Research on Single Frequency Radar Imaging with BP Algorithm

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\textbf{Abstract.} In this letter, we address the Doppler back projection (BP) imaging algorithm for the single frequency continuous wave high resolution imaging. Unlike the existing imaging algorithm for narrow band signal, the Doppler information of targets is used for imaging, which makes up for the lack of range resolution. The concrete formula deduction and algorithm process are presented. By extracting the Doppler information of the target, the phase compensation and back projection imaging, we realize the accurate simulation of single point and multi point targets. The simulation results verify the targets can be well reconstructed. At last, We designed a measurement verification system based on the turntable model. Imaging experiments using measured data based on ideal trajectory has been done. The measurement results indicate the efficiency of the Doppler BP algorithm and implement imaging of point target in small scene.

\textbf{Introduction}

In recent years, the field of radio spectrum resources become increasingly tense, band congestion problems highlighted, which greatly promoted the development of the narrow-band continuous wave imaging. Narrow band continuous wave and single frequency continuous wave imaging is a real development in the past ten years\cite{3,4,5}. At present, there are two main methods for narrow band/ single frequency continuous wave imaging\cite{4,5,6}: The first one, space diversity is used to compensate for the lack of range resolution, placing multiple radars around the target, which is used to detect moving targets\cite{7} and 360 degree imaging\cite{8,9}. Second, the echo Doppler frequency shift information is used to back projection imaging\cite{10}, and the imaging scattering body is located on the Doppler curve, which is suitable for the flight path of arbitrary situation\cite{5,6}. This new imaging system is still in its infancy stage, there is little related literature.

In this paper, the second narrow band imaging method is studied, and the principle and process of Doppler BP imaging algorithm are discussed in this paper. The simulation experiments of single point and multi point targets are carried out to verify the theoretical feasibility of the algorithm. Finally, the measured data are collected for imaging processing, and the single frequency continuous wave Doppler BP imaging algorithm is verified, achieving a small scene imaging.

\textbf{Signal Model}

The single frequency continuous wave radar's transmitting signal is shown in the formula (1).

\[ s_{t}(t) = \exp(j2\pi f_{t}t) \]  

where, the amplitude of the transmitted signal is assumed to be 1. \( t \) is the absolute time variable, that is, the continuous time variable.

The echo signal received by the radar can be expressed as:

\[ s_{r}(t) = A(t)\exp(j2\pi f_{r}(t - \tau(t))) \]  

Here, \( A(t) = |R'(t)| \) is the signal amplitude attenuation index, it is a function changing with continuous time. \( \tau(t) = 2R(t)/c \) is time delay, it also changes with continuous time. Assuming the instantaneous
The position of the sensor is \( \gamma(t) \), the position of the target in the ground imaging area is \( P = (x, y, 0) \), then \( R(t) = |\gamma(t) - P| \) denotes the instantaneous slant distance. \(|P|\) stands for the modulus operation. The received signal is mixed with the transmitted signal, and the signal obtained after mixing is

\[
s_{r \eta}(t) = A(t) \exp\left( -j2\pi f_r \tau(t) \right)
\]  

(3)

The continuous wave radar transmit and receive at the same time, it cannot work in the "stop - go - stop" mode. According to formula (2) and (3), \( \tau(t) \) is related to the continuous time[11]. The mixed signal data above are selected by section. Since the amplitude of the signal has no effect on the phase, the amplitude is ignored, and the signal is reduced to the following form:

\[
s_{r \eta}(t) = \exp\left( -j2\pi f_r \tau(t) \right)
\]  

(4)

After selection is:

\[
s_{r \eta}(t, \eta) = \exp(-j2\pi f_r \tau(t, \eta))
\]  

(5)

Where, \( t_r \) is fast time, \( 0 \leq t_r \leq T_s, \eta \) is slow time, corresponding to the beginning time of each data, we use \( T_s \) as a PRI then \( \eta = n T_s \). Time delay \( \tau(t_r, \eta) \) in continuous wave radar is a function of two dimensional time variable. It has a relationship with both fast time and slow time. As shown in the formula:

\[
\tau(t_r, \eta) = 2R(t_r, \eta)/c = 2|\gamma(t_r, \eta) - P|/c
\]  

(6)

Taking the first paragraph as an example, the general form is obtained. \( \eta = 0 \), the intercepted data is

\[
s_{r \eta}(t, \eta = 0) = \exp(-j2\pi f_r \tau(t, \eta = 0))
\]  

(7)

The Taylor series expansion is performed as shown in the formula (8)[5][10].

\[
\gamma(t_r) = \gamma(0) + \dot{\gamma}(0)t_r + \cdots \approx \gamma(0) + \dot{\gamma}(0)t_r
\]  

(8)

Substitute (8) into slant range [5][10]:

\[
R(t_r, \eta = 0) = |\gamma(0) + \dot{\gamma}(0)t_r - P|
\]  

\[
\approx |\gamma(0) - P| + \left| \dot{\gamma}(0) - \gamma(0)t_r \right|
\]  

(9)

Where, \( |\gamma(0) - P| \) is the slant range at azimuth time \( \eta = 0 \). \( \gamma(0) - P \) is the unit vector pointing from target \( P \) to radar position \( \gamma(0) \). \( \dot{\gamma}(0) - P \) is the projection of the velocity vector in the direction of the slant distance at azimuth time \( \eta = 0 \). Assuming that \( \dot{\gamma}(0)t_r \ll |\gamma(0) - P| \), the approximation above is established[5]. Substitute (9) into the time delay(7):
\[ s_d(t, \eta = 0) = \exp(-j2\pi f, \tau(t, \eta = 0)) \]
\[ = \exp(-j2\pi f, \frac{[\tau(0) - P] + \left(\tau(0) - P\right) \cdot \tau(0) c}{c}) \]
\[ = \exp(-j4\pi \frac{[\tau(0) - P]}{\lambda}) \cdot \exp(-j4\pi \frac{\left(\tau(0) - P\right) \cdot \tau(0)}{\lambda} t) \]

