Analysis of Optimal Hedging Ratio of Copper Futures Based on ECM Model

Qing CAO
Shanghai University, 99 Shang Da Road, Baoshan District, Shanghai, China
1004681506@qq.com

Keywords: ECM Model; Gibbs Sampling; Copper Futures; The Optimal Hedging Ratio.

Abstract. In this paper, on the one hand, we need to prove that there is a cointegration relationship between copper futures price and spot price. Then, using the ECM model to estimate the optimal hedging resale value of copper futures contract in our country, it is based on the frequency statistics method, the optimal hedging strategy, the approach does not consider the existence of risk estimation, and the most natural way to solve this problem is the bayesian method. So, on the other hand, we are under the bayesian framework, Gibbs sampling method is used to the Chinese copper futures market, the optimal hedging strategy has carried on the empirical analysis. By comparing the hedging efficiency, the result of hedging based on Bayesian statistics is better than that based on frequency statistics.

Introduction

Hedging is one of the two important functions of the futures market. China's copper consumption accounts for more than 40% of the world, is a big copper consumption. Some copper spot as raw material enterprises to avoid the impact of spot price volatility, through the copper futures market to hedge. In order to hedge, hedgers will buy and sell a certain percentage of futures contracts, in which case they need to consider how to choose an optimal hedge ratio. Therefore, the optimal hedging rate of copper futures research is still very large practical significance. The ECM model and the ECM model under the Bayesian framework are used to analysis the optimal hedging ratio of copper futures.

Data

In this paper, the data of copper futures price and spot price which we used are derived from Wind. I collected a total of 395 data from May 25, 2015 to December 30, 2016.

In the article, S represents the copper spot price. F represents the copper futures price. LNS and LNF represent the natural logarithmic conversion of copper spot price and futures prices.

The theory of the classical ECM model

Engel, Granger (1987) proposed an error correction model for futures prices and spot prices in the presence of cointegration relationships. Ghosh (1993), based on the study of Engle and Cranger, proposed an error correction model for estimating the hedge ratio with minimum risk (ECM):

\[
\Delta S_t = \theta + \Delta F_t + \gamma \text{ecm}_{t-1} + \sum_{j=1}^{m} \delta_j \Delta F_{t-j} + \sum_{j=1}^{n} \eta_j S_{t-j} + \epsilon_t
\]

(1)

φ is the best hedge ratio. S and F represent copper spot price and copper futures price respectively. M and n respectively stand for the spot price and the futures price changes in the amount of the best lag.

Prior distribution of ECM model

\[
\theta \sim \text{dnorm}(\mu(\theta), 0.5)
\]

(2)

\[
\varphi \sim \text{dnorm}(\mu(\varphi), 0.5)
\]

(3)
\[
\gamma \sim \text{dnorm}(\mu(\gamma), 0.1)
\]

The results obtained by the classical ECM model are the mean of the prior distribution of the parameters of this model, that is \(\mu(\cdot)\).

**Gibbs sampling**

The Gibbs sampling requires that samples be extracted directly from the conditional posterior distribution of each sub-vector of the unknown parameter vector \(\Theta\). The unknown parameter vector \(\Theta\) is divided into three sub-vectors, that is, \(\Theta = (0, \phi, \gamma)\). If the conditional posterior distribution of the parameters of the above parameters is a standard distribution, the Gibbs sampling is performed as follows:

1. Set the initial value for all parameter vectors.
2. The first \(t\) iterative sampling: firstly, sampling from \(p(\theta|\phi, \gamma, X, Y)\) and extract \(\theta^t\), then extract \(\phi^t\) and \(\gamma^t\) at the same way.
3. Repeat step (2) until the sample sequence converges to the stationary distribution independent of the initial value, that is, the target distribution we are trying to simulate.

**Hedging effectiveness**

In order to investigate the efficiency of hedging, we use the traditional measure proposed by Kroner and Sultan (1993):

\[
HE = \frac{(\text{Var}(U) - \text{Var}(H))}{\text{Var}(U)}
\]

(5)

"\(r_u\)" represents the rate of return without hedging. "\(r_h\)" represents the rate of return with hedging.

\[
r_u = \ln S_t - \ln S_{t-1}
\]

(6)

\[
r_h = (\ln S_t - \ln S_{t-1}) - h^*(\ln F_t - \ln F_{t-1})
\]

(7)

\(h\) represents the optimal hedge ratio. \(\text{Var}(U)\) represents the variance of the rate of return without hedging. \(\text{Var}(H)\) represents the variance of the rate of return with hedging.

**Optimal Hedging Ratio of Copper Futures Based on Classical ECM Model**

The price series of futures and spot is often non-stationary, and there may be cointegration relations. Between the futures price and the spot price of both the short-term dynamic relationship and long-term equilibrium relationship.

