A Cooperative Rendezvous Strategy for Co-orbital Spacecraft

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Abstract. In this paper, a new type of time-fixed cooperative rendezvous between two spacecraft in the same circular orbit is proposed and analytical solutions are obtained in consideration of two body model. The analytical solution is used as initial value in the optimization model which solves the rendezvous problem with high precision. The simulation which compares results of two body model and $J_2$ perturbation model shows that this new cooperative rendezvous strategy named Hohmann-Ellipse Cooperative Maneuver (HECM) has great advantage over traditional active-passive rendezvous methods. Given a fixed time, the velocity of cooperative rendezvous can decrease by 30% at most when compared with non-cooperative rendezvous.

Introduction

The problem of spacecraft orbital transfers has been studied extensively for a long time. Most of the reported studies for spacecraft rendezvous missions focus on active-passive (non-cooperative) rendezvous, there are also works on active-active (cooperative) rendezvous. The optimal cooperative rendezvous considering minimum fuel cost between neighboring or coplanar circular orbits have been studied in reference [1,2]. The optimal terminal maneuver of two active satellites which perform cooperative impulsive rendezvous has been studied in reference [3]. Time-fixed cooperative rendezvous which uses primer vector theory has been researched in reference [4]. Multi-spacecraft refueling with cooperative maneuvers based on two body model and $J_2$ perturbation model have been studied in reference [5] and [6] respectively. Two time-fixed cooperative maneuvers have been proposed and utility for Peer-to-Peer (P2P) has been demonstrated in reference [7]. Egalitarian P2P and optimal low thrust P2P have been studied in reference [8] and [9]. Long-range cooperative rendezvous between two coplanar spacecraft on LEO has been theoretically analyzed in reference [10]. Based on extant research, however, the co-orbital rendezvous problem has seldom been studied specifically and when it comes to cooperative rendezvous, the number of literature to be referred to becomes smaller.

In this paper, a new cooperative maneuver strategy named Hohmann-Ellipse cooperative maneuver (HECM) is theoretically studied. The aim is to solve the problem that two spacecraft in the same orbit accomplish rendezvous a fixed time. The rest of this paper is organized as follows. Section 2 mainly introduces the application background and theoretical formulation of Hohmann-Ellipse cooperative maneuver. Section 3 presents the optimization models which contains two body and $J_2$ perturbation dynamic model of HECM. Numerical examples and conclusion are presented in Section 4 and Section 5.

Hohmann-Ellipse Cooperative Maneuver

Mission Scenario

The background is that a spacecraft (the chaser) departs from the Space Station to refuel another spacecraft (the target) in the same orbit and returns to the Space Station after fueling. Fig 1 and Fig 2 show this mission in detail.
The process is listed as follows:

1. The target spacecraft performs Hohmann transfers to complete the cooperative rendezvous. The first impulse of target spacecraft at time $t_0$ is $\Delta v_{t_1}$. After staying on the Hohmann transfer orbit for time $t_{Hohmann}$, the second impulse $\Delta v_{t_2}$ is applied by the target to lift the semi-major axis of the initial circle orbit $r_0$ to $r'_0$ of the target circle orbit.

2. The chaser applies the first impulse $\Delta v_{c_1}$ to transfer to transfer orbit 1 at time $t_0$. After time $t_{c_1}$, the second impulse $\Delta v_{c_2}$ is applied to go to the phasing orbit. After staying on the phasing orbit for $t_{c_2}$ it applies the third impulse $\Delta v_{c_3}$ and enters the transfer orbit 2. It takes $t_{c_3}$ before the chasing spacecraft applying the fourth impulse $\Delta v_{c_4}$. Finally the chaser arrives at the target orbit whose semi-major axis is $r'_0$ where the first rendezvous between the target and the chaser is accomplished.

3. After adding fuel to the target spacecraft, the chasing spacecraft should also apply four impulses to transfer from target circle orbit $r'_0$ to initial circle orbit $r_0$ (the Space Station orbit). The on-orbit service lasts for $T_{serv}$ thereafter the fifth impulse $\Delta v_{c_5}$ is applied and the chaser enters returning transfer orbit 1. After staying for time $t_{c_4}$, the chaser transfers to returning phasing orbit with the help of the sixth impulse $\Delta v_{c_6}, \Delta v_{c_7}$ is applied after phasing for time $t_{c_5}$ and the chaser transfers to returning transfer orbit 2. It takes time $t_{c_6}$ on returning transfer orbit 2 and the last impulse $\Delta v_{c_8}$ helps the chaser return to initial orbit and accomplish rendezvous with the Space Station.

