Intersection Signal Management Based on Multi-Objective Optimization and Decision

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Keywords: Traffic Signal Management, Multi-Objective Optimization, Multi-Attribute Decision.

Abstract. Aiming at solving the multi-objective signal timing optimization and decision making problem for isolated intersections, a multi-objective optimization and decision making integration method is proposed in this paper. Firstly, a multi-objective optimization signal management model is established. Then multi-objective optimization algorithm Hyper-volume Estimation Algorithm (HypE) is used to obtain the signal timing described as Pareto set, and an improved method for uncertain multi-attribute decision problem in which attribute weights are set as interval numbers is proposed to select the satisfactory solution from the Pareto set. Simulation results show that compared with entropy weight method and Technique for Order Preference by Similarity to an Ideal Solution (TOPSIS) method, the proposed method not only can balance the tradeoff between preference demand of decision-makers and objective evaluation information, but also improve comprehensive traffic efficiencies effectively. Moreover, in comparison with the classical Australian Road Research Board (ARRB) and Highway Capacity Manual (HCM) method, the proposed multi-objective optimization and decision making method proposed brings better control effects.

Introduction

Intersection signal control is one of basic ways for urban traffic management. And the isolated control is the foundation of area control in traffic management. In former researches, isolated traffic signal control mainly concentrated on average vehicle delay. Obviously, this kind of traffic signal control procedure can not effectively take account into all of the performance indexes, such as queue length, average stop rate, capacity et al. In modern traffic management, more performance indexes should be considered to obtain better comprehensive traffic benefits [1].

There are two important conventional methods to solve multi-objective optimization problems. One is the trade-off weight method [2], which aggregates the multiple objectives into one objective function by adding a set of trade-off weights. But the control result heavily depend on the selection of trade-off weights. And the result may not be satisfactory ones if unsuitable weights were assigned in this way. Another method is the constraint method, which optimizes the most important control objective and transfroms the others into system constraints. However, it is very difficult for users to determine suitably constraint bounds in a practical problem. In consideration of the shortcomings of conventional methods, this article selects multi-objective evolutionary algorithms (MOEAs) to achieve optimization and coordination of multiple targets.

In addition, it is necessary to choose the most “satisfied” applications after getting the Pareto optimal set. Definitely, choosing a satisfactory timing plan from finite alternatives is a typical multi-attribute decision making (MADM) problem, which has attracted great interest in academia. Literature review presents that latest researches focus on certain decision information of attribute weights and attribute values. However, due to complexity of physical environment, fuzziness thoughts of decision-makers, measuring errors and other factors, decision information is often uncertain in practice. Generally, interval numbers are widely used to describe uncertain decision information. In this paper, the value of traffic performance index is precise information, and the
weight vector is given in the form of interval numbers by decision makers. Thus, it is a multi-attribute decision problem with interval numbers. By means of interval mathematics, traditional approaches firstly transformed attribute weights with interval numbers into attribute values with interval numbers. Then, interval numbers ranking methods were applied to accomplish ranking of alternatives. However, there exist difficulties during ranking alternatives, because interval numbers can cross each other [3]. Several methods assuming the membership functions of interval number have been raised. But, these methods are criticized by insufficient usage of decision matrix’s objective information and limitations in practical applications. Consequently, this article presents a new decision-making method, which primarily develops a simple mathematical optimization model to combine information of subjective interval weights and objective entropy weights. Secondarily, interval weight vector of alternatives is transformed into certain vector by solving the optimization model in the method. In this way, interval multi-attribute decision problem become a certain one.

Multi-objective Optimization Model of Signal Timing

Performance indicators selection

With increasing process of urbanization, purposes of signal control have pursued multiple traffic benefits which include safety, environmental protection and energy utilization. These performance indexes can be divided into vehicle benefits, pedestrian benefits and environmental benefits. It is widely known that, there are mapping relationships between these performances indexes and control parameters. Specifically, if the control parameters change, so did the correlation of performances indexes. For example, the longer cycle time is, the higher road capacity and longer average vehicle delays will be, thereby the energy consumption and pollutant emission will be more serious. Particularly, in isolated control, signal control parameters mainly contains cycle time and green splits, where the cycle time belong to the strategic control parameters, and splits belongs to the tactics parameters [4]. Besides, road capacity is another key index which should be considered. Therefore, without considering the slow mode transportation flows at unsaturated intersections, we select average vehicle delay, road capacity, average vehicle stop rate and queue length as performance indexes to optimize signal timing parameters.