(10)

Let \( f_d(\eta = 0, P) \) be the Doppler frequency shift of sensor and target at \( t, \eta = 0 \). Formula (10) can be simplified as:

\[ s_{dp}(t, \eta = 0) = \exp(-j4\pi R(\eta = 0, P)/\lambda) \]
\[ \cdot \exp(j2\pi f_d(\eta = 0, P)t) \]

(11)

At the beginning of each data, the Taylor series expansion is performed. Formula (5) can be expressed as follows:

\[ s_p(t, \eta = 0) = \exp(-j4\pi R(\eta, P)/\lambda) \]
\[ \cdot \exp(j2\pi f_d(\eta, P)t) \]

(12)

Where, \( Q_p(\eta, P) = \exp(-j4\pi R(\eta, P)/\lambda) \) needs to be compensated during the imaging process. The second phase, which is related to the instantaneous Doppler frequency shift of the target, is used for the back projection imaging of Doppler in this paper.

**Imaging Algorithm**

Perform Fourier transformation on formula (12) to extract the Doppler frequency-shift:

\[ S_{dp}(f, \eta) = \int s_{dp}(t, \eta) \exp(-j2\pi ft) dt \]

(13)

The Doppler frequency shift between the sensor and the target at each azimuth moment is carried out for the back projection. The ground point image reconstructed by the back projection can be obtained by the formula (14).

\[ im(x, y) = \int S_{dp}(f = f_d(\eta, P), \eta) \cdot Q(\eta, P) \eta d\eta \]

(14)

Where, \( Q(\eta, P) \) represents the filter factor, and \( Q(\eta, P) = Q_d(\eta, P) = \exp(j4\pi R(\eta, P)/\lambda) \).

At each sampling time we can find the corresponding Doppler frequency shift for back projection, after the phase compensation, the final imaging results can be obtained by the coherent accumulation of all the points[12,13]. Write the formula (14) in discrete form:

\[ im(x, y) = \sum_{n=0}^{N-1} S_{dp}(f_d(x, y)(n), n) \cdot Q_{d(x, y)}(n) \]
\[ x = 0, 1, \cdots, X-1; \quad y = 0, 1, \cdots, Y-1 \]

(15)

Where, \((x, y)\) is the location of the point \( P \) in the ground imaging area; \( f_d(x, y)(n) \) is the instantaneous Doppler frequency shift; \( Q_{d(x, y)}(n) \) is the phase term to be compensated.
Here, $Q_{(x,y)}(n) = \exp(j4\pi R_{(x,y)}(n)/\lambda) \cdot R_{(x,y)}(n)$ is instantaneous slant range. In this paper, we use the method of filling zeros to time domain data to refine the frequency spectrum, and then find the corresponding Doppler frequency shift by the approach of the nearest point interpolation.

**Simulation**

The simulation model is set as follows:

![Simulation Model Image](image)

The black dot stands for the sensor, whose height is 1km. The speed of sensor is 100m/s. Circle track radius is 3km. The square grid is a target imaging region, the size of which is 1km1km. The carrier frequency of transmit signal is 24GHz, the sampling rate is 8KHz. The observation angle is 10 degrees, and the is 1KHz, is 1ms, the target is located at the center of the ground scene (0, 0). The imaging results are shown in Fig. 2.

![Image Gray Scale](image)

![X Direction Profile](image)

These figures show that the target can be well reconstructed under ideal conditions. The calculation of 3dB bandwidth in Fig.3 shows that the X direction resolution is 0.0334m.

**Measured Data Imaging**

The experimental imaging scene is based on the turntable model with targets fixed on the turntable.
Turntable radius is 0.2m. The distance from the turntable center to radar is 0.355m, the relative linear velocity of the radar is . The size of the target imaging area is set to 0.6m x 0.6m, and the pixel interval is set to 0.003m.

The two targets set at (-0.17, 0), (0, -0.17) are simulated. Result is shown in Fig. 5, which indicates the target can be well reconstructed.

Before using the measured data for imaging, the Moving Targets Indication (MTI) is needed to deal with the measured data, so as to avoid the emergence of direct wave interference.

The imaging processing result of the measured data is shown in Fig. 6 and Fig. 7.

The calculation indicates that the two distance from targets to the image area center are 0.1718m and 0.1651m. The central angle of the two targets is 87.13 degrees.

Because of the randomness of the data segment, we judge whether the imaging is accurate by comparing the distance between the target and the center of the scene and the relative position of the targets.
Through the analysis of the measured images, we can know that the distance error is in the reasonable range. The distance error is less than 0.01M, and the angle error is not more than 5 degrees, compared with the theoretical value of the 0.17m.

**Conclusion**

A single frequency continuous wave Doppler BP imaging algorithm is presented in this paper. The formula of imaging algorithm is derived, and the process of the algorithm is described. Through the simulation of single target and two point targets, the results show that the imaging scene can be well reconstructed and the effectiveness of the algorithm is verified. At last, Doppler BP imaging algorithm is used to dealing with the measured data. The results show that the imaging algorithm can be used to reconstructing targets under different distribution, and the imaging algorithm is validated.

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