (4) ensures that the number of service performing sites is not greater than the number of suppliers. Eq. (5) ensures that \( z_{ij} \) is either 0 or 1. Based on the solution to this model, the sequence problem is reduced to one supplier problem and the sequence can be decided according to First Come First Served rule.

In the following simple example, assume the routings only consist of straight lines and 90° turning. \(\sqrt{(x_j^b - x_i^a)^2 + (y_j^b - y_i^a)^2} \) denotes the distance between customer \( i \) and supplier \( j \). As shown in Fig. 2, locations of 5 customers are: \((1, 3.5), (4, 2), (2.3, 3.5), (1.4, 4), (2.8, 1.6)\). 2 service performing sites are chosen as the center of corresponding customers: \((1.57, 3.67)\) and \((3.4, 1.8)\).
In the following model, we assume that mobile service suppliers travel to some locations on the predetermined routes of customer. Firstly, the scope of the customer migration is partitioned into K regions, the time duration partitioned into T time slots. Statistics of demands are collected according to the time slots and regions. In order to simplify the mathematical model, the service performing time duration is assumed to be less than a time slot. This restriction can be relaxed by using two consecutive time slots to represent a demand. This model considers the time-variant information: demands and locations of mobile service suppliers.

Input variables:
- \( C_k \) is the maximum number of service suppliers that region \( k \) can accommodate, \( k=1\ldots K \)
- \( Q_{kt} \) is the amount of demand in region \( k \) at time slot \( t \), \( k=1\ldots K, \ t=1\ldots T \)
- \( D_{lk} \) is the travel distance from region \( l \) to region \( k \) in one time slot, if \( l=k \), \( D_{lk}=0 \), \( l=1\ldots K, k=1\ldots K \)

Output variables:
- \( z_{jlk} \) represents a traveling plan, i.e. service supplier \( j \) starts from region \( l \) and arrives destination region \( k \) in time slot \( t \), \( j=1\ldots J, k=1\ldots K, \ l \) is given. \( z_{jlk} \) holds the value of 0 or 1, to ensure one service supplier can execute only one plan in one time slot.

Objective functions:
\[
\text{max} \sum_{j,l,k,t} z_{jlk} \quad (6) \\
\text{min} \sum_{j,l,k} (z_{jlk} D_{lk}) \quad (7)
\]
\[
\text{s.t.} \\
\sum_{j,l} z_{jlk} \leq C_k \quad k=1\ldots K, \ t=1\ldots T \quad (8)
\]
\[ \sum_{j} \sum_{l} z_{jlk} \geq Q_{kt} \quad k=1 \ldots K \]  
\[ \sum_{j} \sum_{k} z_{jk} = \sum_{j} \sum_{k} z_{jk(t+1)} \quad t=1 \ldots T \]

\[ z_{jlkt} \in (0, 1) \text{ for any } j, l, k, t \]  

Eq. (6) indicates the maximization of total demands satisfied. Eq. (7) minimizes the total travel distance of mobile recharging facility vehicles. Eq. (8) limits the number of service suppliers in region \( k \) less than it can accommodate. Eq. (9) satisfies the total demands in region \( k \) in time slot \( t \). Eq. (10) constrains the region that the service supplier arrives in time slot \( t \) is the same as the region it starts in time slot \( t+1 \). Eq. (11) ensures one movable service supplier can only execute one plan in the same time slot. This model is a complicated combinatorial model. Sometimes constraints may have to be violated to obtain a solution, for example, Eq. (9).

A simple example shows the feasibility of the method. The activity scope of customers is partitioned into 4 regions. There are 2 service suppliers in this example. The maximum number of service suppliers that region \( k \) can accommodate \( C_{k} = 2, k=1, 2, 3, 4 \). The travel distance from region \( l \) to region \( k \), \( D_{lk} = \begin{bmatrix} 0 & 20 & 20 & 40 \\ 20 & 0 & 40 & 20 \\ 20 & 40 & 0 & 20 \\ 40 & 20 & 20 & 0 \end{bmatrix} \), \( l=1, 2, 3, 4; k=1, 2, 3, 4 \). One time slot lasts 20 minutes.

Table 1 lists the demands in this example. Table 2 shows the resultant dispatch plan which satisfies the demands in Table 1.

### Table 1. The Demands in This Example.

<table>
<thead>
<tr>
<th>Time Slot t</th>
<th>Region k</th>
<th>Demand ( Q_{kt} )</th>
</tr>
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<tbody>
<tr>
<td>1</td>
<td>2</td>
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<td>2</td>
<td>1</td>
<td>1</td>
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<td>3</td>
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<td>3</td>
<td>1</td>
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<tr>
<td>4</td>
<td>4</td>
<td>1</td>
</tr>
</tbody>
</table>

### Table 2. An Example of Dispatch Plan.

<table>
<thead>
<tr>
<th>Service Supplier</th>
<th>Start Region</th>
<th>Destination Region</th>
<th>Time Slot</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>2</td>
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<td>2</td>
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Conclusions

With rapid development of technology, a new type of scheduling problems is identified. This paper tries to analyze this type of problems and build mathematical programming models for the scheduling problem in case of movable service performing sites. The next step of our research will be solving the problem more efficiently and compare solutions.

References


