Channel Estimation and Demodulation for MIMO-OFDM System Under Fractional Timing Offset

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Abstract. For the channel offset caused by the time offset (TO) of the MIMO-OFDM system, we propose a new high-precision and low complexity new channel estimation algorithms, which is based on least squares (LS) method to solve the problem that we need to send a lot of pilot symbols to estimate the channel coefficient again when the fractional timing offset is present, which greatly reduces the complexity of the system. Meanwhile, we combine the new algorithm and Vectoring technology, so that it allows multiple transmitted signals through the down-link pre-coding matrix and the actual channel to each receiver, completely eliminating the multiple user interference caused by transmitted signals of multiple users and accurately extracting the signal of each user. Simulation experiments prove that the new algorithms have good estimation performance.

Introduction

In the wireless communication systems[1], the transmission performance of the system is largely restricted by the wireless channel. MIMO-OFDM[2] is a multiple input and multiple output and multiple carrier transmission technology, which is widely used in wired and wireless networks because of its good anti-interference and high spectrum efficiency. Timing offset (TO)[3] occurs when the sampling time point of transmitter and receiver signal is not consistent, which generally exists in wireless communication system.

For a system with a large number of users, synchronization[4] and channel estimation [5] are needed with each change of TO, which will undoubtedly increase the cost of the system and increase the complexity of the system. In addition, in the MIMO-OFDM system, the received signal at a receiving antenna is the mixture of the multiple transmitted signals after the channel. Traditionally, the signals are demodulated at the receiver[6], in order to separate or eliminate the interference signals generated by the other user, we must know the received signals of other users, which also increases the complexity of the demodulation[7].

To solve this problem, we propose a new method to reconstruct the channel, we can use the estimated value of channel coefficients of the two preceding synchronization channels to fit channel coefficients of all fractional time points. More details is included in chapter 3.

As to the demodulation, we combine the new algorithm and Vectoring technology[8], completely eliminating the multiple user interference caused by transmitted signals of multiple user and accurately extract the signal of each user. The details is included in chapter 4.

Simulation results show that the new algorithm can meet the requirements of MIMO-OFDM system.
New algorithm

In this paper, we complete the channel estimation via the pilot method, which can be divided into three steps: insert pilots; channel estimation estimating; interpolation algorithm. Inserting pilots refers to choose certain kind of pilot symbols and then insert the pilots at certain places, the pilots are designed as reference[9]. The channel estimation frame is depicted as follows, along with the interpolation algorithm which is used to reconstruct the channel information at none pilot positions base on the known pilot positions.

Estimation frame. Assume that an MIMO-OFDM system includes N central offices(CO) and M customer premise equipment(CPE).All CO are designed to transmit independent data streams, denoted by:

\[ S^{(n)}(k) = [s^{(n)}(0),...,s^{(n)}(p-1)]^T, n = 1,...,N, k = 0,...,P-1. \]  

(1)

where \( S^{(n)}(k) \) is limited within a single OFDM symbol. After FFT, is the received signal vector of the mth user, denoted by:

\[ Y_m(k) = [y(0),...,y(p-1)]^T, m = 1,...,N, k = 0,...,P-1. \]  

(2)

where \( P \) is the number of subcarriers within an OFDM symbol.

For the received signal at the mth CPE, it not only includes the mth CO signal after direct channel, but also includes additional N-1 CO signals through the interference channel. Thus, the Pth subcarrier of the mth CPE can be denoted as:

\[ y_m(k) = H^{(n)}_m(k)S^{(n)}(k) + \omega_m(k) \]  

(3)

where \( H^{(n)}_m(k) = \sum_{l=0}^{L-1} h^{(n)}_m(l)e^{-j2\pi kl/p} \), \( h^{(n)}_m(l) \) is the time domain impulse response for the signal from the nth CO to mth CPE.L is the multipath number, \( \omega_m(k) \) is the white Gaussian noise signal, that is

\[ y_m(k) = \sum_{n=0}^{N-1} \left[ \sum_{l=0}^{L-1} h^{(n)}_m(l)e^{-j2\pi kl/p} \right] S^{(n)}(k) + \omega_m(k) \]  

(4)

To estimate the OFDM channel, all the users' pilot tones are uniformly distributed and aligned with each other in frequency domain. Denotes \( S^{(n)}(k_i) \) the pilot sequence:

\[ S^{(n)}(k_i) = [s^{(n)}(k_i),...,s^{(n)}(s^{(n)}(k_i))]^T, n = 1,...,N, k_i = k_1,...,k_I. \]

where \( k_1,...,k_I \) are pilot tones, the FFT demodulated signal at the pilot tones is:

\[ Y_m(k_i:k_i) = [\hat{y}_m(k_i),...,\hat{y}_m(k_i)]^T, \text{then the formula (4) can be rewrite as :} \]

\[ AH_m + \omega_m(k_i:k_i) = Y_m(k_i:k_i) \]  

