Motion Blur Image Restoration Algorithm Based on the Theory of Reaction Diffusion

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Abstract. Image restoration is of great significance in the digital image processing. When imaging equipment obtains the image, if the imaging equipment and the scene occur relative motion in the exposure time, it could lead to actually get blurred image. Due to motion blur, the got image loses real value. So the research of motion blur image restoration has important application value. This paper proposes a motion blur image restoration algorithm based on the theory of reaction diffusion. The algorithm uses the reaction diffusion equations related mathematical knowledge as the background, build the motion blur image restoration model and design motion blur image restoration algorithm. Through the experiment, the algorithm compares with the common motion blur restoration algorithm (RI-BM3D, BM3DDEB, IDD-BM3D and ForWaRD). The subjective and objective restoration results of the proposed image restoration algorithm are obviously improved.

Introduction

Image degradation reasons have the optical system diffraction, the sensor nonlinear distortion, the optical system aberration, the photographic film nonlinear, the atmospheric turbulence, the image motion disturbance effect and geometric distortion etc.[1]. A variety of degraded image restoration can be summed up in one process. The process specifically is building the degradation model. If we use the reverse process to deal with the image, we can obtain the restoration image. The key problem of image restoration processing is that degradation model is established. Assume, the imaging system is the linear time invariant system. To obtain the restored image $g(x, y)$ is expressed as [2]:

$$g(x, y) = I(x, y) \ast H(x, y) + n(x, y)$$ (1)

where, $I(x, y)$ denotes the original image, $n(x, y)$ denotes the additive noise, $H(x, y)$ denotes the point impact response function of the system, $g(x, y)$ denotes the degraded image. Based on the mathematical expression, degradation model established is shown in Figure 1. From the model of figure 1, we can know that a picture of a pure image is due to a system $H$ and joined the foreign additive noise $n(x, y)$, and degrade into a image $g(x, y)$ [3].

![Figure 1. Degradation model of the image.](image)

If the angle of the direction of imaging target motion and imaging plane is $\theta$, the fuzzy operator $H(x, y)$ of the motion blur image model can be represented as:

$$H(x, y) = \begin{cases} \frac{1}{L} \sqrt{x^2 + y^2} \leq \frac{L}{2} \text{ and } \frac{y}{x} = -\tan \theta, \\ 0, \text{ others} \end{cases}$$ (2)
The Application of Reaction Diffusion Theory in Motion Blurred Image Restoration

The heat conduction equation is a common partial differential equation, also known as diffusion equation. It is mainly used to describe the heat transfer process of the material and diffusion process. Heat transfer equation is denoted as follow:

\[ \frac{\partial u}{\partial t} - k \nabla^2 u = 0 \]  \hspace{1cm} (3)

The \( k \) is conduction coefficient or diffusion coefficient.

Reaction diffusion equations derived from partial differential equations, of which the common heat conduction equation is an isotropic diffusion equation. But it does not take into account the location information of the image in the spreading process, and smoothing the image edge information affects the quality of the image processing. Therefore, Perona and Malik put forward PM model. In order to protect the image edge details, the model on the basis of heat conduction equation fully considers the structure characteristics of the image and adopts an-isotropic diffusion method [4-5].

The PM model can be described as:

\[ \begin{align*}
\frac{\partial (x, y, t)}{\partial t} &= \text{div}(g(\nabla I)|\nabla I|) \\
I(x,0) &= I_0(x), \quad x \in \mathbb{R}^2
\end{align*} \]  \hspace{1cm} (4)

where, \( I \) denotes gray image. The \( I(x, y) \) is the gray value of the image in point \( (x, y) \). The \( I(x, y, t) \) denotes the result of image processing at \( t \) time. \( I_0 \) is the original image. Symbol \( \nabla \) denotes gradient operator. \( \text{div} \) denotes divergence operator. \( g(\cdot) \) denotes the diffusion coefficient of monotone decreasing, and it satisfies the formula (5):

\[ g(0) = 1, \quad \lim_{x \to +\infty} g(x) = 0 \]  \hspace{1cm} (5)

