Contour Error Decoupling Compensation for Non-circular Grinding
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Keywords: Non-circular grinding, Error compensation, Decoupling.

Abstract. An off-line decoupling compensation method for Noncircular Grinding contour error is presented in this paper. The contribution of the axis deviation to the non-circular contour error was analyzed by the projection of the axis deviation to the normal direction, the decoupling method of X_C axis independent compensation is given. The off-line compensation experiment shows that the contour error is suppressed and the decoupling method is effective.

Preface
Non circular profile grinding requires two axis linkage envelope to form target contour. Because of the radius of curvature, the location of grinding point, actual grinding rate, tool wear, tracking error, and many other processing parameters are constantly in changing. All of these lead to complex processing and difficult to determine the causes of error [1].
In factories, the grinding process can only be performed according to G code instructions in NC machine, and it is difficult for users to obtain real-time position feedback. Therefore, it is necessary to detect the actual contour error offline and use the information for off-line compensation.
Although there are many factors which cause the contour error, the fundamental incentive of the non-circle contour error is that the relative position relationship between the work piece and the grinding wheel has been changed caused by various factors [2-4]. Therefore, this paper analyzes the influence of each single axis deviation upon the contour error, and then studies the off-line error compensation method.

Motion Model in Noncircular Grinding
The grinding process of non-circular contour is the process that the grinding wheel revolves around the work piece and then the target contour is enveloped. In this process, the center track of the grinding wheel is the equidistant line of the contour curve [5,6]. As shown in Fig. 1, the center point O of the noncircular part is fixed to the origin of xoy coordinate system. While revolving around the point O, the grinding wheel reciprocates along line OO₁, which is the straight line connects the center of the wheel and the part. Currently the grinding wheel is tangent to the target contour at point A.
In fixed coordinate system with point O as non-circular contour origin, its vector expression is shown as follows:

\[ \vec{p} = re^{i\varphi} \]

Where \( \varphi \) and \( \gamma \) are both variables.
Then the unit normal vector of vector $\rho$ is:

$$n = e^{j(\theta + \varphi + \frac{\pi}{2})}$$

(2)

Where $\theta$ is the angle between the vector $\rho$ and its tangent.

Then the vector of the line which connects the two centers is shown as follow:

$$\overline{OO} = \rho(t) \pm R_w n(t) = r e^{j\theta} \pm R_w e^{j(\theta + \varphi + \frac{\pi}{2})}$$

(3)

$X$ is the distance between the center point $O_1$ of Grinding wheel and the center point $O$ of work piece; the included angle $C$ made by line $OO_1$ and axis $x$, shown as follow:

$$\begin{cases}
X = |\overline{OO}| = \sqrt{r^2 + R_w^2 + 2rR_w \cos(\theta + \frac{\pi}{2})} \\
C = \varphi - \cos^{-1}\left(\frac{r^2 + X^2 - R_w^2}{2rX}\right)
\end{cases}$$

(4)

The grinding wheel and the work piece are two-axis linkage, and the target contour is enveloped according to Eq. 4 to realize non-circular grinding. The machining model is suitable for grinding of non-circular work pieces such as internal and external cam, eccentric shaft and crankshaft etc.

**The Influence of Axis Deviation on Profile Error**

The error of moving position between the grinding wheel and the work piece will lead to the deviation of the actual grinding point, resulting in non-circular profile error. It can be seen from the Eq. 4 that there is a complex nonlinear relationship between the profile error and the two axis position deviation.

It is difficult to calculate $X$-$C$ compensation because of the large amount of computation. In view of the fact that the profile is continuously changed and deviations is tinier, it can be approximately considered that the states near the grinding point $A$ can be characterized by a points. The normal displacement caused by the $X$ axis motion is $\Delta X_{xn}$, and the normal displacement caused by the rotation of the axis is $\Delta C_{cn}$. To realize the ideal contour processing, the $X$-$C$ linkage relation should be satisfied with Eq. 5, as shown in figure 2.

$$\Delta X_{xn} = \Delta C_{cn}$$

(5)
Non Circular Profile Error Caused by Deviation of X Axis

Taking account the various factors, \( \Delta X_{\text{cn}} \) and \( \Delta C_{\text{cn}} \) are often inconsistent, "over cut" or "Insufficient cut" phenomenon will occur. That is, the profile of the negative error and positive error will occur.