Multi-objective optimization model

Assuming that vehicle flows arrive at intersections uniformly, this article select minimum average vehicle delay, minimum average vehicle stop rate, maximum road capacity, minimize maximum queue length as optimization objects, and establish a multi-objective optimization model of signal timing at isolated intersections. Taking saturation as a constraint, the model is described as:

\[
\begin{align*}
\min \ F(C, \lambda) &= [f_1(C, \lambda), f_2(C, \lambda), f_3(C, \lambda)] \\
\text{s.t.} \quad & C_{\text{min}} \leq C \leq C_{\text{max}}, \quad C\lambda_j \geq g_{j\text{min}} \\
& \sum_{j=1}^{m} \lambda_j + \frac{L}{C} = 1, \quad j = \{1, 2, \cdots, m\} \\
& \alpha \leq 0.9
\end{align*}
\]

(1)

Where, \( f_1(C, \lambda) = \frac{\bar{d}}{Q} \) represents the ratio of average delay [5] to road capacity (be called ADDRC for short) at single intersection, \( f_2(C, \lambda) = \bar{sp} \) represents average vehicle stop rate [5], \( f_3(C, \lambda) = bq \) is the queue length at intersection in a cycle [5], \( C_{\text{min}}, C_{\text{max}} \) are lower and upper bound of cycle time respectively, \( g_{j\text{min}} \) is minimum green time of phase j. \( L \) is lost time per cycle, \( \alpha \) is the intersection saturation, \( m \) is the number of phases. \( n \) is the number of vehicle flows.
Hyper-volume Estimation Algorithm for Multi-objective Optimization

In the evolution multi-criterion optimization field, the hypervolume indicator, which is the most popular set quality measure, is always used to compare the dominance of Pareto sets. Usually, the larger indicator value is, the greater dominance is. In recent years, the EAs based on hypervolume indicator become a research hotspot. However, the high computational complexity of hypervolume indicator limits its further application. The HypE introduced by Johannes Bader [6] proposed a new fitness assignment strategy as the key technique to handle the obstacle. In Bader’s strategy, when the number of objectives is no more than three, the exact hypervolume values are computed; otherwise, they are estimated by using Monte Carlo methods. Because of only three traffic optimization objects in consideration, this article selects the exact compute hypervolume values method by Johannes Bader.

Multi-attribute Decision Making Analysis of Timing Plans

Based on the correlations between optimization objects and performance indicators, the evaluation information of timing plans could be obtained. Obviously, traffic performance indicators selected in this paper become attributes of decision matrix. For the sake of convenience, let $M = \{M_1, M_2, \ldots, M_m\}$ be a discrete set of feasible timing alternatives, $D = \{D_1, D_2, \ldots, D_n\}$ be a finite set of attributes, $\omega_j = (\omega_{j1}, \omega_{j2}, \ldots, \omega_{jr})^T$ be the subjective weight vector of attributes, with

$$\omega_j = [\omega_{j1}, \omega_{j2}], \sum_{j=1}^{n} \omega_{ij} \leq 1, \sum_{j=1}^{n} \omega_{ij} \geq 1, 0 \leq \omega_{ij} \leq 1.$$  

Decision matrix of timing plans

Supposing $x_{ij}$ $(i = 1, 2, \ldots, m; j = 1, 2, \ldots, n)$ respects the consequence of alternative $M_i$ with respect to attribute $D_j$, the decision matrix $X = (x_{ij})_{m \times n}$ can be formulated as follows:

$$X = \begin{bmatrix} D_1 & D_2 & \cdots & D_n \\ M_1 & x_{11} & x_{12} & \cdots & x_{1n} \\ M_2 & x_{21} & x_{22} & \cdots & x_{2n} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ M_m & x_{m1} & x_{m2} & \cdots & x_{mn} \end{bmatrix}$$  

(2)

In multi-attribute decision making (MADM) problem, the decision matrix allows each attribute to have the same range of measurement. This is achieved by normalizing every element in matrix into a corresponding element in matrix [7].

Comprehensive weight vector $\bar{\omega}$

In order to determine weights $\bar{\omega} = (\bar{\omega}_1, \bar{\omega}_2, \ldots, \bar{\omega}_n)^T$ to reflect both subjective consideration of the decision makers and objective information, we minimize the deviation of the subjective information $\omega_i$ and objective information $\bar{\omega}_i$. Thus, the weights can be obtained by solving the constrained optimization problem:

$$\text{Min } f(\bar{\omega}) = \sum_{j=1}^{n} \frac{(\bar{\omega}_j - \omega_{ij})^2}{W_{ij}} \quad \text{s.t. } \omega_{ij}^l \leq \omega_j \leq \omega_{ij}^u, \sum_{j=1}^{n} \bar{\omega}_j = 1$$  

