Research on Function Determination Method of Aviation Equipment
Spare Part Demands under Small Samples

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Abstract. Acquaintance of fault distribution rules of aviation material spare parts is one of the important measures to conduct scientific management of aviation materials, while data collection is the basis to determine the aviation material fault function. The paper proposes suggestions for collection methods and collection opportunities of fault data of aviation material spare parts. Aiming at the actual situation that the same group of data could easily satisfy multiple distributions under small samples and based on a judgment model with combination of improved fuzzy judgment and improved AHP method, a rational distribution function was obtained with expert experience as reference. Through the research on determination of the spare part demand function in the paper, a foundation was laid for supply, reserve and computation of aviation materials. In this way, aviation material demands could be predicted accurate, and economy of aviation equipment guarantee could be enhanced.

Introduction

During analysis of spare part demands, spare part demanding courses shall be analyzed at first, so that the demand distribution function could be determined. Acquaintance of distribution rules of aviation material spare parts is one of the important measures to conduct scientific management of aviation materials. Aiming at different specific research objects, people can select and use different distribution functions. For example, normal distribution, gamma distribution, Weibull distribution and Poisson distribution functions have been applied widely. Of course, each distribution function has advantages and disadvantages, so it is only suitable for specific cases. For example, negative exponential distribution is suitable for description of large-scale demands and retail demands; normal distribution is suitable for large-scale demands; Poisson distribution is applicable to scattered demand distribution.

Spare parts have many random factors such as service life, use frequency and intensity, so consumption situations of spare parts could hardly be predicted. However, according to mathematical statistics principles, inherent rules involving random phenomena could be revealed. Through processing treatment and statistical inference of mass data consumed actually by various spare parts, we can find the probability distribution of the consumption. During determination of demand function parameters, the corresponding statistical examination shall be conducted. Aiming at the size of sample data, the data could be classified into large sample data and small sample data. However, both the large sample data and small sample data take numerous detailed and accurate data as the basis. Under large samples, the spare part demands have common methods, such as $\chi^2$—fitting inspection method and Kolmogorov fitting inspection. The paper mainly researches the function determination method of spare part demands of aviation equipment under small sample conditions.

Collection of Data

Determination of a spare part demand function requires accumulation of numerous data. However, the most troublesome problems in demand function determination include “lack of data” or “lack of
complete and accurate data”. Without the basis of sufficient data, it would be impossible to determine an accurate demand function.

In order to solve the problem, a LSAR (logistics support analysis record) database shall be established according to GJB 1371 Logistics Support Analysis and GJB3837 Logistics Support Analysis Record of Equipment. In China, GJB1371 and GJB3837 were released very early, but their application is not popularized yet. Hence, the major problem at present is to find how to acquire and accumulate data related to product reliability and fault frequency when the LSAR database is not established or the LSAR database is very imperfect, which is equivalent to data source development. According to the investigation and survey, domestic and foreign spare part workers acquire data mainly from the following sources: (1) historical experience; (2) engineering research and development data; (3) manufacture and quality management data; (4) plant (institution) acceptance check data; (5) environmental authentication and outfield acceptance check data; (6) outfield use data; (7) system data of user fault reports; (8) requirements and related records for guarantee (warranty) period; (9) repair department reports, etc. According to the historical sequence, above 9 data sources involve equipment demonstration, scheme, and initial design, models in plants, sample machines, design finalization, production, acceptance check, and service life of outfield use. It is assumed that the data obtained at each stage is recorded according to LSAR records in GJB3837, then the data could be collected and sorted, so as to guide and support determination of a demand function.

Kolmogorov-Smirnov Two Subsample Inspection

In statistics, Kolmogorov-Smirnov is based on a cumulative distribution function and used to check whether two empirical distributions are different and whether an empirical distribution is different from another ideal distribution. It is assumed that there are two parent bodies with continuous distribution functions $F_1(x)$ and $F_2(y)$. Now, two independent subsamples $\xi_1, \ldots, \xi_n$ and $\eta_1, \ldots, \eta_n$ are extracted from them respectively. The original hypothesis needs to be checked now.

