Experiment of Tensile, Shear Strength Conditions Derived from the Balance Stress of Particle

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Abstract. The tensile-shearing strength condition derived by the stress balance of particle not only theoretically explains that pulling-shearing operation makes it easier to be destroyed than compressing-shearing operation, which the classical elastic theory cannot explain, but also solve the pulling-shearing problem for materials with identical pulling and compressing performance which the Mohr’s theory of failure cannot solve. This experiment aims to verify the new pulling-shearing formula and obtain the error between two pulling-shearing formulas.

Introduction
Fracture accidents are frequent, especially collapse accidents of large beams under tensile-shearing forces. The reasons for these accidents are not all quality problems. What’s more, it’s also a reason that the strength of beam in reality is weaker than the strength designed under the classical elastic theory. In classical elastic theory, the strength condition is constructed by the maximum normal stress of micro-cell balance, and the stress state of micro-body is thought to be the stress state of particle. While in this paper, the stress state of particle is found to be different from the state of micro-cell, and the balance stress of particle is found to be larger than the one of micro-cell under 3 different stress states. This, in our opinion, is the root cause for collapse accidents of bridges under pulling-shearing stress.

Experiment verifies the correctness of the strength theory of particle balance. To step further, the tensile-shearing strength condition derived by the strength theory of particle balance us verified in this experiment.

Experimental Period
2007.04.01-2007.10.15

Experimental Site
National key laboratory of damage mechanics in Tsinghua University, China

Experimental Equipment
PLS-S100 double-shaft four-cylinder servo-controlled testing system, whose maximum static load is ±100kN and maximum dynamic load is ±100kN

Experimental Objective
Verification of the third, fourth strength condition and the new strength condition of particle balance

Experimental Design
1) Material for test
Plastic PVC, whose strength limit is $\sigma_b = 43MPa$
2) Size of test piece
As figure 1 shows,

![Figure 1. Size of test piece.]

3) Design method
Design method is shown in figure 2

![Figure 2. Stretch Shear.]

As figure 2(a) shows, pulling forces are in the direction of x and the shearing forces by parting tool are in the direction of the y axis. The force diagram of shear plane is in figure 2(b). As the testing system can centering slicers up and down, the function areas of shearing stresses up and down are both half of the cross section. And the force diagram of particle is shown in figure 2(c). Strictly speaking, there’s still small normal stress in the direction of y axis on the longitudinal section. The maximum normal stress is

$$\sigma_y = \frac{Q_y}{A} = \frac{4 \times 10^3}{340 \times 10^{-3} \times 9 \times 10^{-3}} = 1.3 \text{MPa}$$

As can be seen, the normal stress $Q_y$ is small and can be neglected.

Meanwhile, the nose of slicer is blunt, and can decrease stress concentration. What’s more, the results will not be affected even there’s stress concentration.

**Data**

Table 1. Unidirectional Stretch and Shear Data Sheet.

<table>
<thead>
<tr>
<th>No</th>
<th>$P_x$/KN</th>
<th>$A = 59 \times 10^{-3} \times 9 \times 10^{-3} / m^2$</th>
<th>$\sigma_s = \frac{P_x}{A}$/MPa</th>
<th>$Q_y$/kN</th>
<th>$\tau_y = \frac{Q_y}{A_t}$/MPa</th>
<th>$\sigma_y$/MPa</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>14.1</td>
<td>531 $\times 10^{-6}$</td>
<td>26.6</td>
<td>2</td>
<td>265.5</td>
<td>7.5</td>
</tr>
<tr>
<td>2</td>
<td>15.2</td>
<td>531 $\times 10^{-6}$</td>
<td>28.6</td>
<td>3</td>
<td>265.5</td>
<td>12</td>
</tr>
<tr>
<td>3</td>
<td>17.2</td>
<td>531 $\times 10^{-6}$</td>
<td>32.4</td>
<td>2.5</td>
<td>265.5</td>
<td>9.4</td>
</tr>
<tr>
<td>4</td>
<td>16</td>
<td>531 $\times 10^{-6}$</td>
<td>30.1</td>
<td>4</td>
<td>265.5</td>
<td>15</td>
</tr>
</tbody>
</table>

Note: No.3 and No.4 are handwritten, as a result of recording instrument failure
Introduction of Strength Conditions \(^{[7]}\)