(3)

Where the physical interpretation of $f(\bar{\omega})$ is the total deviation between the subjective and objective information, the geometric meaning of which is the square of the distance from a fixed-point in solution space to searching space. Take alternatives with two attribute for example: In Figure 2, on the horizontal axis is the weight $\omega_{ij}, \omega_{ij}^l, \omega_{ij}^u$ of one performance index, and on the
vertical axis is the weight $\omega_2, \omega_2', \omega_2^*$ of the other performance index. The aim of the Eq. 3 is to minimize the distance from the fixed-point $E(\omega_1, \omega_2)$ to searching space ABCD, which can be divided into nine sections: I, II, III, IV, V, VI, VII, IX. If $E(\omega_1, \omega_2) \in I$, the length of line $ED$ is the minimal distance from point $E(\omega_1, \omega_2)$ to rectangle ABCD. If $E(\omega_1, \omega_2) \in V$, the minimal distance is the distance from point $E(\omega_1, \omega_2)$ to line $CD$, then $\omega = (\omega_1, \omega_2')$. If $E(\omega_1, \omega_2) \in IX$, the value of objective function is 0, $\omega = (\omega_1, \omega_2^*)$. The other situations can be discussed in the same way.

If there are more than two attributes in the MADM problem, $\bar{\omega}$ is obtained by solving the model (3) with the utility of simple GA.

![Figure 1. Optimization model.](image)

### Selection of decision plan

The weighted decision matrix is constructed based on Eq. 4, sorted by TOPSIS [8] to obtain the optimal plan.

$$r_j = \bar{\omega}_j \cdot v_{ij}$$  \hspace{1cm} (4)

### Case Study

In this section, an interaction of Tianmu Mountains Road and Hengfeng Road in Zhabei District in Shanghai is selected as a case to illustrate the validity of the optimizing and decision making method of signal timing problem presented by this paper. The phase scheme includes four phases, while all approaches of this interaction are channelized. Through practical investigation, the vehicle flow of each approach in peak time is listed in Table 1.

<table>
<thead>
<tr>
<th>Approach</th>
<th>East</th>
<th>West</th>
<th>South</th>
<th>North</th>
</tr>
</thead>
<tbody>
<tr>
<td>Flow direction</td>
<td>Left</td>
<td>Right</td>
<td>Though</td>
<td>Left</td>
</tr>
<tr>
<td>Volume (pcu/h)</td>
<td>331</td>
<td>633</td>
<td>1004</td>
<td>732</td>
</tr>
</tbody>
</table>

Based on the traffic data in Table 1, this paper builds multi-objective optimization model of signal timing, and the related parameters are as follow: $C_{min}$ is 50s, $C_{max}$ is 150s, $L$ is 10s, the minimum green time of each phase is 5s. Meanwhile, the parameters of HypE are initialized as follows: Popsize is 100, Gmax is 500, Crate is 0.95, Mrate is 0.05, and the reference set $R = (2000, 2000, 2000)$.

### Comparison with other decision making methods

Considering performance indicators’ properties and objective demands, this paper classifies road capacity as benefit attribute, average vehicle stop rate and maximum queue length as cost attribute and average vehicle delay as zone attribute with $[34.0s, 36.0s]$ as its optimum zone. After
standardizing the decision matrix of timing plans, weights of performance indicators used in this paper can be calculated. Table 2 displays information of weights in detail.

Applying $\omega_i$ and TOPSIS, the optimizing and decision making method (be called ODMM for short) presented by this paper obtains its corresponding optimum signal timing plan. Similarly, the objective entropy weight method (be called OEWM for short) [9] can get relevant alternative as well by using $\omega_i$ and TOPSIS. Besides, as a typical method to deal with interval MADM problems, the interval TOPSIS [18] can also obtain the ranking of alternatives. The consequences of different decision making methods are shown in Table 3.

Table 2. Information of weights.

<table>
<thead>
<tr>
<th>Weight</th>
<th>Average vehicle delay</th>
<th>Road capacity</th>
<th>Maximum queue length</th>
<th>Average vehicle stop rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\omega_i$</td>
<td>[0.5,0.6]</td>
<td>[0.3,0.5]</td>
<td>[0.1,0.3]</td>
<td>[0.1,0.2]</td>
</tr>
<tr>
<td>$\omega_h$</td>
<td>0.1967</td>
<td>0.2846</td>
<td>0.2341</td>
<td>0.2846</td>
</tr>
<tr>
<td>$\omega$</td>
<td>0.5000</td>
<td>0.3000</td>
<td>0.1000</td>
<td>0.1000</td>
</tr>
</tbody>
</table>

Table 3. Consequences of decision making methods.

