Fractal Prediction Model During the Wear Process Based on Archard Formula

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Abstract. It is very important for the tribological design of the mechanical system to develop a mathematical model to predict the rate of wear depth in a wear process. Archard formula is the basement of wear calculation, it is very important to estimate the adhesive wear life. This paper uses the fractal parameters with scale-independence to characterize surface topography in the wear process by introducing the fractal theory, and takes into account the different shape of asperities during the running-in process and the stable wear process, the revised Archard wear calculation formula is developed. The numerical simulation results show that the rate of wear depth and the wear depth changing with time and the fractal parameters are consistent with that described by classical theories, it will provide a theoretical support for accurate prediction of the friction pairs' wear life.

Introduction

The development of tribology theory can be traced back to 1508, Da Vinci Leonardo firstly proposed the basic concept of friction. Amontons [1] and Couloms [2] put forward the law of friction, that is, the friction force is proportional to contact pressure, it has been used as a scientific law. Tomlinson explained the phenomenon of friction by surface molecular force, and then deduced the coefficient of Amontons' wear formula [3]. Bowden and Tabor have established a relatively complete adhesive wear theory by experiment [4]. Holm deduced the wear volume of the unit sliding displacement according to the interaction between atoms in wear process [5]. In 1953, Archard [6] proposed the adhesive wear calculation formula, it shows that the wear volume is proportional to the load and the sliding distance, and is inversely proportional to the softer surface's yield strength. It is considered that only adhesive wear theory established by Bowden and Archard formula are scientific laws in these well-known wear calculation models. There are a lot of researches and applications based on the Archard formula, Zhang Jiasi deduced the simplified formula of material's wear rate with the condition of the middle and low temperature based on the concept of wear mechanisms [7]; Zhang Yishan established a mathematical model to characterize cylinder liner and piston ring during running-in process by using liner ’s surface roughness and wear rate as two state parameters of the system [8]; Ting and Mayer proposed a wear analysis model which can calculate the wear on piston ring's thrust surface based on Archard theory [9].

Archard model is widely used in the wear calculation, the model assumes that all the bonding points are the hemispherical asperities, it corrected the assumption by introducing adhesion wear constant [3], however, there aren't accurate calculation method for the adhesion wear constant, and the constant is determined by experiment which can range from $10^{-5}$ to $10^{-2}$ [10], so Archard model can only be used to approximately estimate the wear life.

Fractal theory is source on geographical geometry. The core idea is that a part of local (shape, structure, information, function etc.) will exhibit similarity with the overall in all of the phenomena and the process, the theory was proposed by Mandelbort [11], and Mandelbort applied it to research the British coast line. The fractal feature of rough surface has the scale-independence, which can provide all the information of surface topography [12,13]. Scholars such as Thomas [14], Majumdar
and Ganti [16] applied the fractal theory to the study of surface roughness, they used the famous Weierstrass Mandelbrot function (W-M function) to simulate the profile curve of rough surface, used fractal dimension (D) to quantitatively characterize the rough surface. Ge Shirong and Zhu Hua [17] firstly proposed the concept of characteristic roughness by combining fractal dimension D and fractal roughness G. Characteristic roughness parameters can be more sensitive to characterize the surface topography, because it is the combination of the fractal dimension and fractal roughness.

In a word, the contact between surfaces is actually the contact between asperities [3], Archard formula is based on the deformation of the asperity, so it is advantageous to use fractal theory characterizing the asperity, and it will be able to introduce the topography of contact surface and material's properties to wear formula. This paper will use the fractal parameters with scale-independence to characterize surface topography in the wear process by introducing the fractal theory. A wear model based on fractal parameters, operating parameters and material's properties will be established.