When test piece is under unidirectional tension and shear, the third strength condition is

\[
\sigma_{r3} = \sqrt{\sigma^2 + 4\tau^2} \leq \sigma
\]  

(1)

And the forth strength condition is

\[
\sigma_{r4} = \sqrt{\sigma^2 + 3\tau^2} \leq \sigma
\]  

(2)

When safe factor \( n = 1 \), (1) (2) will become

\[
\sigma_{r3} = \sqrt{\sigma^2 + 4\tau^2} = \sigma_s
\]  

(3)

\[
\sigma_{r4} = \sqrt{\sigma^2 + 3\tau^2} = \sigma_s
\]  

(4)

The strength condition of particle balance (when \( n = 1 \)) \(^{[8]}\) is

\[
\sigma_{sr} = \sqrt{\sigma^2 + 2\sigma\tau + 2\tau^2} = \sigma_s
\]  

(5)

In the above formula, \( \sigma_s \) is the yield tensile stress.

Breaking Stress under Three Strength Conditions

Table 2. Breaking stress under three strength conditions.

<table>
<thead>
<tr>
<th>No</th>
<th>( \sigma_x ) / Mpa</th>
<th>( \tau_y ) / Mpa</th>
<th>( \sigma_{r3} = \sqrt{\sigma^2 + 4\tau^2} ) / Mpa</th>
<th>( \sigma_{r4} = \sqrt{\sigma^2 + 3\tau^2} ) / Mpa</th>
<th>( \sigma_{sr} = \sqrt{\sigma^2 + 2\sigma\tau + 2\tau^2} ) / Mpa</th>
<th>( \sigma_b ) / Mpa</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>26.6</td>
<td>7.5</td>
<td>30.6</td>
<td>29.6</td>
<td>40.2</td>
<td>43</td>
</tr>
<tr>
<td>2</td>
<td>28.6</td>
<td>12</td>
<td>37</td>
<td>35</td>
<td>42.1</td>
<td>43</td>
</tr>
<tr>
<td>3</td>
<td>32.4</td>
<td>9.4</td>
<td>37.5</td>
<td>36.3</td>
<td>42.6</td>
<td>43</td>
</tr>
<tr>
<td>4</td>
<td>30.1</td>
<td>15</td>
<td>42.5</td>
<td>39.8</td>
<td>45.1</td>
<td>43</td>
</tr>
<tr>
<td>Mean</td>
<td>36.9</td>
<td>35.2</td>
<td>42.5</td>
<td>39.8</td>
<td>45.1</td>
<td>43</td>
</tr>
<tr>
<td>Error</td>
<td>( \frac{\sigma_b - \sigma_s}{\sigma_b} \times 100% )</td>
<td>14.2%</td>
<td>18.2%</td>
<td>1%</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Conclusion

1. The error of breaking stress between the new strength condition and the tensile strength limit is only 1%, while for the third and fourth strength conditions are 14.2% and 18.2%, respectively. That’s to say, the new strength condition is correct and it’s unsafe to design under the classical strength conditions.

2. The stress under the third, fourth strength theory is the equivalent stress while the stress under the new condition is the practical stress. And the direction of the true stress can be determined by the angle formula of particle-balance stress,

\[
\alpha_s = \arctan \left( \frac{\sigma_x}{\sigma_y} \right) + \arctan \left( \frac{\sin \arctan \frac{\sigma_y}{\sigma_x}}{\sqrt{1 + \left( \frac{\sigma_x}{\sigma_y} \right)^2}} \right)
\]

(6)

Under unidirectional tension-shear, \( \sigma_y = 0 \), (6) will be
\[ \alpha_s = \arctan \frac{\tau}{\tau + \sigma_s} \]  

(7)

The third set of data in table 2 \( \sigma_s = 32.4MPa, \tau = 9.4MPa \) into (7),

\[ \alpha_s = \arctan \frac{9.4}{9.4 + 32.4} \]

= \arctan 0.2249

= 12'40'

(8)

It suggests that the angle between the particle-balance stress and the \( x \) is 12'40’ and the angle between the fractured force and the \( y \) axis is then 12'40’, which agrees perfectly with the fractured surface of test piece.

(3) This experiment again verifies the correctness of the particle balance concept and the new theory.

References