Research on System Control Based on a Novel Theory

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ABSTRACT

A system is combined by itself and the periphery, and the periphery controls the system. The Periphery Theory describes that a system input is dependent on a system gate of the periphery, and the gate is considered as a switch which will adjust the state of the system automatically. In this research, a new logistic model with switch is created to learn how the switch adjust the system. The switch is set to close/open the gate of the system to decrease/increase the input, and it is related to the final states of the system. By comparing this new model with the original one, the switch also improve the complexity of the system, which changes the distribution of final system states, making the positions of the transition and bifurcation points occur earlier. Moreover, a new Feigenbaum constant for the new logistic model is 4.68274650, and it is totally different from the original one. In addition, a quantitative relationship between the probability of system switch and the final system states shows the protection provided by the gate to the system.

INTRODUCTION

According to periphery theory[1-4], each system consists of an interior and an exterior part and a system boundary connecting the two. The boundary is the only...
connection between the internal and external systems and consists of a wall and a gate. The wall is closed to prevent exchange between the internal and external systems, whereas the gate is open to allow exchange.

Periphery theory has been widely used in many fields[5-6]. In 2001, Cao et al. studied periphery theory and developed it further. On this basis, using a simple biological model (the logistic model), this research has used periphery theory to study how to achieve exchange between the internal and external parts of a system through the system boundary.

The logistic model, as a biological model, can be used to describe the transition of a population from one state to another. Based on the model parameters, the model describes the increase or decrease of the population[7-8]. In fact, the model represents changes in population according to periphery theory. Assuming that the birth rate is equal to the death rate, population change will take place only by immigration or emigration through the boundary gate. In this case, a switch parameter is adjusted in the logistic model to control the population. Numerical results show that the probability of the switch parameter being turned on determines the final states of the population. These states could be stable, cyclic, or chaotic.

THEORY AND METHOD

Periphery Theory

Boundary shells are widely found in nature and in human society, such as national boundaries, fences, biological shells, the atmosphere, the Earth’s crust, firewalls, clothes, etc. Periphery theory was created to investigate boundary shells and how they work in various fields such as information, control, environment, biology, physics, management, aesthetics, and philosophy.

A system is always divided into two parts, one being the system itself, the other being the external environment. The two parts exchange through the boundary shell, which means that the system impacts the environment and also is affected by the environment. In traditional studies, it has been assumed that the environment is unrestricted in its ability to impact the system, ignoring the perimeter’s intermediary role between the environment and the system. Periphery theory is based on the assumption that any exchange between the system and its environment through the boundary is controlled by the boundary and that the states of the system are restricted by the boundary. The boundary plays a vital role to the survival of the system. Boundary shells are also restraints in the material and spiritual worlds. Taking a national boundary as an example, only things and people that obtain permission from customs can pass through the national boundary. Castles in Europe and walls in China have been used against enemy invasion, while the gate in the boundary is an aisle for people. Castles and walls were national or regional traditional security products and indeed protected human civilization. Houses are built to provide shelter from rain and thieves, whereas the door is opened for people. Beijing courtyard and Hakka Tulou have the same function. All boundary shells are similar to each other.
Periphery theory aims to study boundary shells with the following characteristics: (1) protecting the system; (2) exchange between the system and the environment.

The boundary consists of a wall and a gate. The wall is used to protect the system itself, and the gate is used for exchange. For a closed system, building a gate will be helpful in controlling population exchange.

**Logistic Model**

The logistic model[9] is used to describe changes in population and has been widely used in biological fields[10-11]. The model can be written as follows:

\[
x' = \mu x (1-x)
\]

where the parameter \( \mu \) is a control parameter and state 0 and state 1 are the two stable states of the system. Equation 1 can be replaced by its differential form:

\[
x_{n+1} = \mu x_n (1-x_n)
\]

Equation (2) means that any state of the system is related only to its previous state. Numerical testing (May, 1976) identified two situations. (1) If the initial state belongs to (0,1) and the control parameter \( \mu \) is less than a threshold, the final state will be fixed, which means that the final state is not sensitive to the initial state or to the control parameter. (2) If the initial state belongs to (0,1), the final state of the system is determined by the control parameter. As shown in Figure 1, when \( \mu \) is less than approximately one, the final state is always 0. However, when \( \mu \) is greater than approximately one, the final state is dependent on it.