The normal displacement error caused by X axis deviation \( \Delta X \) is \( \varepsilon_x \).

\[
\varepsilon_x = \Delta X \times \cos \alpha_A = \frac{\Delta X (X^2 + R_w^2 - \rho^2)}{2XR_w}
\]

(6)

Wherein, \( \alpha_A \) is the angle between the normal line of the point and the straight line which connects the two centers: \( \alpha_A = \tan^{-1} \frac{r \cos \theta}{r \sin \theta + R_w} \)

Non Circular Profile Error Caused by Feed Deviation of C Axis

The normal displacement error caused by C axis deviation \( \Delta C \) is \( \varepsilon_c \), as shown in figure 2.

\[
\varepsilon_c = X \Delta C \sin \alpha_A = X \sqrt{1-\left(\frac{X^2 + R_w^2 - \rho^2}{2XR_w}\right)^2} \Delta C
\]

(7)

Since the normal errors caused by two-axis displacement are both on the normal line, the two normal errors can be linear superimposed as:

\[
\varepsilon = \varepsilon_x + \varepsilon_c = \frac{\Delta X (X^2 + R_w^2 - \rho^2)}{2XR_w} + X \sqrt{1-\left(\frac{X^2 + R_w^2 - \rho^2}{2XR_w}\right)^2} \Delta C
\]

(8)

Figure 2. The profile normal errors caused by axis position deviation.

Taking the camshaft of code # 53006645 as an example, the influence facts of X-direction and C-direction constant deviations upon the cam profile error are calculated respectively. As shown in Figure 3 and Figure 4, which reflect the sensitivity of the uniaxial error to the profile error.
It can be seen that the normal error of the profile caused by $\Delta X$ is the largest at the base circle, and the contour error is directly expressed as $\Delta X$, and $\Delta C$ has no effect at the base circle instead. And the normal error caused by $\Delta X$ in the profile with the curvature change is sine-like, the most sensitive section is at the inflection point.

**Off-line Decoupling Method for Contour Error Compensation**

Three coordinate or cam detector is used for contour error detection of non-circular work piece. When measuring, the contact measuring head is spherical or plane probe, and it is tangent to the outline contact point of non-circular work piece. As shown in Figure 5, it can be seen that the measured contour error directly reflects the deviation of the actual contour of the point in the normal direction. This normal error $\delta$ can be thought to be caused by the X-C two axis deviations. Here is a discussion about how to achieve the two axis decomposition of the error $\delta$ by decoupling in $\triangle OO_1O_1$, as shown in figure 6.
Set the contour normal error $\delta$, the center of the grinding wheel is compensated to point $O'$ along the direction of the normal line. The $OO_1'$ vector is given as:

$$\overrightarrow{OO_1'} = \overrightarrow{OO_1} + \delta \overrightarrow{n}$$

Then:

$$|\overrightarrow{OO_1'}|^2 = |\overrightarrow{OO_1}|^2 + |\overrightarrow{OO_1'}|^2$$

In $\triangle OO_1O_1'$, the reciprocating compensation amount of the grinding wheel and the angle compensation amount of the cam rotation are decoupled as:

$$\begin{cases} \Delta X = |\overrightarrow{OO_1}|-|\overrightarrow{OO_1'}| \\ \Delta \Theta = \sin(\delta) \cos^{-1}\left(\frac{|\overrightarrow{OO_1}|^2 + |\overrightarrow{OO_1'}|^2 - \delta^2}{2|\overrightarrow{OO_1}|\overrightarrow{OO_1'}}\right) \end{cases}$$

It can be seen that the normal line is collinear with the center line $OO_1$ in the base circle section, where the cam compensation angle $\Delta \Theta$ is zero, which is consistent with the previous analysis.

**Error Compensation Experiment Offline**

Taking constant angular speed $1 \text{rad/s}$, grinding wheel radius $250 \text{mm}$ and grinding wheel line speed $60 \text{m/s}$, grinding experiments were carried out on the M131-CAM camshaft grinding experimental device. The result of grinding work piece inspection is shown in Figure 7a.

After the error data of NO5011380 flat cam is densified, the compensation amount in both directions of $X$ and $C$ is calculated by the Eq.11, and the theoretical G code is superimposed one by one to generate the new G Code for compensation. The results of grinding work piece inspection are shown in Figure 7a and Figure 7b respectively.

Experiments show that, in the case of NC smooth working condition, there are no major...
changes in the factors that produce the deviation, the grinding state is stable and the error tends to be stable. Using the error information obtained by the trial grinding, the data interpolation and densification are carried out, and the contour error is decoupled to realize the non-circular contour compensation control.

**Conclusion**

The influence of the two-axis deviation on the contour error is related to the rate of change of the radius of curvature. Where the curvature change is large, the C-direction deviation caused by the normal deviation of the profile is more sensitive; where curvature change small, contour error is directly reflected in the X-axis deviation. The uncorrected contour error is decoupled into X and C two independent compensation quantities, which can effectively compensate for the contour error and improve the grinding accuracy.

**Acknowledgements**

This research is funded by National Natural Science Foundation Project of China (Grant No. 51375056 and 51405026), the Beijing Municipal Education Commission Project (Grant No. KM201711232001), and the Beijing Municipal Science and Technology Project (Grant No. Z161100001516002).

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