Reliability-based Design of Pile Diameter in Loess Area
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ABSTRACT: It is pretty difficult to design the piles diameter because of the existing of collapsible loess based on reliability theory. This paper will give the formula of reliability index for vertical capacity consider collapsible. A target reliability index of 3.2 is presented as the criterion for vertical the bearing capacity of pile according to worldwide specifications and references. Based on target reliability of piles, the effective pile diameter is calculated, and the pile spacing is deduced according to effective pile diameter. Finally, through case study, the designed pile diameter using the presented method is more scientific and reasonable because it considers the influences of various uncertainties on collapsible loess, which has great engineering significance.

Keywords: reliability design; pile diameter; target reliability index; compaction effect; effective pile diameter

INTRODUCTION

In loess area, due to collapsible loess, the foundation cannot meet the requirement of superstructure, therefore the foundation need to been strengthened[1]. Pile foundation is widely using in loess area because of its high capacity, small settlement and easy construction. However, the parameters may not safe using traditional factor of safety design method due to the existing of various uncertainties. How to design pile diameter can assure the safety, reliability design considering the influences of uncertainties in fact engineering, which solves the above problems[2].

Dynamic compaction (DC) method is an effective ground treatment technique that is widely used for a variety of soil types and conditions, particularly in loess areas. However, DC is rarely applied to ground where thick collapsible loess is present[3]. Mosallamy[4] investigates the collapse potential of loess soil in Egypt, the factors affecting the dynamic properties of this soil and establish relationships between them. Whether engineered or natural, may exist in seismic zones; or may be subjected to small strain vibrations, therefore, there is a need to assess the wetting dynamic properties of collapsible soils. Jiang[5] presents a distinct element method (DEM) to investigate its macro- and micro-mechanical behaviour (compression and collapse behaviour) under one-dimensional (1D) compression condition. Yuan[6] analyzes the similarity and difference between collapsibility and seismic settlement of loess, which shows that both are based on the metastable microstructure of loess, but their triggering mechanisms are different. Pei examines physicochemical and index properties of loess modified by adding lime and fly ash piles[7].

This paper will employ reliability theory and compaction effect to analyze vertical capacity piles in loess area. Computation formula of pile spacing is deduced according to pile group effect.
EFFECTIVE PILOT DIAMETER FORMULA

Formula of reliability index

According to reliability theory, the performance function of pile foundation is

\[ g = R - S \]  

Where \( R \) and \( S \) is the ultimately and load vertical capacity of piles, respectively.

Resuming the load over the pile tip is \( p \), and the design pile diameter is \( d \), the following formula can be obtained

\[ S = p / \pi (d / 2)^2 = 4p / \pi d^2 \]  

In engineering fact, various factors have significant influence on \( p \) and \( d \), \( p \) and \( d \) should be random variables. In order to easily study, using Taylor theory, \( S \) can be written as

\[ S = \mu_s + \frac{\partial S}{\partial p} (p - \mu_p) + \frac{\partial S}{\partial d} (d - \mu_d) \]  

Where, \( \mu_p \) and \( \mu_d \) are the means of load and pile diameter, respectively.

Normal distribution is widely used to fit the probability distribution, for the random variables \( p \) and \( d \), this paper will employ normal distribution to fit the probability distribution.

The mean and variance of \( p \) and \( d \) are

\[ \mu_s = 4\mu_p / \pi\mu_d^2 \]  

\[ \text{var}(S) = 4\text{var}(p) / \pi\mu_d^2 - 8\mu_p \text{var}(d) / \pi\mu_d^2 \]  

In formula (1), the ultimate vertical capacity of piles can be written as

\[ R = B_i \sum q_{si} l_i + q_{si} A_i \]  

Where, \( B_i \) is the length of piles; \( q_{si} \) is standard value of ultimate shaft resistance which can be computed using static cone penetration resistance; \( l_i \) is the thick of \( i \)-layer soil; \( q_{si} \) is the standard value of ultimate tip resistance; \( A_i \) is area of tip. \( B_i \), \( l_i \) and \( A_i \) can be considered as constant when the coefficients of variation of them are pretty small. \( q_{si} \) and \( q_{si} \) are mutual independence random variables. Therefore, the ultimate vertical capacity of pile obeys normal distribution. The mean and variance of \( R \) are

\[ \mu_r = B_i \sum \mu_{si} l_i + \mu_{si} A_i \]  

\[ \sigma_r = B_i \sum \sigma_{si} l_i + \sigma_{si} A_i \]  

According to first order second moment, the reliability index can be computed using following formula

\[ \beta = \frac{\mu_r - \mu_s}{\sqrt{\sigma_r^2 + \sigma_s^2}} \]  