4. The target spacecraft returns to original position by means of Hohmann transfer.

**HECM Formulation**

The semi-major axis of initial orbit is $r_0$ and the semi-major axis of target orbit is $r'_0$. Rendezvous between two spacecraft happens on the target orbit. There exists an initial phase angle $\theta_H$ between the target and the chaser on the initial orbit. The time permitted to accomplishing rendezvous is $T_G$ and time permitted to coming back to the Space Station is $T_B$. The duration of refueling lasts for time $T_{serv}$. In order to transfer from $r_0$ to $r'_0$ by means of Hohmann Transfer, the velocity and time needed are shown as follows:

\[
\Delta V_{t_1} = \frac{\mu}{r_0} \sqrt{\frac{2r'_0}{r'_0 + r_0} - 1}
\]

\[
\Delta V_{t_2} = \frac{\mu}{r_0} (1 - \frac{2r'_0}{r'_0 + r_0})
\]

\[
T_{Hohmann} = \pi \frac{(r'_0 + r_0)^3}{8 \mu}
\]
where $\Delta V_{t1}$ and $\Delta V_{t2}$ stand for the first and second impulse applied by the target respectively and it takes $T_{\text{Hohmann}}$ to finish Hohmann Transfer. After arriving at the target orbit, the target spacecraft will stay on it for time of $T_G - T_{\text{Hohmann}}$

Supposing the semi-major axis of the phasing orbit is $r_1$, four impulses given by the chasing spacecraft are

\[
\Delta V_{c1} = \sqrt{\frac{\mu}{r_0}} \left( \sqrt{\frac{2r_1}{r_0 + r_1}} - 1 \right)
\]
\[
\Delta V_{c2} = \sqrt{\frac{\mu}{r_1}} \left( 1 - \sqrt{\frac{2r_0}{r_0' + r_1}} \right)
\]
\[
\Delta V_{c3} = \sqrt{\frac{\mu}{r_1}} \left( \sqrt{\frac{2r_1}{r_0' + r_1}} - 1 \right)
\]
\[
\Delta V_{c4} = \sqrt{\frac{\mu}{r_0'}} \left( 1 - \sqrt{\frac{2r_1}{r_0' + r_1}} \right)
\]

The duration of each impulse $\Delta t_{c1}$, $\Delta t_{c2}$, $\Delta t_{c3}$ are

\[
T_{\text{tran}} = \pi \sqrt{\frac{(r_1 + r_0)^3}{8\mu}}
\]
\[
T_S = T_G - T_{\text{tran1}} - T_{\text{tran2}}
\]
\[
T_{\text{tran2}} = \pi \sqrt{\frac{(r_1 + r_0')^3}{8\mu}}
\]

$T_S$ represents the time staying on the phasing orbit which must satisfy the equality constrains

\[
2\pi T_S + 2\pi = \pi + 2\pi T_G - T_{\text{Hohmann}} + \theta_H
\]

The left of the equation stands for the variation of chaser’s phase angle which is equal to the sum of the variation of target’s phase angle with initial phase angle $\theta_H$.

After accomplishing rendezvous with the target spacecraft, the phase angle between the chaser spacecraft and the Space Station is $\theta_S$. It takes $T_{\text{serv}}$ to finish on-orbit refueling and then the chaser spacecraft returns to the Space Station. During the period of returning, the Space Station becomes the new target and the time allowed to come back is $T_B$. $\theta_S$ is given by

\[
\theta_S = \theta_H + \pi + 2\pi \frac{T_G - T_{\text{Hohmann}} + T_{\text{serv}}}{\sqrt{\frac{r_0^3}{\mu}}} - \frac{T_G + T_{\text{serv}}}{\sqrt{\frac{r_0^3}{\mu}}}
\]