(5)

where \( A \) is a \( I \times (NL) \) matrix, that is \( A = [A^{(0)},...,A^{(I-1)},...,A^{(N-1)},...,A^{(L-1)}] \),the sub-matrix of \( A \) is \( A^{(n)} = [s^{(n)}(k_i)e^{-j2\pi kl/p},...,s^{(n)}(k_i)e^{-j2\pi kl/p}], n = 1,...,N, l = 0,...,L-1 \), \( I \) is the pilot tones.

\[ H_m = [h^{(0)}_m(0),...,h^{(I)}_m(L-1),...,h^{(N)}_m(0),...,h^{(N)}_m(L-1)]^T, \text{then ,we estimate the matrix } H_m : \]

\[ \hat{H}_m = A^+ Y_m(k_i:k_i), m = 1,...,N \]  

(6)

where \( A^+ \) is the pseudo-inverse of \( A \),to insure that \( \hat{H}_m \) has a unique solution, it's usually require that \( I \geq (NL) \) and \( A \) is column full rank, rewrite formula (5), we'll get:
\[ H_n = A^* Y_n(k_i : k_i) - A^* \omega_n(k_i : k_i) \]  \hspace{1cm} (7)

similar to reference[3], the mean square error of channel can be expressed as:

\[ MSE = \frac{1}{NL} E\|H_n - \hat{H}_n\|^2 \]  \hspace{1cm} (8)

combine formula (5)(6)(7), we have:

\[ MSE(m) = \frac{1}{NL} E\|A^* \omega_n(k_i : k_i)\| \]
\[ = \frac{1}{NL} tr\{A^* E\{\omega_n(k_i : k_i)\} A^* A\} \]  \hspace{1cm} (9)

where \[ \| \] denotes the European norm, \[ tr(\cdot) \] denotes the matrix trace, \[ E\{\cdot\} \] denotes the mathematical exception, \[ \omega_n \] is the zero mean Gaussian white noise,

\[ E\{\omega_n(k_i : k_i)\} = \delta^2 I' \]  \hspace{1cm} (10)

where \[ \delta^2 \] is the variance of Gaussian white noise, \[ I' \] is the unit array, thus:

\[ MSE = \frac{\delta^2}{NL} tr\{A^* A\} \]  \hspace{1cm} (11)

where we can see \[ MSE \] is determined by \[ A \], in order to gain the minimum value of \[ MSE \], it's required that \[ A^* A = c I' \], where \[ c \] is a constant, \[ I' \] is a unit array.

Reconstruct the channel. Assume the nth central office(CO) after the constellation mapping is \[ S^{(n)}(K) \], the kth subcarrier of the receiving signal on the mth customer premise equipment(CPE) is \[ y_n(m)(K) \], after oversampling twice on the receiving signal, we gain the signal from one of the sampling point which can be denoted as:

\[ y_n(k) = \sum_{l=0}^{L-1} h^{(\omega)}_{m,n}(l) * e^{-j2\pi f_l} * S^{(\omega)}(k) + \omega_n(k) \]  \hspace{1cm} (12)

From the other sampling point, we'll get:

\[ y_n(k) = \sum_{l=0}^{L-1} h^{(\omega)}_{m,n}(l) * e^{-j2\pi f_l} * S^{(\omega)}(k) + \omega_n(k) \]  \hspace{1cm} (13)

Where \[ h^{(\omega)}_{m,n}(l) \] is the corresponding channel coefficients after sampling the receiving signal under fractional timing offset. \[ h^{(\omega)}_{m,n}(l) \] represents the time point of the timing offset.

To estimate the channel coefficients, we provide a new algorithm based on the least square method, the core idea for which is to fit the new channel coefficients at any time point of TO via the coefficients after twice oversampling the receiving signals at the given two time points of TO. The procedure can be described as follows in Fig
Figure 3.3 the procedure of two sampling points fitting the channel coefficients under timing offset.

The procedure of new algorithm are described as follows

a. gain the channel coefficients matrix at the fractional offset time point $x$:

$$\hat{h}_{m,x}^{(0)}(l) = [\hat{h}_{m,x}^{(0)}(0), \hat{h}_{m,x}^{(0)}(1), ..., \hat{h}_{m,x}^{(0)}(l-1)]^T, \ x = 0, 0.1, ..., 0.9. \quad (14)$$

b. gain the channel coefficients matrix at the fractional offset time point $y$:

$$\hat{h}_{m,y}^{(0)}(l) = [\hat{h}_{m,y}^{(0)}(0), \hat{h}_{m,y}^{(0)}(1), ..., \hat{h}_{m,y}^{(0)}(l-1)]^T, \ y = 0, 0.1, ..., 0.9. \quad (15)$$

