A Novel Method for Doppler and DOD-DOA Jointly Estimation Based on FRFT in Bistatic MIMO Radar System

Derui Song, Li Li, Jianli Zhang, Xindi Jing and Xianghai Wang

ABSTRACT

This paper studies the problem of parameter estimation jointly in bistatic Multiple-Input Multiple-Output (MIMO) radar system. For moving target, the echo often contains time-varied Doppler frequency. Thus, this paper proposes a new signal model and a novel method for parameter estimation in bistatic MIMO radar system. Firstly, an extended signal model is presented in bistatic MIMO radar. Secondly, two Doppler parameters are jointly estimated by searching the peak of the fractional correlation function. Finally, MUSIC algorithm and ESPRIT algorithm are used to estimate DODs and DOAs. Simulation results are presented to verify the effectiveness of the proposed method.

INTRODUCTION

Multiple-Input Multiple-Output (MIMO) system has attracted more and more attention for its ability to enhance system performance [1-4]. A MIMO radar system consists of both transmit and receive sensors. Transmit antennas emit orthogonal waveforms while echoes are orthogonal, thus separable, at the receiver. MIMO processing may be performed to achieve spatial and signal waveform diversity.

The multi-target parameter estimation and localization is one of the most important aspects in bistatic MIMO radar. Most existing algorithms are divided into two categories. One ignores Doppler frequency and only estimated DOA and DOD as shown in [2-4]. The other estimates not only DOA and DOD but also time-invariant Doppler frequency. In fact, the received signals contain time-variant Doppler frequency. In this case, these existing methods cannot effectively solve this problem and provide an optimal solution. So, this paper presents a new signal model with time-variant Doppler frequency and proposes a new method based on FCF to estimate Doppler, DOD-DOA.
THE PROPOSED SIGNAL MODEL

We assume that there are \( Q \) closely spaced transmit antennas and \( N \) closely spaced receive antennas, and \( L \) targets. Fig.1 illustrates a bistatic MIMO radar system, with half-wavelength space between adjacent elements used for both transmit array and receive array. The transmit antennas emit orthogonal waveforms \( x_q(t) \) for \( q = 1, \ldots, Q \). This paper proposes a new signal model of bistatic MIMO radar system. The received signal from the \( n \)th antenna \( y_n(t) \) (\( n = 1, \ldots, N \)) can be expressed as

\[
y_n(t) = \sum_{l=1}^{L} \sum_{q=1}^{Q} \left\{ \sigma_l x_q(t) \exp\left( j2\pi \left( f_l t + \mu_l t^2 / 2 \right) \right) \right\} A_q(\varphi_l) B_n(\theta_l) + w_n(t), \quad 0 \leq t \leq T \quad (1)
\]

where \( \sigma_l \) denotes the radar cross-section corresponding to the \( l \)th target. \( f_l \) and \( \mu_l \) denote the initial Doppler frequency (IDF) and Doppler frequency rate (DFR) corresponding to the \( l \)th target, respectively. \( A_q(\varphi_l) = \exp\left( j2\pi (q-1)d_l \sin \varphi_l / \lambda \right) \) is the transmitter steering vector, \( B_n(\theta_l) = \exp\left( j2\pi (n-1)d_l \sin \theta_l / \lambda \right) \) is the receiver steering vector. The noise \( w(t) \) is assumed to be independent, zero-mean Gaussian white noise.

Since the transmitted waves are orthogonal with each other, there are \( \langle x_q, x_k \rangle = 0, q \neq k \) and \( \| x_q \|^2 = 1 \), for \( k = 1, \ldots, Q \) and \( q = 1, \ldots, Q \). At each receiving antenna, these orthogonal waveforms can be extracted by \( Q \) matched filters. The extracted signals from the \( q \)th matched filter can be expressed as

\[
y_{q,n}(t) = \sum_{l=1}^{L} \left\{ \sigma_l \exp\left( j2\pi \left( f_l t + \mu_l t^2 / 2 \right) \right) \right\} A_q(\varphi_l) B_n(\theta_l) + w_n(t), \quad 0 \leq t \leq T \quad (2)
\]

FRACTIONAL CORRELATION FUNCTION

In recent years, a new time-frequency analysis tool, the Fractional Fourier transform (FRFT)\(^{[5-7]}\), attracts increasingly more attention in signal processing society and is widely applied in detection, parameter estimation and direction of arrival estimation.