Eq 5 means that the change of chasing spacecraft’s phase angle minus the change of Space Station’s phase angle is equal to $\theta_S$. Provided that the semi-major axis of chaser’s phasing orbit is $r_1$, velocity is obtained by
\[ \Delta V_{c5} = \frac{\mu}{r_0'} \left( \sqrt{\frac{2r_2}{r_0' + r_2}} - 1 \right) \]
\[ \Delta V_{c6} = \frac{\mu}{r_2} \left( 1 - \sqrt{\frac{2r_0'}{r_0' + r_2}} \right) \]
\[ \Delta V_{c7} = \frac{\mu}{r_0} \left( \sqrt{\frac{2r_0'}{r_0 + r_1}} - 1 \right) \]
\[ \Delta V_{c8} = \frac{\mu}{r_0} \left( 1 - \sqrt{\frac{2r_2}{r_0 + r_2}} \right) \]  \hspace{1cm} (6)

The duration of each impulse \( \Delta t_{c4}, \Delta t_{c5}, \Delta t_{c6} \) are
\[ T'_{\text{tran1}} = \pi \sqrt{\frac{(r_2 + r_0')^3}{8\mu}} \]
\[ T'_{S} = T_B - T'_{\text{tran1}} - T'_{\text{tran2}} \]  \hspace{1cm} (7)
\[ T'_{\text{tran2}} = \pi \sqrt{\frac{(r_2 + r_0)^3}{8\mu}} \]

\( T'_S \) must be constrained to the equation below
\[ 2\pi \frac{T_B}{\sqrt[3]{r_0'^3}} = 2\pi + 2\pi \frac{T'_S}{\sqrt[3]{r_2^3}} + \theta_S \]  \hspace{1cm} (8)

**Optimization Model**

The optimization model can be divided into two parts. The first part is two body dynamic optimization model and the other part is the one considering \( J_2 \) perturbation. Differences of the two parts are only reflected in design variables, objective functions. Objective function

**Design Variables**

Two body optimization model contains 3 design variables which are \( r_0', r_1 \) and \( r_2 \) respectively. Design variables are given by \( X_1, X_2 \) and \( X_3 \) in \( J_2 \) perturbation optimization model. They are described as below.

\[
X_1 = \begin{cases} 
    x_1 = (\Delta v_{c1}, \Delta v_{c2}) \\
    x_2 = (\Delta v_{c1}, \Delta v_{c2}, \Delta v_{c3}, \Delta v_{c4}) \\
    x_3 = (\Delta v_{c5}, \Delta v_{c6}, \Delta v_{c7}, \Delta v_{c8})
\end{cases}
\]

\[
X_2 = \begin{cases} 
    x_4 = (r_0', r_1, r_2)
\end{cases}
\]

\[
X_3 = \begin{cases} 
    x_5 = (T_{\text{Hohmann}}, T_S, T'_{\text{tran1}}, T'_{\text{tran2}}) \\
    x_6 = (T_S, T'_{\text{tran1}}, T'_{\text{tran2}})
\end{cases}
\]  \hspace{1cm} (9)
where $x_1$ is the velocity increment of the target spacecraft, $x_2$ is the set of velocity increment of the chasing spacecraft to realize rendezvous with the target spacecraft, $x_3$ is the set of velocity increment of the chasing spacecraft to return to the Space Station, $x_4$ is the set of semi-major axis, $x_5$ is the set of transfer time which contains target’s and chaser’s transfer time in refueling orbit, $x_6$ is the set of transfer time which only contains chaser’s transfer time in returning orbit.

**Objective Function**

In the rendezvous mission, the chaser and target spacecraft both execute orbital transfers which consumes some propellant. Two different situations are considered here. In the first situation, the propellant is sufficient to finish the rendezvous. However, the amount of propellant is limited in another situation so that the velocity increment cannot exceed the maximal velocity increment $v_{max}$.

The objective is to minimize the total velocity increment.

$$\min f(x) = \text{sum}(|\Delta V_t| + |\Delta V_c|)$$

(10)

where $|\Delta V_t|$ stands for the total impulse applied by the target spacecraft, $|\Delta V_c|$ stands for the total impulse applied by the chasing spacecraft.

**Constraints**

For the whole mission, the total mission time is limited. In this cooperative rendezvous, $T_G$, $T_B$ and $T_{serv}$ are all determined.

$$[T_{total} - (T_G + T_{serv} + T_B)] \rightarrow 0$$

(11)

where $T_G$ is the total transfer time for refueling orbit, $T_B$ is the total transfer time for returning orbit, $T_{serv}$ is the on-orbit servicing time, $T_{total}$ is the entire time of the whole mission.

