Nonlinear Mechanical Behaviors for System of Bearing on the Top of First Story Column

Zhi-dan LIN, Hua SHI and Lu XUE
City Institute, Dalian Univ. of Tech., Dalian, China

Keywords: First-floor isolation, Natural frequency, Seismic analysis, Geometric nonlinear.

Abstract. According to the system of bearing on the top of first story column, the mathematical model of isolation system based on geometric nonlinear is investigated by using Hamilton's principle. The natural frequency of the system is investigated, and the effects of axial pressure and difference column size to the isolation system in seismic response are discussed. The isolation system dynamics model based on geometric nonlinear is established considering the cross section rotated and the influence of the shear deformation and axial pressure. The differential quadrature element method is employed for discrete processing on governing equations and boundary conditions. The natural frequency and seismic response of isolation system are solved numerically. Results show that: the axial force will reduce the lower order natural frequencies significantly. With increase of slenderness ratio of cantilever column, stiffness degradation of isolation system decreases, lateral displacement of the top of cantilever column increases, which seriously influences the stability of the isolation system and decrease the security of isolated structure, finally.

Introduction

The main application of isolation technology is base isolation, which the isolation layer locates on the top of the foundation. In recent seismic isolation projects, a new design method of seismic isolation that installs laminated rubber bearing onto top of the column in basement is highlighted and gradually attracts the attention of the project field, shown in Figure 1. The design guarantees the sufficient use of internal and lower space and saves costs.

The early studies of rubber bearing were based on the Haringx theory by Gent [1]. Kelly [2-3] studied the horizontal stiffness, vertical stiffness and tension buckling of rubber bearings, based on the theory of beam. Zhou et al. [4-5] set up the analytic model of the serial system, deduced the practical calculation formulas and analyzed the stability. Ma et al. [6] proposed an iteration-free algorithm for the dynamic response analysis of structures with isolators on the top of the columns, and studied the influences of P-Δ effects on the responses of the substructure. However, there is less
report on the dynamic performance of the serial system of bearing with column. Taking into account the effects of the cross-section rotation, shear distortion and compression axial force, the natural frequency of serial system of bearing with column was studied less. Qi [7-8] et al. discussed the design method of first-floor structure capital seismic isolation technique and tested dynamic characteristics of a five-floor framework structure that applies first-floor structure capital seismic isolation technique under environment inspiration and initial displacement condition. Thus, in order to further explore dynamic characteristics of serially connected seismic isolation system, in addition to overall analysis on dynamic response of serially connected seismic isolation structure, detailed analysis should be conducted on the seismic isolation system of laminated rubber bearing and column serially connected.

On the basis of homogeneous laminated rubber bearing theory, the dynamic response control equation of the isolation system based on geometric nonlinearity is deduced; with application of differential quadrature element method (DQEM) [9-11], the equation and boundary conditions are dispersed; finally, the earthquake response to the system is solved numerically. Corresponding observation object is obtained and the influence of axial force and slenderness ratio of suspended column on seismic isolation functions of serially connected seismic isolation system is discussed by using numerical results. These results provide foundation for further researches on the dynamic behaviors of serially connected seismic isolation structure.

**Governing Differential Equation**

As shown in Figure 2, considering that the total height of the isolation system is H with laminated rubber seismic isolation bearing on the top, the laminated structure consisting of steel plates and rubber sheets is simplified as equivalent, continuous and uniform columns. Meanwhile, considering the influence of bend deformation and shear deformation, the cross section is $A_r$; the diameter is $d$; equivalent density is $\rho_r$; modified bend elastic modulus is $E_r$; shear modulus is $G_r$; moment of inertia of cross section is $I_r$. The lower part is reinforced concrete column, to which the seismic isolation bearing is connected at the height of $h$. The cross section is $A_c$; equivalent density is $\rho_c$; elastic modulus is $E_c$; shear modulus is $G_c$ and moment of inertia of cross section is $I_c$. The other side $y=0$ is connected to the foundation.

![Figure 2. Differential element.](image)

The centre line of homogeneous column is y-axis and the symmetry axis of cross section is x-axis, according to first-class shear deformation beam theory, the displacement fields are listed as below:
\[
\begin{align*}
U_i &= u(y,t) - x\varphi(y,t) \\
U_i &= v(y,t)
\end{align*}
\]  
(1)

In the equation, u and v refer to the vertical and horizontal displacement on the column center line, respectively; \(\varphi\) refers to the normal angle of cross section after deformation.