Assume that the signal \( s(t) \) is modeled as

\[
s(t) = b_0 \exp\left( j2\pi \left( a_1 t + a_2 t^2 / 2 \right) \right)
\]

where \( b_0 \) is the signal amplitude, \( a_1 \) is the initial frequency and \( a_2 \) is frequency rate.
Fractional correlation function (FCF) \( \hat{R}_s^a(t) \) of the signal \( s(t) \) is defined by
\[
\hat{R}_s^a(t) = \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} R_s(t + \tau) \exp{(j \tau \cot \alpha)} dt
\] (4)
where \( R_s(t + \tau) \) is the correlation function of the signal \( s(t) \), \( \tau \) denotes time delay, and \( \alpha \) is the rotation angle in FRFT domain. With the use of (3), (4) can be expressed as
\[
\hat{R}_s^a(t) = \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} \exp{(j(2\alpha a_\tau + \cot \alpha)\tau)} \exp{(j2\pi(a_\tau + a_\tau^2/2))} dt
\] (5)
When \( \cot \alpha = -2\pi a_\tau \), \( \hat{R}_s^{a_0}(t) \) has the best energy-concentrated property. Therefore, we can obtain the following expression as
\[
\alpha_0 = -\arccot(2\pi a_\tau)
\] (6)

**JOINT PARAMETER ESTIMATION BASED ON FRACTIONAL CORRELATION FUNCTION**

In this section, study of parameter estimation is made by taking the signal \( y_{qnl}(t) \) as an example. The signal \( y_{qnl}(t) \) denotes the extracted signals \( y_{qnl}(t) \) corresponding to the \( l \)th target. \( y_{qnl}(t) \) can be expressed as
\[
y_{qnl}(t) = \sigma_i \exp{(j2\pi(f_i t + \mu_l t^2/2))} A_q(\phi_l) B_n(\theta_l) + w_q(t)
\] (7)

According to (5) and (7), the fractional correlation of \( y_{qnl}(t) \) can be written as
\[
\hat{R}_{y_{qnl}}^a(t) = \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} R_{y_{qnl}}(t + \tau) \exp{(j \tau \cot \alpha)} dt
\] (8)
where \( R_{y_{qnl}}(t + \tau) = E\{y_{qnl}(t + \tau)y_{qnl}^*(t)\} \). When cot \( \alpha_l = -2\pi \mu_l \), \( \hat{R}_{y_{qnl}}^{a_l}(t) \) has the best energy-concentrated property. Therefore, we can obtain the following expression as
\[
\hat{\mu}_l = -\cot \alpha_l / (2\pi)
\] (9)

Define variable \( y_i(t) \) as
\[
y_i(t) = y_{qnl}(t) \cdot \exp{(-j2\pi((\hat{\mu}_l/2)t^2)}
\] (10)

Assumed \( R_i(u) \) as the Fourier transform of \( y_i(t) \). So, the IDF \( f_i \) can be estimated by
\[
\hat{f}_i = \arg \max_u \{ R_i(u) \}
\] (11)

According to (2), (9) and (11), both receive subarrays \( R_1 \) and \( R_2 \) constructed in this paper can be expressed by
\[
R_1 = \begin{bmatrix} y_{i1} & y_{i2} & \cdots & y_{iL} \end{bmatrix}^T = BG + N_1
\] (12)
\[
R_2 = \begin{bmatrix} y_{q1} & y_{q2} & \cdots & y_{qL} \end{bmatrix}^T = BAG + N_2, \quad q \neq 1
\] (13)
where \( B = [B_1 \cdots B_L] \), \( B_q = [B_q(\phi_1) \cdots B_q(\phi_L)]^T \), \( A = \text{diag}\{A_q(\phi_1), \cdots, A_q(\phi_L)\} \), \( G = [g_1(t) \ g_2(t) \ \cdots \ g_L(t)]^T \), \( g_i(t) = \sigma_i \exp{(j2\pi(\hat{f}_i t + \hat{\mu}_l t^2/2))} \), \( (\cdot)^T \) and \( \text{diag}(\cdot) \) denote transpose and diagonal matrix respectively.