**Optimization Process**

Firstly, calculate the result of two body model. By means of differential evolution algorithm, the two equations (Eq 4 and Eq 5) can be satisfied and minimized objectives are obtained. Thus the optimal velocity increment and transfer time are derived.

Secondly, initialize the $J_2$ perturbation model. The result of velocity increment and transfer time are substituted into the $J_2$ perturbation model as the initial value.

Finally, calculate the result of $J_2$ perturbation model. By means of sequential quadratic programming (SQP), the time constraint and propellant constraint are satisfied and minimized objectives are obtained. Thus the optimal velocity increment and transfer time under consideration of $J_2$ perturbation are derived.

**Numerical Examples and Analyses**

**Problem Configuration**

There are two spacecraft on an LEO orbit whose semi-major axis is 7000km. In this chapter, both cooperative and non-cooperative strategies are calculated and compared to each other. The total rendezvous time $T_{total}$ is 5 days, with $T_G = 1$ day, $T_{serv} = 2$ days and $T_B = 2$ days. The constants used in trajectory calculations are earth radius $R_e = 6378.137km$, $J_2 = 1.082626 \times 10^{-3}$ and $\mu_e = 3.986004418 \times 10^{14}$. 
Results of Two Body Model

In this case, the result derived from two body model is researched. The transfer time is 72h which includes $T_G$ and $T_B$.

![Figure 3. Comparison of velocity increment with different phase angle.](image)

<table>
<thead>
<tr>
<th>Rendezvous Type</th>
<th>$\theta_H$ (deg)</th>
<th>$r'_0 - r_0$ (km)</th>
<th>$\Delta V$ (m/s)</th>
<th>$\Delta T_G + \Delta T_B$ (h)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cooperative</td>
<td>30</td>
<td>6.62</td>
<td>36.37</td>
<td>72</td>
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<td>60</td>
<td>13.26</td>
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</tr>
<tr>
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<td>120</td>
<td>26.58</td>
<td>145.19</td>
<td>72</td>
</tr>
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<td>150</td>
<td>33.27</td>
<td>181.37</td>
<td>72</td>
</tr>
<tr>
<td></td>
<td>180</td>
<td>39.97</td>
<td>217.497</td>
<td>72</td>
</tr>
<tr>
<td>Non-cooperative</td>
<td>30</td>
<td>-</td>
<td>43.61</td>
<td>72</td>
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<td></td>
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<td>72</td>
</tr>
<tr>
<td></td>
<td>90</td>
<td></td>
<td>130.57</td>
<td>72</td>
</tr>
<tr>
<td></td>
<td>120</td>
<td></td>
<td>173.92</td>
<td>72</td>
</tr>
<tr>
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<td>150</td>
<td></td>
<td>217.19</td>
<td>72</td>
</tr>
<tr>
<td></td>
<td>180</td>
<td></td>
<td>260.38</td>
<td>72</td>
</tr>
</tbody>
</table>

From Table 1 and Fig 4 it can be concluded that the velocity increment of cooperative rendezvous is smaller than that of non-cooperative rendezvous in two body model. Furthermore, the rising height $r'_0 - r_0$ increases with the increase of phase angle $\theta_H$.

Results of $J_2$ Perturbation Model

The optimal result considering $J_2$ perturbation is shown in Table 2.

<table>
<thead>
<tr>
<th>Rendezvous Type</th>
<th>$\Delta \theta_H$ (deg)</th>
<th>$\Delta V$ (m/s)</th>
<th>$\Delta T_G + \Delta T_B$ (h)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cooperative</td>
<td>30</td>
<td>39.37</td>
<td>72</td>
</tr>
<tr>
<td></td>
<td>60</td>
<td>78.82</td>
<td>72</td>
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<tr>
<td></td>
<td>90</td>
<td>123.13</td>
<td>72</td>
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<tr>
<td></td>
<td>120</td>
<td>179.64</td>
<td>72</td>
</tr>
<tr>
<td></td>
<td>150</td>
<td>220.21</td>
<td>72</td>
</tr>
<tr>
<td></td>
<td>180</td>
<td>264.75</td>
<td>72</td>
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<tr>
<td>Non-cooperative</td>
<td>30</td>
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<td>72</td>
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<td>90</td>
<td>124.94</td>
<td>72</td>
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<td></td>
<td>120</td>
<td>176.23</td>
<td>72</td>
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<tr>
<td></td>
<td>150</td>
<td>220.87</td>
<td>72</td>
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<tr>
<td></td>
<td>180</td>
<td>257.46</td>
<td>72</td>
</tr>
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</table>
The result shows that the velocity increment of HECM is very similar to the result of non-cooperative maneuver. The reason why results of $J_2$ perturbation model are different from what are obtained from two body model is that extra normal velocity increment in $J_2$ perturbation model is reduced to a large scale by setting suitable deviations of RAAN. According to Gauss perturbation equation, the right ascension of ascending node will change during co-planar rendezvous process, which leads to existence of normal velocity. In order to cut down the unnecessary velocity increment, the initial orbital RAAN of the chaser spacecraft and the target spacecraft are a little different. If the total rendezvous time is long enough which means the overall velocity increment is not very large, the normal velocity being cut down plays an important role in the total velocity increment. Thus, the velocity increment cost by non-cooperative maneuver is smaller than cooperative maneuver.