Fractal wear formula

W.Yan and K.Komvopoulos [18] proposed a 3D W-M function based on the Ausloos-Berman (A-B) function [19], which can describe the microscopic characteristics of three-dimensional fractal surface i.e:

$$z(x,y)=L \left( \frac{G}{L} \right)^{D_S-2} \left( \frac{\ln \gamma}{M} \right)^{1/2} \sum_{m=1}^{n_{max}} \sum_{n=0}^{(D_S-3)n} \left[ \cos \phi_{m,n} - \cos \left( \frac{2\pi^2 \sqrt{x^2+y^2}}{L} \cos \left( \arctan \frac{y}{x} - \frac{\pi m}{M} \right) + \phi_{m,n} \right) \right]$$

(1)

Where L is the sampling length; G is the fractal roughness which is a height scale parameter independent with frequency; D_S is the surface fractal dimension; The parameter M represents quantity and intensity of peak composed of the surface; \( \lambda_n \) is the reciprocal of the random profile’s space frequency, \( \gamma=1/\lambda_n \), \( \gamma \) denotes the profile density, which is a constant greater than 1, it’s appropriate for random surface that obeys the normal distribution with \( \gamma=1.5 \) (suitable for high spectral density and random phase); is random phase in the interval \([0,2\pi]\); \( n_{max}=\text{int}[\log(L/L_S)/\log(\gamma)] \), L_S is a cut-off length.

In order to clarify the physical significance of the fractal parameters in Eq.1, we simulate the isotropic surface as shown in Fig.1 with \( L=0.02\text{mm}, \gamma=1.5 \). Fig.1(a) and Fig.1(c) show that surface is rougher when G is larger; Fig.1(a) and Fig.1(b) show that surface is smoother when D is larger; The quantity and intensity of peaks increase with the increasing of M, surfaces possess cylindrical corrugations with M=1 shown as Fig.(c) and Fig.1(d).
Fig. 1 shows that a typical contact plane can be simulated by Eq. 1, and a contour curve on the plane can be expressed by [18]:

$$z(x) = L \left( \frac{G}{L} \right)^{\frac{D_n}{2} - 2} \left( \ln \gamma \right)^2 \sum_{n=0}^{\infty} \frac{1}{(2n+1)^2} \left\{ \cos \phi_n \cos \left[ \frac{2\pi n}{L} - \phi_n \right] \right\}$$

(2)
Fig. 2 shows that when two rough, nominally flat surfaces are brought together, surface roughness causes contact to occur at discrete asperities [20], and the shapes of asperities in the running-in process and stable wear process are different. It assumes that a surface is a rigid smooth plane. $x$ is the interfacial gap; the actual wear depth of each asperity are not identical, the wear depth is range from zero to $w$, $w$ is the complete deformation value of asperity [18]; $r$ is the actual contact radius of the asperity. The asperity will wear when the asperity is plastic deformation and the force at a microcontact point is given by:

$$ p = \pi r^2 \sigma_s $$

Where $\sigma_s$ is the yield strength of softer surface.

It is assumed that the asperity is conical shape in the running-in process shown as Fig.2 (a), so the wear volume corresponding to the sliding distance $(2r)$ is $\pi r^2 \delta/3$, then the rate of wear volume can be expressed as:

$$ \frac{dV}{ds} = \frac{1}{3} \pi r^2 \delta $$

Where $s$ is the sliding distance, $\delta$ is the asperity's actual wear depth, $r$ is the radius of asperity, $\delta$ can be given by [19]:

$$ \delta = 2(G)^{D_s-2}(\ln \gamma)^{1/2}(2r)^{(3-D_s)} $$

Where $G$ is the fractal roughness, $D_s$ is the surface fractal dimension, $r$ is the radius of asperity, $\gamma$ denotes the profile density.

Substituting Eq.5 into Eq.4, then Eq.4 can be expressed as:

$$ \frac{dV}{ds} = \frac{\pi G^{D_s-2}(\ln \gamma)^{1/2} s^{4-D_s}}{6} $$

Submitting Eq.3 to Eq.6, Eq.6 can be expressed as:

$$ \frac{dV}{ds} = \frac{2}{3} G^{D_s-2}(\ln \gamma)^{1/2} s^{2-D_s} \frac{p}{\sigma_s} $$
\[
\frac{dV}{dt} = \frac{1}{3} dh \cdot \pi r^3 \cdot \frac{1}{3} \frac{dr}{ds} = \frac{1}{3} \pi r^2 \cdot \frac{dh}{dt} \cdot v^4
\]  
\tag{8}
\]

Where \( v \) is the sliding speed.