$$H_0: F_1(x) = F_2(x), -\infty < x < \infty$$

The empirical distribution function of two subsamples as follows:

$$F_{n_1}(x) = \begin{cases} 0, & \text{when } x < x_{(1)} \\ \frac{n_1(x)}{n_1}, & \text{when } x_{(j)} \leq x < x_{(j+1)}, j = 1, \ldots, n_1 \\ 1, & \text{when } x \geq x_{(n_1)} \end{cases}$$

$$F_{n_2}(y) = \begin{cases} 0, & \text{when } y < y_{(1)} \\ \frac{n_2(y)}{n_2}, & \text{when } y_{(l)} \leq y < y_{(l+1)}, l = 1, \ldots, n_2 \\ 1, & \text{when } y \geq y_{(n_2)} \end{cases}$$

Are used to construct the statistic amount:

$$D_{n_1n_2} = \sup_x |F_{n_1}(x) - F_{n_2}(x)|, -\infty < x < \infty$$

Theorem 1: When the sample sizes $n_1$ and $n_2$ tend to $\infty$, the statistic amount $D_{n_1n_2} = \sup_x |F_{n_1}(x) - F_{n_2}(x)|$ has a limit distribution function:
\[
\begin{align*}
P \left\{ \frac{n_1 n_2}{n_1 + n_2} D_{n_1 n_2} < \lambda \right\} & \to K(\lambda) \\
&= \sum_{j=1}^{\infty} (-1)^j \exp(-2 j^2 \lambda^2), \text{ when } \lambda > 0 \\
&= 0, \text{ when } \lambda \leq 0
\end{align*}
\]

Solution steps:
The hypothesis is checked by the following steps:
① Empirical distribution functions \( F_{n_1}(x) \) and \( F_{n_2}(y) \) are computed.

② The maximum value \( \sup_x \left| F_{n_1}(x) - F_{n_2}(y) \right| \) is computed and found.

③ The level of significance \( \alpha \) was given and checked out from a critical value table of Kolmogorov inspection.

Critical value \( D_{n_1 n_2, \alpha} \) of \( P \left\{ \frac{n_1 n_2}{n_1 + n_2} D_{n_1 n_2} < \lambda \right\} = \alpha \)

④ If \( \sup_x \left| F_{n_1}(x) - F_{n_2}(y) \right| > D_{n_1 n_2, \alpha} \) is worked out by ③, the original hypothesis could be rejected; the hypothesis will be accepted if \( \sup_x \left| F_{n_1}(x) - F_{n_2}(y) \right| < D_{n_1 n_2, \alpha} \). Hence, it is deemed that the empirical distribution is equal to ideal distribution.

**Problems and Solutions of Small Sample Inspection**

Small sample inspection is different from large sample inspection which could obtain a unique and explicit distribution hypothesis. Due to the small sample size, the same group of data may satisfy multiple distributions at the same time during data fitting. Hence, it is necessary to adopt other analysis processing methods to screen the results. As for rarely used spare parts, due to the small demand amount, Poisson distribution is often used to describe the demanding courses, and classification attributes and service life characteristics of spare parts are often neglected. Strictly speaking, it is not scientific. Under the small spare part receiving data size, equipment management workers may believe that it is more suitable to adopt other distribution functions rather than non-Poisson distribution to describe demanding courses of spare parts. Scientifically speaking, during determination of the demand distribution function, it is necessary to consider service life characteristics of spare parts and make full use of knowledge and experience of equipment management workers so as to make decision making more objective and scientific. Hence, the paper used a judgment model with combination of improved fuzzy judgment and improved AHP method. Through expert score making, experience of equipment management workers is fully listened and referred to; the complexity degree, inspection errors and other factors of the distribution function are taken as the judgment basis; finally, the demand distribution of spare parts could be determined. The algorithm has the following processes:
Small Samples

Data

K-S testing

Is there only one distribution that can be tested?

Y

N

the judgment model with combination of the improved fuzzy judgment and improved AHP method

Result

Figure 1. Processes of distribution inspection under small samples.

Next, the judgment model with combination of the improved fuzzy judgment and improved AHP method will be used to describe specific solution steps of spare part consumption distribution:

1. Determination of standby selective set

Statistical inspection is conducted to small sample data. It is assumed that \( n \) probability models pass the inspection. \( A_1, A_2, \ldots, A_n \) denote specific models, so the judgment objects constitute a standby selective set \( A = (A_1, A_2, \ldots, A_n) \).