In order to prove the validity and advantage of HECM in $J_2$ perturbation model, the total maneuver time is shortened to 18 hours where $T_G = 6$ h, $T_B = 12$ h and $T_{serv}$ remains the same. The result are shown in Table 3 and Fig 5.

Table 3. Optimal solution with considering $J_2$ perturbation in 18h.

<table>
<thead>
<tr>
<th>Rendezvous Type</th>
<th>$\Delta \theta_H$ (deg)</th>
<th>$\Delta V$ (m/s)</th>
<th>$\Delta T_G + \Delta T_B$ (h)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cooperative</td>
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<td></td>
<td></td>
</tr>
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<td>142.71</td>
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<tr>
<td>60</td>
<td>290.06</td>
<td>18</td>
<td></td>
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<tr>
<td>90</td>
<td>439.96</td>
<td>18</td>
<td></td>
</tr>
<tr>
<td>120</td>
<td>585.24</td>
<td>18</td>
<td></td>
</tr>
<tr>
<td>150</td>
<td>682.64</td>
<td>18</td>
<td></td>
</tr>
<tr>
<td>180</td>
<td>855.32</td>
<td>18</td>
<td></td>
</tr>
<tr>
<td>Non-cooperative</td>
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<td></td>
</tr>
<tr>
<td>30</td>
<td>170.37</td>
<td>18</td>
<td></td>
</tr>
<tr>
<td>60</td>
<td>387.04</td>
<td>18</td>
<td></td>
</tr>
<tr>
<td>90</td>
<td>550.34</td>
<td>18</td>
<td></td>
</tr>
<tr>
<td>120</td>
<td>693.43</td>
<td>18</td>
<td></td>
</tr>
<tr>
<td>150</td>
<td>874.25</td>
<td>18</td>
<td></td>
</tr>
<tr>
<td>180</td>
<td>1056.04</td>
<td>18</td>
<td></td>
</tr>
</tbody>
</table>

Figure 4. Comparison of the percentage of velocity decreased with different phase angle.

From Fig 5 we can see, when the total maneuver time is reduced to 18 hours, the optimal velocity increment of HECM is better than non-cooperative results at any initial phase angles. It is because that the proportion of normal velocity in the total velocity increment is decreased when time is shorter.
Therefore, the advantage built up by modifying RAAN deviation for non-cooperative maneuver is reduced and the remarkable advantage offered by HECM appears.

It can be concluded that HECM also has more advantages than traditional non-cooperative maneuvers strategies using the dynamic model with J₂ perturbation.

**Conclusions**

A new cooperative maneuver named Hohmann-Ellipse cooperative maneuver considering the two body as well as J₂ perturbation dynamic model is studied in this paper. Not only minimum velocity increment but minimum propellant cost is obtained by means of optimization model.

Numerical examples have been used to testify the effectiveness of the proposed cooperative strategy and a major conclusion is drawn. HECM can reduce the total velocity increment to a large scale in both two body and J₂ perturbation dynamic models. Results given by HECM is even better than results which has modified unnecessary normal velocities and the advantage is more remarkable when the rendezvous is accomplished in a short time.

All in all, the proposed cooperative strategy in this study can apply to co-orbital refueling and other on-orbit service missions. The cooperative rendezvous strategy has advantages over the traditional non-cooperative strategy.

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**References**