Spatial spectrum of MUSIC can be got, which can be expressed as
\[
P(\theta) = \frac{1}{B^H(\theta)U_q U_q^H B(\theta)}
\] (14)
Searching spectral peak of $P(\theta)$, we can get the DOA estimator $\theta_j$. We define $C_{11} = R_{BB} - \sigma^2 I = BR_{GG}B^H$ and $C_{12} = R_{BB} - \sigma^2 Z = BAR_{GG}B^H$, where $Z$ is showed as $Z = \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 1 & \ldots \\ 0 & 1 & 0 \end{bmatrix}$. According to $C_{11}$ and $C_{12}$, we can get $A = B^aC_{12}C_{11}^aB$.

where $(\cdot)^a$ denotes the Moore-Penrose pseudo-inverse. Therefore, the DOD estimator $\phi_i$ is estimated by

$$\phi_i = \arcsin\left(\arg\left(a_i\right)/\left(q - 1\right)\pi\right)$$

(15)

where $a_i$ is the element of the principal diagonal of matrix $A$, $\arg(a_i)$ stands for the phase of $a_i$.

SIMULATION RESULTS

The considered bistatic MIMO radar is composed of $Q = 4$ transmit antennas and $N = 6$ receive antennas. Supposed the target locates at the positions $(\phi_1, \theta_1) = (30^\circ, 20^\circ)$, $(\phi_2, \theta_2) = (60^\circ, 50^\circ)$ and Doppler parameters are $f_1 = 8$, $\mu_1 = 1.2$, $f_2 = 10$, $\mu_2 = 2$ respectively. The number of snapshots is 1000. The number of Monte Carlo iterations is 500 in all simulations. In the following simulation experiments, we study the resolution capability and estimation accuracy of the proposed method, Parallel factor (PARAFAC) method[3] and ESPRIT using the rotational factor produced (RFP-ESPRIT) method[8].

Simulation 1: Signal to Noise Ratio

Fig.2 depicts the root-mean-square error (RMSE) as a function of Signal-to-Noise Ratio (SNR) when $M = 1000$. From Fig.2, we find that the performance of the proposed method is significantly better than that of the PARAFAC method and RFP-ESPRIT method.

Fig.3 shows RMSE curves for DODs and DOAs estimation of the proposed method, the PARAFAC method and RFP-ESPRIT method versus SNR. In this simulation, and are set. As we can see, the proposed method gives better performance than the PARAFAC method and RFP-ESPRIT method in the condition of low SNR. From these figures, we also find that the estimation performance of the Doppler frequency parameters affects the estimation performances of the DOD and DOA.
Fig. 4 illustrates the scatter grams of the DOAs and DODs estimated by the proposed method, the PARAFAC method and RFP-ESPRIT method based on 100 independent trials under the hypothesis that SNR is equal to 7dB and other simulation conditions are exactly the same as those described in Simulation 2. From Fig. 4, we can observe that the proposed method provides a more precise location estimate than the PARAFAC method and RFP-ESPRIT method.

Simulation 2: The estimated result of DOAs and DODs

CONCLUSIONS

For moving target, the echo often contains time-varied Doppler frequency which results in difficulties in the parameters estimation. Thus, this paper proposes a novel method for estimating time-varied Doppler frequency and DOD-DOA in bistatic MIMO radar system. Firstly, Doppler parameters are estimated by the fractional correlation algorithm. Secondly, two subarray models are constructed and two algorithms are presented to estimate DOD and DOA. Simulation results demonstrate that the proposed method still has good performance when poor SNR condition exists. The next step of research focus is how to improve the performance of parameter estimation in the impulsive noise environment.

This work was partly supported by the National Science Foundation of China under Grants 61401055, and the Public science and technology research funds projects of ocean under Grants 201005011. The authors also gratefully acknowledge the helpful comments and suggestions of the reviewers, which have improved the presentation.
REFERENCES