2. Determination of factor set and judgment set

The following three influential factors are taken into account: it is assumed that \( u_1 \) denotes the complexity degree of the distribution function formula of spare part consumption; \( u_2 \) denotes the size of error of the distribution function obtained after fitting; \( u_3 \) denotes the conformity degree between actual consumption rules of spare parts and the selected distribution function (determined by voting of several experienced equipment management workers). Hence, the influential factor set can be represented by \( U = \{u_1, u_2, u_3\} \). Influential factors can be divided into different grades, so a judgment set could be established: \( V = \{v_1, v_2, v_3, v_4, v_5\} \). The factor set and judgment set could be obtained, as shown in Table 1.

<table>
<thead>
<tr>
<th>Factor set ( U )</th>
<th>Judgment set ( V )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>High</td>
</tr>
<tr>
<td>( u_1 ) complexity degree</td>
<td>( C_{11} )</td>
</tr>
<tr>
<td>( u_2 ) error degree</td>
<td>( C_{21} )</td>
</tr>
<tr>
<td>( u_3 ) conformity with actual situation</td>
<td>( C_{31} )</td>
</tr>
</tbody>
</table>

3. Establishment of single-factor judgment matrix

Each member in the expert group evaluates the judgment object and expresses personal evaluation in the form of score marking or voting. It is assumed that there are 7 members in the expert group. They make scores by the inspected models \( A_1 \) and \( A_2 \) respectively. As for \( A_i \), 4 members deem its complexity degree to be high; 2 members deem it to be relatively high; 1 member deems it to be common. Evaluations of other factors are similar. Voting results are filled into Table 1.

In Table 1, \( C_{ij} (i=1,2,3; j=1,2,3,4,5) \) is the number of votes which deem the \( i \)-th factor \( u_i (i=1,2,3) \) to have the \( j \)-th grade \( v_j (j=1,2,3,4,5) \). It is set that:
\[ r_j = \frac{C_u}{\sum_{j=1}^{3} C_{ij}} (i = 1, 2, 3) \]

Hence, the single-factor judgment matrix is obtained as follows:
\[ R = \begin{bmatrix} r_{11} & r_{12} & r_{13} & r_{14} & r_{15} \\ r_{21} & r_{22} & r_{23} & r_{24} & r_{25} \\ r_{31} & r_{32} & r_{33} & r_{34} & r_{35} \end{bmatrix} \]

4) Comprehensive judgment

According to scores made by the expert group to the factor set of each distribution function, the consistency improvement algorithm based on interval number optimization of judgment matrix is used to analyze and process score results. Finally, the weight \( \alpha = (\omega_1, \omega_2, \omega_3) \) related to the factor set \( U \) is obtained. Comprehensive judgment results are as follows:
\[ B = \alpha \circ R = (b_1, b_2, b_3, b_4, b_5) \]

Finally, the distribution with the highest rate of “high” and “relatively high” is selected from the results. It could be deemed that the hypothesis distribution is the most suitable probability function. The conformity degrees between its complexity degree and error degree and the actual situations satisfy the requirements.

**Analysis of Calculating Examples**

Historical data of a spare part in an aviation equipment aviation-material storehouse in recent 7 years was counted and sorted. With 0.5 year as a time interval, 13 pieces of data (10, 11, 6, 15, 8, 6, 9, 7, 14, 15, 17, 11, 13) were obtained. Inspection hypotheses of normal distribution, Poisson distribution and exponential distribution were conducted to the data. Statistical software SPSS was used for data fitting. Fitting results are shown in the following table.