![Figure 1. Relationship between the control parameter and the final system states.](image)

Note that the system is a bifurcation model and that its Feigenbaum constant is 4.669 201 609 1.
A new Logistic Model Based on Periphery Theory

If the control parameter is less than 2.998 507 2, a system transition can be expected, in which the system shifts smoothly from one stable state to another, as shown in Figure 1. Based on this phenomenon, Feng and Cao[12] built a new model—the logistic model with switch, known as the population-immigration model. It can be written as:

\[ x_{n+1} = (\mu + \delta) x_n (1 - x_n) \]  

(3)

\[ \delta = \begin{cases} 
1 + \gamma & \text{if } (x_{n+1} - x_n) \leq C \\
0 & \text{if } (x_{n+1} - x_n) > C 
\end{cases} \]  

(4)

where \( \delta \) is the switch parameter. If the difference between two states in the present moment is less than that in the previous moment, the switch is turned on; otherwise, the switch is turned off. The parameter \( \gamma = I_{in} - I_{ex} \) is defined as the change in immigration, where \( I_{in} \) represents input and \( I_{ex} \) represents output. For convenience in calculation, the change in immigration is bounded by one, and the parameter \( C \) is also bounded.

Specifying the parameter \( C \) would make the model too subjective, and therefore a new logistic model was developed which can adjust the population of the system automatically. Accordingly, Equation (4) can be rewritten as:

\[ \delta = \begin{cases} 
1 & (x_{n+1} - x_n) / (x_n - x_{n-1}) < 1 \\
0 & (x_{n+1} - x_n) / (x_n - x_{n-1}) \geq 1 
\end{cases} \]  

(5)

It is obvious that if the magnitude of immigration in the present moment is greater than in the previous moment, then the switch will be turned off (parameter \( \delta = 0 \)) to stop this increase. On the contrary, if the magnitude of immigration in the present moment is less than in the previous moment, the switch will be turned on to allow more immigrants to come in.

CALCULATION AND ANALYSIS

Structure of the New Model

In the new model, the initial value \( (x_0=0.01) \) and the number of iterations \( (=100,000) \) are fixed. The last 10 states of the system with control parameter \( \mu \) are shown in Figure 2a. By compared to Figure 1, the new model displays a totally different character with the original model.

(1) The transition point is earlier than before. In the original model, when the control parameter was less than 0.9998, the system remained in state 0, whereas in the new model, the control parameter remains at 0.4655.

(2) The bifurcation point is also earlier. The original system starts to bifurcate at position 2.9995, but the new system does so at 1.9999.

(3) The states between the transition and bifurcation points of the new model are different than previously. In the original model, there is only one stable state, whereas the new model bifurcates into more states, but these states approach a bifurcation point which is similar to the single state of the original model.
Figure 2b shows the probability of the switch turning on or off. The black dots are the probability of the switch turning on (δ=1), and the gray dots represent the probability that the switch is turned off (δ=0). The probability and the control parameter have a relationship with the states pictured in Figure 2a. Note that the switch parameter stays almost constant (δ≈0) at the beginning, which means that the switch is turned off. Afterwards, the switch almost always stays on until it shuts off at about position 2.8. According to the switch parameter, the time series can be divided into five periods.

**Figure 2.** The new logistic model: (a) the relationship between the control parameter and the system states, with the x-axis representing the control parameter and the y-axis the system states; (b) the relationship between the control parameter and the switch parameter, with the x-axis representing the control parameter and the y-axis the probability of the switch turning on.

1. When the control parameter is in the range 0.000 000 00 to 0.374 999 99, the time series of the switch is an arc, which represents a transition of the switch from the on to the off state. By increasing the number of iterations (from 100,000 now to 200,000 or more), the arc would become a horizontal line, which would mean that the switch would remain off. If the system stayed in state 0, this would also indicate that the switch would not be turned on.

2. When the control parameter is in the range 0.374 999 99 to 0.788 927 65, the probability that the switch is turned on is greater than the probability that it is turned off, with a value of about 66.7%. During this period, the system bifurcates into three states.

3. When the control parameter is in the range 0.788 927 65 to 1.811 050 00, the probability that the switch is turned on is also greater than the probability that it is turned off. Two situations are possible: (a), the switch remains turned on; (b), the switch status curve will decrease and then increase, with the change position at about μ=1.000 000 00. During this period, the states of the system are bifurcated, and the states have the same characteristics as the original states.

4. When the control parameter is in the range 1.811 050 00 to 2.802 131 24, the probability of the switch being turned on is about 99.0%, which is much greater
than the probability that it is turned off. During this period, the states are similar to
the original states. However, the new control parameter $\mu'$ is the difference between
the original control parameter $\mu$ and the switch parameter $\delta$, which can be
expressed as $\mu' = \mu - \delta$. This means that setting the new control parameter to a small
value would yield the same system states as setting the original control parameter
to a large value when the switch is turned on ($\delta=1$).

(5) When the control parameter is in the range $2.802\,131\,24$ to $300\,000\,00$, the
probability that the switch is turned on is less than one, but it is much greater than
the probability of the switch turning off. This leads to a phenomenon in which the
tail of the time series drops by comparison with the original time series and the
values are disorderly. The changes in the system are unpredictable because its
behavior is chaotic during this period.

