Research for Unsteady Seepage Flow of Asymmetrical Fractured Vertical Wells in Coalbed

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ABSTRACT

In this study, based on Green function and Laplace transformation, a pressure transient analysis semi-analytical evaluation model which can be used for coalbed reservoirs of finite conductivity asymmetrically fractured vertical well was established. The mathematical model considers asymmetrically fracture and finite conductivity. Fick law was used to describe gas diffusion in spherical matrix and Lagrange function was used to depict the unsteady desorption of coalbed gas. The influences of related parameters, such as artificial fracture conductivity, asymmetry factor, storativity ratio, cross-flow factor, dimensionless drainage radius and so forth, on the seepage flow were analyzed by using the established model. The results show that The percolation process include 6 stages (a) artificial fracture flow with the effect of wellbore storage effect; (b) transition flow; (c) linear flow between matrix and artificial fracture; (d) cross-flow between matrix and fracture in double medium fractures; (e) the pseudoradial flow stage of whole system; (f) closed boundary flow. Artificial fracture conductivity has great impact on whole production cycle. With the artificial fracture conductivity increases, the raise of capacity is very significant, especially for the production period after the influence of wellbore storage effect. Asymmetry factor is also important to whole production cycle. The bigger asymmetry factor is, the lower capacity is. Storativity ratio and cross-flow factor influence the degree and occurrence time of cross-flow between matrix and

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fractures respectively. The smaller storativity ratio is, the more observer cross-flow is. The smaller cross-flow factor is, the earlier cross-flow will happen. The dimensionless drainage radius has only an effect on the later production. The smaller dimensionless drainage radius is, the earlier the closed boundary flow will happen.

INTRODUCTION

Hydraulic fracturing plays an important role on the exploitation of unconventional oil and gas reservoirs[1-3], as the same as the shale reservoirs, coal seam reservoirs are the double porosity media structure composed of pore and fracture, and the face cleat and butt cleat shearing each other to form a fracture network, three-phase media including matrix of coal, water and gas is coexistent[4-6]. These special properties make the mechanical properties of coal and rock mass extremely complex, and the fracture cracks have strong uncertainty and uncontrollable in the three-dimensional extension and distribution. Xuejun Shan[7] explained the fracture geometry of the 21 CBM fracturing wells, the results show that there are three kinds of cracks in the form of expansion: single-wing cracks, asymmetric double-wing cracks, vertical and horizontal cracks symbiotic type, of which asymmetric two-wing cracks accounted for more than 90%. Many scholars at home and abroad have done a lot of research on this aspect[8-24], the domestic aspects, Wei Chen[8] established the evaluation model of finite conductivity and symmetrical fractured vertical wells in coalbed methane reservoirs; Weiping Ouyang[9] uses the finite element method to establish the numerical well test model of vertical fractured well of coalbed methane infinite conductivity, and the double logarithmic well test theory chart is drawn; Haitao Cao[13] used the theory of point source function to deduce modern decreasing plate of CBM symmetrically fractured well production; Baojun Cao[14] established a model for asymmetric fracture productivity of volcanic rocks based on the principle of conformal transformation and equivalent filtrational resistance; Wenjuan Wu[15] took the Chang 6 oil reservoir in Ordos Basin as an example, using the logging data to carry out the geological modeling and studying the three-dimensional stress field established the numerical simulation model of the asymmetric fracturing in the ultra-low permeability oil and gas reservoirs; Jian Xiong[16] derived productivity prediction model of finite conductivity asymmetric vertical fractured wells in low permeability gas reservoirs based on the steady flow theory and conformal transformation. The foreign aspects, Benjamin, J. and Barker[17] studied the pressure dynamics of finite conductivity symmetrical fractured vertical well in coalbed methane with confined boundaries based on the assumption of two dimensional single-phase Darcy flow; K.H.Guppy[18] established a numerical and semi-analytical model to analyze the high velocity non-Darcy's flow behavior of the finite conductivity fractured wells in coalbed methane reservoirs; Fernando Rodriguez[19] established a semi-analytical model of finite conductivity asymmetrical fractured well in oil reservoirs based on a new solution to the dynamic analysis of quasi-linear flow and bilinear flow pressure;