<table>
<thead>
<tr>
<th>Sample size</th>
<th>N</th>
<th>13</th>
</tr>
</thead>
<tbody>
<tr>
<td>Normal parameters ( a, b )</td>
<td>Mean</td>
<td>10.9231</td>
</tr>
<tr>
<td></td>
<td>Standard deviation</td>
<td>3.66200</td>
</tr>
<tr>
<td></td>
<td>Absolute value</td>
<td>.107</td>
</tr>
<tr>
<td>Extreme difference</td>
<td>Positive</td>
<td>.107</td>
</tr>
<tr>
<td></td>
<td>Negative</td>
<td>-.107</td>
</tr>
<tr>
<td></td>
<td>Kolmogorov-Smirnov Z</td>
<td>.387</td>
</tr>
<tr>
<td>Inspection result</td>
<td>Approximation significance (two sides)</td>
<td>.998</td>
</tr>
<tr>
<td></td>
<td>Accurate significance (two sides)</td>
<td>.994</td>
</tr>
<tr>
<td></td>
<td>Point probability</td>
<td>.000</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Sample size</th>
<th>N</th>
<th>13</th>
</tr>
</thead>
<tbody>
<tr>
<td>Poisson parameters ( a, b )</td>
<td>Mean</td>
<td>10.9231</td>
</tr>
<tr>
<td></td>
<td>Absolute value</td>
<td>.096</td>
</tr>
<tr>
<td>Extreme different</td>
<td>Positive</td>
<td>.083</td>
</tr>
<tr>
<td></td>
<td>Negative</td>
<td>-.096</td>
</tr>
<tr>
<td></td>
<td>Kolmogorov-Smirnov Z</td>
<td>.346</td>
</tr>
<tr>
<td>Inspection result</td>
<td>Approximation significance (two sides)</td>
<td>1.000</td>
</tr>
<tr>
<td></td>
<td>Accurate significance (two sides)</td>
<td>1.000</td>
</tr>
<tr>
<td></td>
<td>Point probability</td>
<td>.000</td>
</tr>
</tbody>
</table>
If the accurate significance is larger than 0.05, the inspection hypothesis is deemed to be satisfied. If it is smaller than 0.05, the hypothesis is deemed to be rejected. According to the data obtained in statistical table, we can find data of the group satisfies Poisson distribution and normal distribution at the same time, but rejects the exponential distribution. Hence, screening of the distribution function was continued according to inspection processes in Fig.1. 7 experts were selected to constitute a judgment group. The group judged whether the spare part was suitable to Poisson distribution or normal distribution. They made scores according to judgment factors. Fuzzy judgment matrixes of Poisson distribution and normal distribution were obtained respectively, as follows.

\[
R_{\text{normal}} = \begin{bmatrix}
0.57 & 0.29 & 0.14 & 0 & 0 \\
0.86 & 0.14 & 0 & 0 & 0 \\
0 & 0 & 0.71 & 0.14 & 0.14
\end{bmatrix}
\]

\[
R_{\text{Poisson}} = \begin{bmatrix}
0 & 0.86 & 0.14 & 0 & 0 \\
0.57 & 0.43 & 0 & 0 & 0 \\
0.29 & 0.29 & 0.14 & 0.14 & 0.14
\end{bmatrix}
\]

Based on the AHP algorithm of consistency improvement of judgment matrix interval number optimization, the weight of factor set was given through experts’ score making:

\[\alpha = (0.25, 0.45, 0.30)\]

The \(M(\wedge, \otimes)\) method established in previous section was used to obtain:

\[
B_{\text{normal}} = \alpha \circ R_{\text{normal}} = (0.45, 0.3, 0.14, 0.14, 0.14)
\]

\[
B_{\text{Poisson}} = \alpha \circ R_{\text{Poisson}} = (0.14, 0.43, 0.29, 0.14, 0.14)
\]

Based on the results, we can clearly find that: the probability of using normal distribution to describe “high” and “relatively high” of the hypothesis distribution is 75%, which is higher than the probability 57% of Poisson distribution. Hence, we can find that it is suitable to describe the hypothesis distribution using normal distribution.

### Summarization

Acquaintance of fault distribution rules of aviation material spare parts is one of the important measures to conduct scientific management of aviation materials, while data collection is the basis to determine the aviation material fault function. The chapter proposed suggestions for method and opportunities of data collection; analyzed characteristics of the common methods \(\chi^2\)—distribution and \(D_n\) distribution in large sample statistics, and established the inspection hypothesis method of data under the large sample conditions through combination of \(\chi^2\)—distribution and \(D_n\) distribution; and selected the (K-S) inspection method suitable for small sample inspection to conduct statistical analysis of small sample data.
Aiming at the actual situation that the same group of data under small sample conditions would easily satisfy multiple distributions, the judgment model based on combination of improved fuzzy judgment and improved AHP method, and expert experience was used to obtain a rational distribution function. Through research on the spare part demand function determination proposed in the paper, a basis could be laid for subsequent supply, reserve and computation of aviation materials.

References