So the rate of wear depth can be expressed as:
\[
\frac{dh}{dt} = \frac{8G^{D_s-2} \left( \ln \gamma \right)^2 s^{-D_s} p \nu}{\pi \sigma_s}
\]  
\tag{9}
\]

The wear depth can be given by integrating Eq. 9:
\[
h = 2G^{D_s-2} \left( \ln \gamma \right)^2 s^{(3-D_s)} \ln t
\]  
\tag{10}
\]

In the stable wear process, asperity's peak can be reduced by 65%~75%\[3\], so it is reasonable to assume that the asperity is a cylinder shape shown as Fig. 2(b), the rate of wear volume corresponding to the sliding distance (2r) is given by:
\[
\frac{dV}{ds} = \frac{\pi r^2 \delta}{2r} = \frac{\pi r \delta}{2}
\]  
\tag{11}
\]

And
\[
\frac{dV}{ds} = \frac{dV}{dt} \cdot \frac{dr}{ds} = \pi r^2 dh \cdot \frac{1}{3} \frac{dr}{ds} = \frac{dh \cdot \pi r^2}{v}
\]  
\tag{12}
\]

Where \( v \) is the sliding speed.

Combining Eq. 5, Eq. 11 and Eq. 12, the rate of wear depth can be expressed as:
\[
\frac{dh}{dt} = \frac{8G^{D_s-2} \left( \ln \gamma \right)^2 \nu^{1-D_s} \cdot pt^{-D_s}}{\pi \sigma_s}
\]  
\tag{13}
\]

The rate of wear depth in the running-in process and the stable wear process are similar in form. However, the fractal parameters (\( D_s, G \)) and operating parameters (\( V, p \)) are constantly changing in the wear process, fractal roughness \( G \) decreases and fractal dimension \( D_s \) increases with the time \[5\]. Therefore, the rate of wear depth in running-in process and stable wear process is continuously changeable and complicated.

**Numerical simulation**

It is assumed that the sliding speed and the force at adhesion point does not change with time. Numerical simulations of Eq.9 and Eq.10 was carried out for with \( G=10^{-6}mm, \gamma=1.5, s=0.08mm \). Fig.3(a) shows that the wear depth in running-in process is much larger than that in stable wear process at the same fractal dimension; the rate of wear depth in running-in process decreases with time, the rate of wear depth remained almost unchanged in stable wear process after the running-in process. The fractal dimension in the stable wear process is larger than that in running-in process because the surface is smoother, so it is reasonable to take different fractal dimensions to compare the rate of wear depth in the two processes. The rate of wear depth is lower when the fractal dimension is larger, which is consistent with that described by classical wear theory, so it shows that the fractal wear formula is correct.
The wear depth increases with the increasing of fractal roughness and decreases with the increasing of the fractal dimension at the same parameters, because the increment of the dimension and the decrement of the fractal roughness make surface more smooth.
Figure 4. The wear depth changing with fractal parameters.

Figure 5 shows that the rate of wear depth decreases with the increasing of fractal dimension no matter what value of G, the variation of wear rate becomes more little when the fractal dimension is larger, and this is consistent with that described by classical theories[3]; the rate of wear depth increases with the increasing of fractal roughness when the fractal dimension is greater than 2.05, and the fractal roughness has smaller influence on the wear rate when the fractal dimension is larger, because the surface becomes more smooth when the fractal dimension is larger, molecular interaction is the main cause of wear, so the influence of G on the rate of wear depth is relatively weakened, it's consistent with the conclusions by professor Ge Shirong [5], it further demonstrates the correctness of the fractal wear formula.

Figure 5. the rate of wear depth changing with fractal parameters.
Conclusions

Due to the scale-independence of the fractal parameters, the fractal geometry is very good for the characterization of rough surface. Fractal parameters can effectively characterize the change of surface topography during the wear process. In the wear process, the changes of fractal parameters and the changes of surface topography or the rate of wear depth have a good corresponding relationship. This paper establishes fractal wear model by combining the fractal theory and the classical wear theory, and considers the different shapes of asperities in the running-in process and stable wear process, considers the influence of the material properties, surface topography and operating parameters on wear, solves the problem that the adhesive wear constant is difficult to determine in Arhard model. The model is mathematically rigorous and comprehensive, and the validity of the model is verified by numerical simulation.

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