<table>
<thead>
<tr>
<th>Cycle length(s)</th>
<th>OEWM</th>
<th>Interval-TOPSIS</th>
<th>ODMM</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average vehicle delay(s/pcu)</td>
<td>48.6924</td>
<td>37.8062</td>
<td>38.4239</td>
</tr>
<tr>
<td>Road capacity(pc/h)</td>
<td>6396.7916</td>
<td>6088.9392</td>
<td>6119.4292</td>
</tr>
<tr>
<td>Average vehicle stop rate</td>
<td>0.9325</td>
<td>0.9468</td>
<td>0.9454</td>
</tr>
<tr>
<td>Maximum queue length(pcu)</td>
<td>36.7337</td>
<td>25.2690</td>
<td>26.0376</td>
</tr>
</tbody>
</table>

Table 3 suggests that the effectiveness of ODMM is parallel with that of interval TOPSIS, which implies that ODMM can get pretty good traffic efficiencies in off-line control system. Compared with OEWM, ODMM decreases average vehicle delay by 21.09%, reduces maximum queue length by 29.12%, while it slightly increases average vehicle stop rate by 1.38% and decreases road capacity by 4.43%. Thus, we assert that ODMM not only improves comprehensive traffic efficiencies effectively, but also selects the signal timing parameters flexibly to fit objective and subjective requirements.

Table 4. Results of different signal timing methods.

<table>
<thead>
<tr>
<th>Cycle length(s)</th>
<th>HCM</th>
<th>ARRB</th>
<th>ODEM</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average vehicle delay(s/pcu)</td>
<td>43.9089</td>
<td>42.7713</td>
<td>38.4239</td>
</tr>
<tr>
<td>Road capacity(pc/h)</td>
<td>6299.7000</td>
<td>6270.5347</td>
<td>6119.4292</td>
</tr>
<tr>
<td>Average vehicle stop rate</td>
<td>0.9370</td>
<td>0.9384</td>
<td>0.9454</td>
</tr>
<tr>
<td>Maximum queue length(pcu)</td>
<td>32.0060</td>
<td>32.8349</td>
<td>26.0376</td>
</tr>
</tbody>
</table>
**Comparison with other signal timing methods**

In this study, HCM signal timing method is available, because the saturation constraint is below 0.9. However, the cycle length is determined by Synchor timing examination owing to the result of HCM method dissatisfies the cycle constraint, while other parameters still computed by HCM method. Besides, ARRB method is also a feasible choice for signal timing of this case which makes the traditional Webster method more acceptable by introducing the stop penalty parameter. Table 4 lists the results of different methods. It indicates that ODMM can get better traffic effects than HCM and ARRB method. Compared with HCM method, ODMM decreases average vehicle delay by 12.49% and reduces maximum queue length by 18.65%, although it makes average vehicle stop rate increased slightly by 0.89% with only 2.86% decrease of road capacity. Compared with HCM method, ODMM decreases average vehicle delay by 10.16%, reduces maximum queue length by 20.7%, while it slightly increases average vehicle stop rate by 2.14% and decreases road capacity by 0.75%. In conclusion, ODMM improves vehicle delay performance and queue length performance at the expense of a small amount of vehicle stop rate and road capacity, which accomplish better comprehensive traffic efficiencies.

**Conclusions**

This paper proposes a novel optimizing and decision making method to deal with the signal timing problem. Firstly, establishing a multi-objective optimization model of signal timing. Secondly, HypE is introduced as the multi-objective optimization method to solve this model directly and obtain the signal timing plans. Thirdly, with the purposes of select the final satisfactory signal timing plan, a simple optimization model which integrates subjective and objective weight information is presented. Particularly, a numerical case study demonstrates several conclusions as follows:

- Compared with interval TOPSIS and OEWM, it can figure out that ODMM can not only balance decision-makers’ needs and objective evaluation information flexibly, but also acquire satisfactory traffic comprehensive benefits at the same time;
- Compared with ARRB and HCM method which are widely used in practice, ODMM brings better comprehensive efficiencies as well.

Aiming at the signal timing problem for isolated intersections under unsaturated status, this paper analyses the relevant optimization and decision making method. In further study, more traffic compositions including pedestrian flows and bicycle flows will be analyzed while taking oversaturated traffic network into consideration.

**Acknowledgment**

This work is financially supported by the National Natural Science Fund Projects (61104166), Shanghai City Board of education scientific research innovation projects (12YZ029).

**Reference**