Switch Parameter and the Periodic Solution of the New Model

Note that the solution of the new model is related to the switch. This section
will describe how the switch influences the system states. Figure 3 illustrates the
analysis of four conditions after the system leaves state 0.

In Figure 3a, when the control parameter is in the range $0.375\,000\,00$ to $0.788\,927\,65$, three states are bifurcated. These three states are $0.02520$, $0.03685$, and
$0.05324$ when the control parameter is $0.5$. When it is $0.6$, the three states take on
values of $0.08851$, $0.12908$, and $0.17987$, and when the control parameter is $0.7$, the
three states are $0.14017$, $0.20489$, and $0.27695$. The three states are arranged in
a cycle in ascending order, which gives the switch an opportunity to be turned on
with a probability of about $66.7\%$.

Figure 3b represents the condition in which the control parameter is in the range
$0.788\,927\,65$ to $1.811\,050\,00$. Then the switch is almost always turned on (the
probability is greater than $99\%$), and the states bifurcate into several different parts,
which exhibit periodic variation. (1) When $\mu=0.8$, the period is $27$; (2) when $\mu=0.9$,
the period is $20$; (3) when $\mu=1.0$, the period is $13$; (4) when $\mu=1.2$, the period is $26$.
The progression is totally different from that shown in Figure 3a. The system stays
in one state for a long time and then deviates and decreases. In some cases, such as
$\mu=0.95$, the system stays in a constant state, which means that the cycle does not
exist.

In Figure 3c, when the control parameter is in the range $1.811\,050\,00$ to $2.802\,131\,24$, the system becomes bifurcated, but the switch is still almost always turned
on (probability greater than $99\%$). The bifurcation occurs step by step, and the
periods of the various steps are different. (1) Before bifurcation, the system evolves
to a single constant state. For example, when $\mu=1.9$, there is no cycle; (2) When the
system bifurcates for the first time, one state becomes two states. The period is $2$
when $\mu=2.2$; (3) When two states become four states, the period is $4$ when $\mu=2.5$;
(4) As the control parameter increases, the number of states becomes $8$, and the
period would be $8$ also when $\mu=2.55$. The number of cycles would increase to $2n$ if
the control parameter continued to increase.
Figure 3. Progression of system states with various control parameter settings. The x-axis represents time, and the y-axis represents the system state.

In Figure 3d, the system has become chaotic, and no cycles exist. As shown, the control parameters are 2.85, 2.9, and 2.95, and the progression of states is disorderly.

New Feigenbaum Constant of the new Model

When $\mu > 1.998\ 542\ 76$, the system becomes bifurcated and chaotic, with a fractal structure similar to the original model. In the bifurcation position, the control parameter can be calculated, as shown in Table 1.

In the bifurcation position, when the control parameters are calculated, they are found to approach a constant ($\mu_m \approx 2.57$). According to the definition of the Feigenbaum constant[13] in Equation (6), a new constant could be calculated:

$$\delta = \lim_{n \to \infty} \frac{\mu_n - \mu_{n+1}}{\mu_{n+1} - \mu_{n+2}}$$

which is about 4.682 746 50. The new Feigenbaum constant is totally different from the original one because the switch parameter was imported into the model.

<table>
<thead>
<tr>
<th>Control parameter</th>
<th>Bifurcation position</th>
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<tbody>
<tr>
<td>$\mu_1$</td>
<td>1.99852476</td>
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<tr>
<td>$\mu_2$</td>
<td>2.44891596</td>
</tr>
<tr>
<td>$\mu_3$</td>
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**SUMMARY**

Based on periphery theory, a switch has been imported into a logistic model, building a new model which represents changes in population. The model expresses a vital relationship between population change in a system and its boundary. When input and output through the boundary are considered, population changes in the system become complex, and the nature of these changes depends on the switch.

The new model exhibits the following differences from the original model:

1. In the new model, the control condition moves the transition point earlier.
2. The bifurcation point is also earlier, meaning that the model could become bifurcated at smaller values of the new control parameter.
3. The states between the transition and bifurcation points are different. A number of states replace the original single state, which means that the system no longer stays in just one state, but transitions from one state to another, with the states forming a cycle.
4. The new model also changes the states of the system during chaotic phases. In the tail, the system could stay in chaotic states when the new control value is smaller. In that case, the population could become stable at a low value of the control parameter.
5. As in the original model, a limit parameter \( \mu_m \) exists. When \( \mu > \mu_m \), the system becomes chaotic. A new Feigenbaum constant was calculated, \( \sigma' = 4.682 \, 746 \, 50 \).

By improving the control conditions of the original logistic model with switch, the switch could adjust itself to adapt the system to changing conditions. The new model shows the protection provided by the gate to the system, based on periphery theory.

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**REFERENCES**