Based on the displacement fields mentioned above, under the framework of limited deformation, the nonlinear geometric equation can be obtained as below:

\[
\begin{align*}
\varepsilon_y &= \frac{\partial u}{\partial y} + \frac{1}{2} \left( \frac{\partial v}{\partial y} \right)^2 - x \frac{\partial \varphi}{\partial y} \\
\gamma_{ys} &= \frac{\partial v}{\partial y} - \varphi
\end{align*}
\]  
(2)

It can be seen from the beam theory that as for homogeneous column, if linear and isotropic elastic materials are assumed, the constitutive equation is:

\[
\begin{align*}
\sigma_y &= E \varepsilon_y, \quad \tau_{ys} = G \gamma_{ys}, \quad G = \frac{E}{2(1+\mu)}
\end{align*}
\]  
(3)

In the equation, \(E\) refers to the modified bend elastic modulus of laminated rubber bearing; \(G\) refers to shear modulus; \(\mu\) refers to Poisson ratio.

In order to deduce the motion differential equation, boundary conditions and initial condition that displacement, u, v and \(\varphi\), of geometric nonlinear elastic homogeneous column structure satisfy, Hamilton variation principle is applied. In satisfying geometric nonlinear equation, displacement and boundary conditions and enabling all the possible displacement of designated motion at the initial and termination moment, the actual displacement, u, v and \(\varphi\), make the functional select stationary values.

\[
\Pi = \int_0^T H dt = \int_0^T (U - W - T) dt
\]  
(4)

In the equation, \(H = -(U - W - T)\) refers to Hamilton function and T refers to kinetic energy of the structure; \(U=U_1+U_2\) refers to strain energy and \(U_1\) and \(U_2\) refer to the strain energy raised by normal strain and shear strain respectively; \(W\) refers to the work applied by horizontal external load and axial force.

With introduction of the concept of finite element, serially connected seismic isolation system can be divided into two units and \(\bar{e}\) that is marked in the variable’s upper right means the unit No. e. \(\bar{e}=1\) refers to the steel concrete column unit in the lower part and \(\bar{e}=2\) refers to the laminated rubber seismic isolation bearing unit in the upper part. \(v^\bar{1}(y,t), u^\bar{1}(y,t)\) and \(\theta^\bar{1}(y,t)\) are recorded as the cross section angles that is initiated by horizontal displacement, vertical displacement and bending of steel concrete column in the lower part, respectively; meanwhile, \(v^\bar{2}(y,t), u^\bar{2}(y,t)\) and \(\theta^\bar{2}(y,t)\) refer to the cross section angles that is initiated by horizontal displacement, vertical displacement and bending of laminated rubber bearing in the upper part; thus, the motion control equation of geometric nonlinear serially connected seismic isolation system is:
\[
\begin{align*}
\rho^* A^* u^* & - E^* A^* \frac{\partial^2 u^*}{\partial y^2} - E^* A^* \frac{\partial^2 v^*}{\partial y^2} - \frac{3}{2} E^* A^*(\frac{\partial v^*}{\partial y} \frac{\partial^2 \phi^*}{\partial y^2}) - p = 0 & 0 < y < h \\
\rho^* A^* \frac{\partial^3 v^*}{\partial t^2} + p \frac{\partial^2 v^*}{\partial y^2} - E^* A^* \frac{\partial^3 u^*}{\partial t^2} & - E^* A^* \frac{\partial^2 u^*}{\partial y^2} - \frac{3}{2} E^* A^*(\frac{\partial v^*}{\partial y} \frac{\partial^2 \phi^*}{\partial y^2}) - p = 0 & 0 < y < h \\
-\kappa^* G^* A^* \frac{\partial^2 \phi^*}{\partial y^2} - \frac{\partial \phi^*}{\partial y} & - q = 0 & 0 < y < h \\
\rho^* I^y & \frac{\partial \phi^*}{\partial y} - E^* I^y \frac{\partial^2 \phi^*}{\partial t^2} - E^* I^y \frac{\partial^2 \psi^*}{\partial y^2} - \frac{3}{2} E^* I^y (\frac{\partial v^*}{\partial y} \frac{\partial^2 \phi^*}{\partial y^2}) & y = h \\
-\kappa^* G^* A^* \frac{\partial^2 \phi^*}{\partial y^2} - \frac{\partial \phi^*}{\partial y} & - q = 0 & h < y < H \\
\rho^* I^y & \frac{\partial \phi^*}{\partial y} - E^* I^y \frac{\partial^2 \phi^*}{\partial t^2} - \kappa^* G^* A^*(\frac{\partial \phi^*}{\partial y} - \phi^* - \phi^*) & h < y < H \\
\end{align*}
\]

Coordination conditions:

There are internal boundary conditions (coordination conditions) including displacement coordination conditions and internal force balance conditions between steel concrete column and laminated rubber bearing:

\[
\begin{align*}
u^* = u^* , v^* = v^* , \phi^* = \phi^* & \quad y = h \\
M^x = M^x & , Q^y = Q^y & N^z = N^z & y = h \\
\end{align*}
\]

Initial conditions:

If homogeneous column remains in natural status when \( t < 0 \), the following initial conditions can be satisfied when \( t \geq 0 \):

\[
\begin{align*}
u|_{t=0} &= 0 , \quad \dot{u}|_{t=0} = 0; \\
v|_{t=0} &= 0 , \quad \dot{v}|_{t=0} = 0; \\
\phi|_{t=0} &= 0 , \quad \dot{\phi}|_{t=0} = 0; \\
\end{align*}
\]

Boundary conditions are:

\[
\begin{align*}
u &= 0 , \quad v = 0 , \quad \phi = 0 & \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \; \;}
Numerical Analysis and Discussion

Frequency of Isolated System

The parameter of basement columns: concrete strength grade of C30, the side length of square section is 1050 mm, the height is 3.0 m; The bearing parameter is shown in table 1.

Table 1. Parameters of rubber bearings (LRB500).

<table>
<thead>
<tr>
<th>Diameter (mm)</th>
<th>Lead's Diameter (mm)</th>
<th>Rubber underlayer thickness (mm)</th>
<th>Rubber underlayer number</th>
<th>Total thickness of rubber underlayer (mm)</th>
<th>Steel thickness (mm)</th>
<th>Steel number</th>
<th>Top steel thickness (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>500</td>
<td>100</td>
<td>4.8</td>
<td>20</td>
<td>96</td>
<td>2</td>
<td>19</td>
<td>15</td>
</tr>
</tbody>
</table>

The first and fifth dimensionless frequency of GZP500 with the applied compressive axial load $p$ are shown in Figure 3. It can be seen that the first and fifth frequency of the isolation system gradually decreased with the increase of load, when the vertical load is close to the critical load of the system, the first frequency tends to zero, the isolation system is to be instability.

The variations of the first-mode natural frequency with the applied compressive axial load $p$ are shown in Figure 4 for three different bearings (GZP600, 700, 800). The critical axial load is found when the first natural frequency $\omega = 0$. The buckling dimensionless loads are: for GZP600, $p = 0.0569$; GZP700, $p = 0.0684$; GZP800, $p = 0.0779$. In the frequency steady descending segment, the first frequency decreases with the bearing models. Then, it also shows that the critical load of serial system increases with the bearing models.

Seismic Analysis

In order to analyze the response principle of the isolation system under the earthquake, the far-field (the distance from epicentre is larger than 10km) earthquake record that is recommended by ATC-63 is applied. With application of MATLAB language, programs are edited. In numerical calculation, the peak value of earthquake wave is adjusted to a rare level of 0.2g for each grade, that is, 400cm/s$^2$.

In Figure 5, the curves of average value on the top of seismic isolation bearing under the effect of earthquake and the axial load of 10MPa are given. It can be seen from Figure 5 that the top displacement of serially connected seismic isolation system bearing apparently increases as the slenderness ratio increases. Slope change rate of each curve has proved it. When the slenderness ratio is determined, the higher the column is, the gentler the system appears. Thus, the peak value of
displacement is larger. It will greatly decrease the safety of serially connected seismic isolation system.

In Figure 6, the effect of slenderness ratio on dynamic response of concrete columns is re-examined from the angle of current specification, and stability of series seismic isolation system is evaluated from the collapse state of concrete frame vertical members. The interlayer displacement angle of concrete column is derived from its lateral displacement, the characteristic of which is consistent with displacement of cantilever column. It can be seen from Figure 6 that slenderness ratio taking 5.3 as its limit, when slenderness ratio is less than 5.3, the interlayer displacement angles are basically superposed, indicating the series seismic isolation system is in steady state; when it’s greater than 5.3, four curves immediately disperse, showing their own characteristics.

Figure 5. Average displacement curves of rubber bearing at top point.  Figure 6. Average drift curves of column at top point.

Summary

The dynamical control equations of seismic isolation system based on geometric nonlinearity are deduced by Hamilton variation principle; kinematic equation and boundary conditions of rubber bearing unit and cantilever column unit are established based on finite element method; numerical solution is conducted by using different quadrature element method, the relatively mature mathematical method. The results show that, the natural frequency decreased with the vertical load, when the vertical load is close to the critical load, the natural frequency tends to zero; in ground motions, with increase of slenderness ratio of cantilever column, stiffness degradation of series seismic isolation system decreases, lateral displacement of the top of cantilever column increases. It influences the stability of series seismic isolation system seriously and finally results in decrease of integral security of isolated structure.

References