Usually, the PM model can be preset two diffusion function as follows:

\[ g(|x|) = \frac{1}{1 + \mu |x|^2}, \quad \text{or} \quad g(|x|) = \exp(-\mu |x|) \]  \hspace{1cm} (6)

Reaction diffusion equation (RDE) is applied to image restoration. Its the basic idea is to remove the influence of images qualitative degradation factors and at the same time to protect and enhance image details. Based on the PM model RDE can be described as [5]:

\[ \frac{\partial (x, y, t)}{\partial t} = \text{div}(g(|\nabla I||\nabla I|) + f(I) \]  \hspace{1cm} (7)

Restoration Model of Motion Blur Image

PM model based on diffusion process can effectively recover quality of degradation image, but details of restored image are not ideal. In order to further restore and enhance the recovery image details, the model of Reaction Diffusion Equation Based Image Restoration (RDER) is proposed. This model not only can be used to remove the image motion blur but also has certain robustness for noise introduced (including the impulse noise, Gaussian noise and mixed noise) when the capture image. RDER model can be described as:

\[ \frac{\partial I(x, y, t)}{\partial t} = \nabla(g(|\nabla G_{\sigma} \ast I||\nabla I|) + Pf(I) \]  \hspace{1cm} (8)

where, \( P \) is constant, \( P = 1/|\nabla I| \). It can adaptively enhance the image detail information (such as edge, texture, etc.). Formulas (8) and formula (7) have the same initial conditions. \( G_{\sigma} \) denotes Gaussian smooth kernel function. It is
\[ G_{\sigma}(x, y) = \frac{1}{2\pi\sigma^2} \exp\left(-\frac{x^2 + y^2}{2\sigma^2}\right). \]  

(9)

where, \( \sigma \) is smooth scale factor. \( x \) and \( y \) denote the image pixel location. Diffusion function \( g(|x|) \) satisfy the condition that is \( g(|x|) = \frac{1}{1 + \mu x^2} \) and \( \mu = |\nabla G_{\sigma}| \).

In formula (8), the reaction item \( f(I) \) describes a quantitative process. In order to improve the efficiency of algorithm, it can be controlled by the parameter \( P \), and the parameters can be adaptively adjusted. So the \( Pf(I) \) can be described as follows: When the gradient is bigger, the image contains more edges and details and quantitative process is slow. When the gradient is small, the image smooth part is more, so quantitative process is faster.

Because the image gray value is a certain value, it belongs to the Gaussian distribution, so we adopt Lloyd maximization method to design the quantizer \( (Q_s) \) of the image [6]. Here, we use \( f(x) \) to describe the \( Q_s \). \( f(x) \) is

\[
f(x) = \begin{cases} 
  x - n_i, & x \leq n_i \\
  \exp(-x) \sin((x-n_i)(x-m_j)) \ast \cos((x-n_i)(x-m_j)), & x \in [n_i, m_j) \\
  m_j - n_i, & x \in [m_j, n_{i+1}) \\
  \exp(-x) \sin((x-n_{i+1})(x-m_j)) \ast \cos((x-n_{i+1})(x-m_j)), & x \in [n_{i+1}, m_j) \\
  -x + n_{i+1}, & x \geq n_{i+1}.
\end{cases}
\]

(10)

In the \( f(x) \), \( s \) is the quantitative series of the \( Q_s \), here, \( s = 3 \). Code word and separate item of the \( Q_s \) meet \( n_1 < m_1 < n_2 < m_2 < \cdots < n_s < m_s < n_{s+1} \). The RDER model is discrete. It is

\[
\frac{\partial I}{\partial t} = g(\nabla G_{\sigma} \ast I^t) \partial I + \nabla g(\nabla G_{\sigma} \ast I^t) \nabla I + Pf(I). \]

(11)