**Unit root test of data**

Before the analysis, firstly, we need to test the stationarity of the data. If we regress the non-stationary data directly, there will be a "pseudo regression" phenomenon. We were testing the stationarity of the copper futures price and spot price respectively. Because the log processing can eliminate heteroscedasticity, and it will not affect the cointegration relationship between the original data. Following the logarithmic transformation of the spot price and futures prices and their differential sequence, we used the ADF test. The ADF test results are shown in the following table:

<table>
<thead>
<tr>
<th>variable</th>
<th>t-Statistic</th>
<th>P value</th>
<th>1% level</th>
<th>5% level</th>
<th>10% level</th>
<th>conclusion</th>
</tr>
</thead>
<tbody>
<tr>
<td>LNS</td>
<td>-1.4641</td>
<td>0.5509</td>
<td>-3.4468</td>
<td>-2.8687</td>
<td>-2.5706</td>
<td>non-stationary</td>
</tr>
<tr>
<td>LNF</td>
<td>-1.5783</td>
<td>0.4122</td>
<td>-3.4468</td>
<td>-2.8687</td>
<td>-2.5706</td>
<td>non-stationary</td>
</tr>
<tr>
<td>△LNS</td>
<td>-19.2128</td>
<td>0.0000</td>
<td>-3.4468</td>
<td>-2.8687</td>
<td>-2.5706</td>
<td>stationary</td>
</tr>
<tr>
<td>△LNF</td>
<td>-16.8756</td>
<td>0.0000</td>
<td>-3.4468</td>
<td>-2.8687</td>
<td>-2.5706</td>
<td>stationary</td>
</tr>
</tbody>
</table>
We can get the first order difference sequence of LNS and LNF are smooth. The I (1) process is consistent with the logarithmic spot price series and the logarithm futures price series.

**Cointegration analysis**

There is a cointegration relationship between the two variables, which means that there is a long-term stable equilibrium relationship between the two variables. We used the E-G two-step method for cointegration testing. The E-G two step method of analysis is as follows: firstly, we make the cointegration regression, and then check the stability of the residual sequence. If the residual sequence is a stationary sequence, there is a co integration relationship between the two variables. The LNS and LNF sequences are estimated by OLS, and the unit root test of residual e is performed:

Table 2. ADF test results of residual sequence.

<table>
<thead>
<tr>
<th>variable</th>
<th>t-Statistic</th>
<th>P value</th>
<th>1% level</th>
<th>5% level</th>
<th>10% level</th>
<th>conclusion</th>
</tr>
</thead>
<tbody>
<tr>
<td>e</td>
<td>-5.4780</td>
<td>0.0000</td>
<td>-3.4468</td>
<td>-2.8687</td>
<td>-2.5706</td>
<td>stationary</td>
</tr>
</tbody>
</table>

From Table 2, we can see that the t statistic of the residual is -5.4780, and the p value is 0.0000 less than 1% of the test level. Therefore, the hypothesis of the unit root of residual is rejected. The residual sequence is considered smooth. There is a long-term equilibrium relationship between copper futures and spot prices.

**Classical error correction model**

The following error correction model (ECM) is established to analyze the adjustment speed when the deviation from the long-term equilibrium. The estimated results of the ECM model for copper spot prices and futures prices are as follows:

$$\Delta LNS = -0.000178 + 0.730582\Delta LNF - 0.671155 ecm(-1) + \epsilon,$$  \hspace{1cm} (8)

The estimated parameters of the model are significant. P value is very small. Thus, it shows that the model is significant overall estimation. The coefficient of D (LNF) is very significant, and the coefficient of error correction is also very significant.

To sum up, we can get the optimal hedge ratio of copper futures, namely $h=0.7306$.

**Optimal Hedging Ratio of Copper Futures Based on Bayesian Statistics**

A complete Bayesian analysis model consists of the combined prior distribution density of all unobservable variables and the likelihood function of the model. Bayesian inference is based on the joint posterior distribution of unobservable variables. According to the Bayesian theorem, the combined posterior density distribution of the unobservable variables is proportional to the union a priori of the unobservable variables multiplied by the likelihood function of the model.

**Doodle diagram of ECM model**

![Doodle diagram](image)
Bayesian estimation of parameters

In the operation of the model, a set of initial values are input, and the Gibbs iteration is performed for 10000 times. The simulation results of the iterative chain locus and the posterior distribution of the parameters are obtained. The Bayesian estimation results for the corresponding parameters are given in Table 3.

![Iteration chain track of parameters of ECM model.](image)

Table 3. Bayesian Estimation of ECM Model.

<table>
<thead>
<tr>
<th>parameter</th>
<th>mean</th>
<th>variance</th>
<th>2.5%</th>
<th>Median</th>
<th>97.5%</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\theta$</td>
<td>-0.001551</td>
<td>0.09006</td>
<td>-0.1777</td>
<td>-0.001861</td>
<td>0.1746</td>
</tr>
<tr>
<td>$\varphi$</td>
<td>0.627</td>
<td>8.738</td>
<td>-16.34</td>
<td>0.5517</td>
<td>18.18</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>-0.6493</td>
<td>3.06</td>
<td>-6.704</td>
<td>-0.6539</td>
<td>5.286</td>
</tr>
</tbody>
</table>

The Bayesian estimates for the ECM model parameters are given in Table 3. The Bayesian estimates for $\theta$ are -0.0016, and the 95% Bayesian interval for $\theta$ is (-0.1777, 0.1746). The Bayesian estimate of $\varphi$ is 0.627, and the 95% Bayesian interval for $\varphi$ is (-1.6.34, 18.18); the Bayesian estimate of $\gamma$ is -0.6493, and the 95% Bayesian interval for $\gamma$ is (-6.704, 5.286).

Thus, the model can be written as:

$$y = -0.0016 + 0.627x - 0.6493ecm + \epsilon$$

To sum up, we can get the optimal hedge ratio of copper futures, namely $h=0.627$.

Summary

According to the formula of hedging efficiency, we can calculate the hedging efficiency based on frequency statistics is 43.05%. While under the hedging efficiency based on the Bayesian statistics is 44.15%. Therefore, we can conclude that the effectiveness of hedging based on Bayesian statistics is better than that based on frequency statistics.

References