Based on formula(4)～(7), the reliability index of vertical capacity for piles can be computed using following formula

\[ \beta = \frac{4\mu_r / \pi\mu_d^2 - (B_i \sum \mu_{si} l_i + \mu_{si} A_i)}{\sqrt{4\text{var}(p) / \pi\mu_d^2 - 8\mu_p \text{var}(d) / \pi\mu_d^2 + B_i \sum \sigma_{si} l_i + \sigma_{si} A_i}} \]
Considering the variability of pile diameter, the variance of pile diameter can be obtained as

$$\text{var}(d) = \left(\frac{1}{6}k\mu_d\right)^2$$

(9)

Where, $k$ is a random variable, which describes concave and convex fluctuation around the mean of pile, $k$ can be obtained in fact engineering.

Therefore, $\mu_d$ can be solved as

$$\mu_d = 2\left[3(B, \sum \mu_{q_a} + \mu_{q_a} A_i) \mu_p + \beta \left\{9(B, \sum \sigma_{q_a} + \sigma_{q_a} A_i) \mu_p^2 + \left[\left(\sum \mu_{q_a} + \mu_{q_a} A_i\right)^2 + 9 \text{var}(p)\right]\right\}^{\frac{1}{2}} \right]$$

(10)

$$\{3(B, \sum \mu_{q_a} + \mu_{q_a} A_i)^2 - \left(B, \sum \sigma_{q_a} + \sigma_{q_a} A_i\right) \beta^2\}$$

In Eq.(10), if the target reliability index is given, pile diameter can be competed.

The mean of pile diameter is actually effective pile diameter.

**Obtaining the target reliability index of piles in loess area**

The target reliability of the pile foundation should match that of the superstructure and of other types of foundations. Phoon[9], Zhang[10], Paikowsky[12] and others have studied the target reliability of foundations based on calibration of the ASD practice. A target reliability index, $\beta_{TS}$, between 3.0 and 3.5 appears to be suitable for pile foundations.

The reliability of a pile group can be significantly higher than that of single piles. Since proof tests are mostly conducted on single piles, it is necessary to find a target reliability for single piles that matches the target reliability of the foundation system. Based on calibration of the ASD practice, Withiamet[12] recommended target reliability index values of $\beta_{TS} = 2.0-2.5$ for single driven piles and $\beta_{TS} = 2.5-3.0$ for single drilled shafts. Zhang[10] attributed the increased reliability of pile foundation systems to group effects and system effects. They calculated $\beta_{TS}$ values for single driven piles to achieve a $\beta_{TS}$ value of 3.0. For pile groups larger than four piles, the calibrated $\beta_{TS}$ values are mostly in the range of 2.0-2.8 if no system effects are considered. The $\beta_{TS}$ values decrease to 1.7-2.5 when a system factor of 1.25 is considered, and further to 1.5-2.0 when a larger system factor of 1.5 is considered. For a structure supported by four or fewer far-apart piles, the pile group effect and system effect may not be dependable. The target reliability of single piles should therefore be the same as that of the foundation system, say $\beta_{TS} = 3.0$.

Compare with single capacity in comment soil, the target reliability index of piles in loess area should be more large because the capacity of natural foundation is relatively small caused by collapsible. Appropriately increasing the target reliability index can offset the deficiency of natural foundation. This paper will select 3.2 as the target reliability index of vertical capacity for piles in loess area, which is base whether destroy of piles in loess area.

**2 Determining pile spacing**

Compaction effect of pile in loess area is pretty importance because soil around piles can be laterally compressed when soil is inlayed with piles, which can be strengthened the foundation. Due to compression effect, natural soil around soil is
compressed, disturbed and remolded, causing plastic region and elastic region around soil shown in Fig.1.

![Figure 1. The expansion of the hole for pile.](image1)

In Fig.1, Plastic region is the influence of piles on soil. In plastic region, the soil can be more compressed when pile is near to center of piles. Resuming the dry density of soil is $\gamma_{d_{\text{max}}}$ and $\gamma_{d_{0}}$ when the compressions are greatest and smallest, respectively. $\gamma_{d_{0}}$ is factually near natural soil. Therefore, compression degree should be a random variable which is described as $D(r)$, $r$ is the influence diameter of pile on soil. To conveniently study, compression degree is considered as monotone decreasing exponential function which can be written as

$$f(r) = D_0 \exp[(-\frac{r}{D_0})]$$

(11)

Where $\mu_p$ is effective pile diameter, $D_0$ is the original value which can be computed using $D_0 = (\gamma_{d_{\text{max}}} / \gamma_{d_{0}}) - 1$.