By the difference scheme is derived, the formula (8) can be converted into discrete RDER model. It is described as follows:

\[
\frac{I_{i,j}^{n+1} - I_{i,j}^n}{\tau} = \frac{(g^\ast)_i^j}{h} \{2 \ast [(I_{i-1,j}^n + I_{i+1,j}^n) + (I_{i,j-1}^n + I_{i,j+1}^n)]
+ (I_{i,j}^{n+1} + I_{i,j}^{n-1}) + (I_{i+1,j}^{n+1} + I_{i-1,j}^{n-1}) - 12I_{i,j}^n + \frac{(g^\ast)_y^x}{h} (I_{i,j}^{n+1} + I_{i,j}^{n-1} + g^\ast)_y^x (I_{i,j}^{n+1} + I_{i,j}^{n-1} + Pf(I_{i,j}^n)) \}
\]

(12)

where, the \( \tau \) and \( h \) respectively denote the time step and space step length factor. The truncation error is \( O(\tau + h^3) \). Difference scheme satisfy the \( L^\ast \) stability. Formula (12) is established the mathematical model of motion blur image based on RDER. \( (I_x)_i,j \) and \( (I_y)_i,j \) are defined as follows:

\[
(I_x)_{i,j} = \frac{2(I_{i+1,j}^n - I_{i-1,j}^n) + I_{i+1,j+1}^n - I_{i+1,j-1}^n + I_{i-1,j+1}^n - I_{i-1,j-1}^n}{4}
\]

(13)

\[
(I_y)_{i,j} = \frac{2(I_{i,j+1}^n - I_{i,j-1}^n) + I_{i+1,j+1}^n - I_{i+1,j-1}^n + I_{i-1,j+1}^n - I_{i-1,j-1}^n}{4}
\]

(14)

**The Restoration Algorithm of Motion Blur Drop Quality Image**

The motion blur image restoration algorithm flow chart based on the theory of reaction diffusion is as follows Figure 2.
Experiments

(1) Objective evaluation index of image restoration effect ISNR. In order to evaluate objectively the effect of the image's recovery, we use the improved peak signal noise ratio (ISNR) to measure restoration effect.

\[
ISNR = 10 \log_{10} \frac{\sum_{i,j} [g(i,j) - f(i,j)]^2}{\sum_{i,j} [f(i,j) - \hat{f}(i,j)]^2}
\]  

where, \( g(i,j) \) is degraded image, \( f(i,j) \) is original image, \( \hat{f}(i,j) \) is restoration image. \( \sum_{i,j} \) denotes the summation of the whole image. If the value of the ISNR is the greater, the image restoration effect is better.

(2) Algorithm comparison experiments. Through the experiments, we can get subjective evaluation of the recovery effect comparing the our algorithm and RI-BM3D[7],BM3DDEB[7],IDD-BM3D[8], ForWaRD[9] algorithm. The Lena.jpg is tested images in the experiments. All numerical calculations are implemented in Windows 7 platform and Matlab R2012b. The experiment computer is the desktop computer of Intel(R) Pentium(R) Dual CUP E2220 2.40GHz. The point spread function (PSF) is the \( h(x, y) = \text{fspecial('motion',71,56)} \). The restoration results of the five kinds of restoration algorithm are shown in Figure 3:
(3) ISNR value comparison. Through the experiments, we can get objective evaluation of the recovery effect comparing the our algorithm and RI-BM3D, BM3DDEB, IDD-BM3D, ForWaRD algorithm. In five different point spread function (PSF), recovery experiment is did and the value of their ISNR is measured. The ISNR value is as shown in Figure 4.

Conclusions

Comparing figure 3 shows that image restoration effect of our algorithm is superior to restoration effect of RI-BM3D BM3DDEB, IDD-BM3D and ForWaRD. Comparing figure 4 shows that the ISNR values of our algorithm are higher than the ISNR values of other four recovery algorithms. Thus we can conclude: Motion blur image restoration algorithm based on the reaction diffusion theory compares with RI-BM3D, BM3DDEB, IDD-BM3D and ForWaRD recovery algorithm. Our algorithm in image restoration subjective and objective results has certain to improve.

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