c. merge the two matrix into one: $n = x, y, x+1, y+1, ..., x+L-1, y+L-1$

$$\hat{h}_{m,x+y}^{(0)}(l) = [\hat{h}_{m,x}^{(0)}(0), \hat{h}_{m,y}^{(0)}(0), \hat{h}_{m,x}^{(0)}(1), \hat{h}_{m,y}^{(0)}(1), ..., \hat{h}_{m,x}^{(0)}(l-1), \hat{h}_{m,y}^{(0)}(l-1)]^T \quad (16)$$

d. combing $n$ and $\hat{h}_{m,x+y}^{(0)}(l)$, fit the real channel impulse response in the order of $r$, we'll gain the optimal approximation function $P_m^{(a)}(t)$, where, $r = 1, 2, ..., 2L-1$.

e. use $P_m^{(a)}(t)$ to fit the channel coefficients of the fractional offset time point:

$$\hat{h}_{m,v}^{(0)}(l) = [\hat{h}_{m,v}^{(0)}(0), \hat{h}_{m,v}^{(0)}(1), ..., \hat{h}_{m,v}^{(0)}(l-1)]^T, \ v = 0, 0.1, ..., 0.9. \quad (17)$$

Demodulate the signal. Combine the N receiving signal at the CPE with the $N \times N$ channel estimating information, we'll get the transmitting matrix:

$$\hat{S}(k) = ([\hat{H}(k)]_{m,s})^{-1}Y_m, k = 0, ..., P-1. \quad (18)$$

where $(\cdot)^{-1}$ denotes the inverse of the matrix, $[\hat{H}(k)]_{m,s}$ is a $N \times N$ matrix, $[\hat{H}(k)]_{m,s} = \sum_{l=0}^{L} \hat{h}_{m}^{(0)}(l)e^{-j2\pi kl/P}$ ( $\hat{h}_{m}^{(0)}(l)$ is the estimate of the channel from the nth CO to the mth CPE), $\hat{S}(k) = [s^{(1)}(k), ..., s^{(N)}(k)]^T$. Note that the matrix $[\hat{H}(k)]_{m,s}$ is column full rank, thus $\hat{S}(k)$ has a unique solution, the pre-coding matrix is:

$$H(k)_{pre\text{-coding}} = [\hat{H}(k)]^{-1} \quad (19)$$
Simulation results

In experiment, an OFDM system with CO=5 transmitting signals and CPE=5 receivers is used as the experiment model. Each user transmitting data stream using 16QAM signals with the symbol rate 1M baud and the OFDM symbol-size I=512. The additive noise is chosen to be a Gaussian and white random process. The channel order is chosen to be L=10, and the CP length exactly equals to the channel order, i.e. $L = L = 10$. The polynomial fitting parameters are as follows in Table 1.

The effect for different TO values on the channel property is shown in Fig 4.1, in which we simply introduce the timing offset. By using the new algorithm in Fig 4.2, we can see that the average MSE approximates to the condition without TO, i.e. $T=0$ which is in Fig 4.1, if we choose the proper TO values.

![Graph showing the effect of different TO values on channel property](image)

<table>
<thead>
<tr>
<th>SNR/dB</th>
<th>Mean Square Error MSE</th>
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<tbody>
<tr>
<td></td>
<td>$T=0$</td>
</tr>
<tr>
<td></td>
<td>$T=0.1$</td>
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<tr>
<td></td>
<td>$T=0.3$</td>
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<tr>
<td></td>
<td>$T=0.5$</td>
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<tr>
<td></td>
<td>$T=0.7$</td>
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<tr>
<td></td>
<td>$T=0.9$</td>
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Table 1.

<table>
<thead>
<tr>
<th>Sampling point</th>
<th>Sampling point</th>
<th>Fitting point</th>
<th>Fitting order</th>
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<tbody>
<tr>
<td>x</td>
<td>y</td>
<td>v</td>
<td>r</td>
</tr>
<tr>
<td>0.3</td>
<td>0.7</td>
<td>0.5</td>
<td>2/4/6/8/10</td>
</tr>
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From Fig 4.3 and Fig 4.4 shows that the optimal fitting order for the direct channel the different polynomial order r is 8 and 4 respectively.

![Graph showing the optimal fitting order for direct channel](image)

After the channel estimation, we can then recover the transmitted data using the pre-coding matrix. As shown in Fig 5(a,b) and Fig 6(a,b), the new channel estimator and the transmitted recovery are successful in recovering the data. This means that the TO effects on the demodulator have been weakened.
Conclusions

In this paper, we offer a new slow channel estimation algorithm with high precision and low complexity based on the least squares fitting, meanwhile, with which we implement the correct demodulation for MIMO-OFDM system under fractional timing offset.

References


