Numerical Simulation on Propagation of a Ubiquitiform Crack in Rock Materials

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ABSTRACT

In this paper, quasi-static crack propagation in the rock materials is modelled using the extended element method (X-FEM). In the proposed model, the heterogeneity of material properties was characterized by the Weibull distribution. The complexity of the ubiquitiform crack can then be determined by the box counting dimension. A ubiquitiform fracture surface which is resulted from the heterogeneity of the material properties is obtained, which is more consistent with experimental result than the fracture surface obtained in the homogenous materials. Moreover, the calculated numerical result of the complexity is found to be in good agreement with previous experimental data.

INTRODUCTION

In the past decades, the fractality of the fracture surfaces in various kinds of materials such as concrete[1, 2], steel[3, 4], ceramic[5, 6] and rock[7, 8] has been verified experimentally, which has lead gradually to the establishment of the emergent fractal fracture mechanics. Moreover, as is pointed out further by Ou et al.[9], recently, a real physical or geometrical object in nature should be ubiquitiformal rather than fractal. Comparably, few algorithms are available to effectively reproduce tortuous trajectory and rough surface of realistic cracks. As is well known, a real rock material is always filled with various weak defects such as weak interfaces, voids and micro-cracks. Therefore, in the opinion of the authors of this paper, a ubiquitiformal fracture surface is resulted from the heterogeneity of the material properties under consideration.

There are basically two approaches in characterising the heterogeneity in materials numerically: the direct approach [10,11,12] and the indirect approach [13,14,15]. In the direct approach, the details about the microstructure can be measured and translated directly into a finite element mesh and their material properties are directly assigned to the elements. In the indirect approach, it could attempt to translate the microstructural information into a statistical distribution that
is subsequently used to assign strength and stiffness properties to different elements in a finite element discretization, so different phases are implicitly modelled.

The present study will therefore develop a numerical model to simulate the extension of a ubiquitiform crack in rock materials. In the numerical model, the Weibull distribution is used to describe the heterogeneous material properties, and then the ubiquitiformal profile of a fracture surface can be simulated by using the ABAQUS software together with an XFEM-based cohesive segments method. The complexity of the ubiquitiform crack is determined by using the box-counting dimension method.

**NUMERICAL MODELS**

Simulating the propagation of cracks using traditional finite element methods is challenging because the topology of the domain changes continuously. The extended finite element method (XFEM) has been used very successfully to model cracks because the finite element mesh can be created independent from the crack geometry, and in particular the domain does not have to be remeshed as the crack propagates. The presence of discontinuities is ensured by the special enriched functions in conjunction with additional degrees of freedom. However, the finite element framework and its properties such as sparsity and symmetry are retained. The XFEM is based on the concept of partition of unity [16], by which the trial displacement function $u^h(x)$ can be written approximately as following [17]:

$$u^h(x) = \sum_{j=1}^{N} N_j(x) \left[ u_j + H(x) a_j + \sum_{\alpha=1}^{4} F_{\alpha}(x) b_{\alpha}^j \right]$$

(1)

where $j$ is the set of nodes; $N_j(x)$ and $u_j$ ($j = 1, 2, \ldots, N$) are the traditional nodal shape functions and displacement vectors associated with the continuous part of the finite element solution, respectively; $H(x)$ is the enriched function associated with the discontinuous jump across the crack surface, and $F_{\alpha}(x)$ are the enriched functions associated with elastic asymptotic crack tip functions; $a_j$ and $b_{\alpha}^j$ are the nodal enriched degree of freedom vector.

In this paper, the well-known Weibull distribution is taken to characterize the tensile strength. The statistic density function of the random variable $w$ obeying the Weibull distribution is defined as

$$f(w) = \frac{m}{w_0} \left( \frac{w}{w_0} \right)^{m-1} \exp \left( - \frac{w}{w_0} \right)^m$$

(2)

Where $m$ is the shape parameter or the homogeneity index [19] characterizing the degree of material homogeneity and $w_0$ is the scaling parameter.

The numerical simulations are carried out under uniaxial tension by using a two-dimensional rectangular linear elastic plate in unit thickness, as shown in Fig.1, in which the displacement boundary condition is adopted. The height and the width of the computational domain are $L = 20$mm and $W = 12$mm, respectively, and the length of the initial crack is taken to be $a = 1$mm. For the heterogeneous material, the heterogeneous material properties is described by the Weibull distribution. The Weibull distribution parameters are: the degree of material homogeneity $m = 6$, the scaling parameter is 5.0MPa. The other material properties are: Young’s modulus $E$
= 65Gpa, the fracture energy \(G_F = 0.03\)N/m and Poisson’s ratio \(\nu = 0.16\). Meanwhile, the homogeneous material properties is same to the heterogeneous materia except tensile strength \(f_t = 5.0\)Mpa.

In the numerical computation by using the ABAQUS software, the CPE4 elements are used as shown in Fig. 2. In the region ahead of the initial crack, the side length of each element is taken to be 50.0\(\mu\)m. To save the computation time, the side lengths of the elements far from the initial crack are taken to be 100.0\(\mu\)m. Constitutive relation is the linear elastic traction-separation cohesive crack model used to describe the enriched region.

RESULTS AND DISCUSSION

As has been described in the introduction, the fracture surface should be ubiquitiformal due to the heterogeneity of the rock properties. Based on the Weibull distribution and the XFEM-based cohesive segments model, the extension of crack in the rock materials as shown in Fig.3a, instead of continuous change of the crack propagation direction in a homogeneous material (Fig.3b), the crack propagation direction can change abruptly in a heterogeneous material because that the material strength are different from one spatial point to another. Moreover, the fracture surfaces are obtained in the heterogeneous and homogeneous materials, respectivly, in Fig.4a and Fig.4b. It can be seen that the numerical fracture profile in heterogeneous materials is consistent with experimental result[20] and more realistic. Therefore, the heterogeneity in rock materials results in ubiquitiformal crack.
The box-counting dimension method is used to determine the complexity of the ubiquitiformal crack, with the calculated results as shown in Fig.5, in which $N(\delta)$ is the number of squares of size $\delta$ covered the crack profile. It can be seen from Fig.5 that the fitting curve can fit well the numerical results used for calculating the box-counting dimension, with the correlation coefficient $R = 0.9916$, which gives the complexity of the ubiquitiformal crack $C = 1.231$, which is in agreement with the experimental data[21].
CONCLUSION

A computational model has been developed to simulate the crack propagation process in rock materials with heterogeneous and homogeneous fracture properties. The numerical simulation reveals that crucial fracture phenomena such as the tortuosity in crack trajectories, which can be effectively captured by the heterogeneous model. Therefore, a ubiquitiformal fracture surface is resulted from the heterogeneity of the material properties under consideration. Moreover, the complexity of the ubiquitiformal crack is determined by the box counting dimension. The calculated numerical result of the complexity is found to be in good agreement with previous experimental data.

REFERENCES