If pile group foundation is 2-dimension, and piles are described as $(p_1, p_2, \ldots, p_n)$, the influence of each pile on soil can be considering as same random variables. Cumulative impact process of single pile on soil obeys Poission distribution. For pile group foundation, there is an extrusion overlap region shown in Fig.2 between each piles when pile spacing $L$ is larger than $2r$.

![Figure 2. Overlapped compression area between two piles.](image2)
In Fig.2, cumulative impact process of $p_i$ and $p_{i+1}$ on soil between piles obeys Poisson distribution. Therefore, consuming extrusion can superimpose, dry density of soil between piles can be written as

$$\gamma_d = \gamma_d(l/2 + 1)$$

$$f(l/2) = \begin{cases} D_0 \exp\left(-\frac{l}{2} - \frac{\mu_d}{2}\right) & l > \mu_d \\ 0 & l < \mu_d \end{cases}$$

(12)

According to Eq.(12), pile spacing meets following formula

$$l = -\frac{2}{\alpha} \ln f(l/2)\mu_d + \mu_d$$

(13)

Based on Eq.10~13, pile spacing can be competed using following formula

$$l = -\frac{2}{\alpha} \ln f(l/2)\left(2(B, \sum \mu_{\mu/\mu}, + \mu_{\mu/\mu}, A, \mu_p)\right)$$

$$+ \beta \sqrt{\frac{9(B, \sum \mu_{\mu/\mu}, + \mu_{\mu/\mu}, A, \mu_p^2 + (B, \sum \mu_{\mu/\mu}, + \mu_{\mu/\mu}, A, \beta^2 \sigma_{\mu/\mu}) \mu_I}{l^2}}$$

$$\left[3\pi(B, \sum \mu_{\mu/\mu}, + \mu_{\mu/\mu}, A, \beta^2 \sigma_{\mu/\mu}) \mu_I \right]$$

$$+ 2(B, \sum \mu_{\mu/\mu}, + \mu_{\mu/\mu}, A, \mu_p)$$

$$+ \beta \sqrt{\frac{9(B, \sum \mu_{\mu/\mu}, + \mu_{\mu/\mu}, A, \mu_p^2 + (B, \sum \mu_{\mu/\mu}, + \mu_{\mu/\mu}, A, \beta^2 \sigma_{\mu/\mu}) \mu_I}{l^2}}$$

$$\left[3\pi(B, \sum \mu_{\mu/\mu}, + \mu_{\mu/\mu}, A, \beta^2 \sigma_{\mu/\mu}) \mu_I \right]$$

(14)

Eq.(14) employ reliability theory to compute pile space, which overcome influence of various uncertainties in Geotechnical engineering. Thus Eq.(14) is more scientific and reasonable than the computation formula based on factor of safety method.

**CASE STUDY**

The foundation of a shopping mail in Xi’an city lacks of capacity because its soil has great collapsibility, and the type of collapsibility is self-collapsibility. Accordingly, the foundation need to be strengthened\[13\]. This paper will use driven piles to improve the foundation. Luo\[14\] collected 128 measured data of driven piles, which mean and variance are $\mu_C = 3180.81 \text{kPa}$, and $\sigma_C = 1754.47 \text{kPa}$, respectively. In order to conveniently study, resuming the mean and variance of load on tip pile are $\bar{p} = 300 \text{kPa}$ and $\sqrt{\sigma_p} = 6 \text{kPa}$, respectively. According to Eq.(10), the design effective diameter of drive pile is

$$\mu_d = \frac{2 \sqrt{3756476 + \sqrt{3.2(454251.23+35612459.55k^2)}}}{2451214.8}$$

(15)

In Eq.(15), $k$ is obtained based on measured data. However, the report about reliability analysis of piles in loess area is scarcely any. To verify the presented method,
$k = 0.047$ is employed to compute effective pile diameter according to Jiang’s research. Therefore, the effective diameter of driven piles is $\mu_d = 0.6432$ m according to Eq.(15). Based on Eq.(14), the spacing of driven piles is 6.24 m.

**CONCLUSIONS**

This paper discussed the influences of piles on soil, and given the computation formula of reliability index for piles considering collapsible of loess. Through case study, the conclusions can be obtained.

1. Reliability design of piles in loess area overcomes various certainties in pile foundation engineering, therefore the design parameters are more scientific and reasonable.
2. The presented method can improve pile foundation design in loess area because reliability theory is introduced in pile foundation.
3. Due to rare data about reliability analysis of piles in loess area, reliability theory in pile foundation needs to been further research.

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**REFERENCES**