In this paper, based on Green function and Laplace transformation, a pressure transient analysis semi-analytical evaluation model which can be used for coalbed reservoirs of finite conductivity asymmetrically fractured vertical well was established. The mathematical model considers asymmetrically fracture, finite conductivity, diffusion of coalbed methane and desorption analysis, it can reflect the influence of different parameters on the dynamic propagation of pressure easily, and it can consider the influence of different parameters such as wellbore storage effect and skin effect, and it can give the explanation of the asymmetrical fractured vertical well reservoir and fracture parameters of coalbed methane.

MODEL BUILDING

Physical Model

The different maceral and the degree of coalification correspond to different mechanical properties, under the combined action of tectonic stress and gelatinous material shrinkage, there are face cleat, butt cleat, shear fractures, tension fracture and cleavage in the coal interior, they are intertwined to form a complex percolation system (shown in Figure 1). The entire seepage process can be simply divided into three processes: (1) The fracture-wellbore stage, the production wells asymmetric fracturing, with the production, the wellbore pressure is reduced, under the pressure differential, gas stored in artificial fractures flow into the wellbore, the process to comply with Darcy's flow law; (2) Matrix crack-artificial fracture stage, with the artificial crack pressure decreased, the gas in matrix crack system into the artificial cracks, the flowing behavior of coalbed gas in the matrix cracks also can be described by Darcy's flow law; (3) Matrix-matrix fracture stage, with the gas in the matrix cracks is discharged, the pressure decreases, the gas adsorbed by the matrix is resolved and diffused to the matrix fracture system under the action of concentration difference. In this paper, the classical Lagrange isothermal equation is used to describe the adsorption and desorption of gas on the surface of matrix approximately, the Fick law is used to characterize gaseous diffusion. To sum up, the basic assumptions of the model include:

(1) The upper and lower layers of coal reservoir are closed, isotropic, the thickness is uniform and incompressible;
(2) Only the gas single-phase seepage is considered, and the slightly compressible is considered; (3) Considering the finite conductivity, the fracture and reservoir parameters do not change with the pressure change in the asymmetrically fractured vertical well; (4) Isothermal production, the flow of gas in artificial fractures and matrix fracture systems follows the Darcy's percolation Isothermal production, the flow of gas in artificial fractures and matrix fracture systems follows the Darcy's flow law.

![Fracture network reconstruction vertical well.](image)

**Mathematical Model**

It is known that there is an asymmetrically fractured vertical well in the center of the circular coalbed gas reservoir, and the production is determined (the unit of physical quantity in the process of deduction follows the standard unit of SI).

**SIMULATION OF ARTIFICIAL FRACTURE**

Dimensionless definition:

\[
\frac{x_{wD}}{x_f} = \frac{x_w}{x_f}; \quad \frac{q_{ID}}{q_{sc}} = \frac{2x_tq_t}{x_f}; \quad \frac{x_D}{x_f} = \frac{x}{x_f}; \quad \frac{y_D}{x_f} = \frac{y}{x_f}; \\
p_{ID} = \frac{p_i^2 - p_r^2}{p_i^2q_D}; \quad q_D = \frac{T\mu Z}{khp_i^2q_{sc}}; \quad C_{ID} = \frac{k_iw}{kx_f}
\]
Based on dimensionless treatment, the governing equation of the producing under the set production fracture of asymmetrically fractured well is:

\[
\frac{\partial^2 p_D}{\partial x_D^2} + \frac{2}{C_{\text{Mov}}} \frac{\partial p_D}{\partial y_D} \bigg|_{y_D=0} + \frac{2\pi}{C_{\text{Mov}}} \delta(x_D - x_{wD}) = 0
\]  

The production of artificial fracture satisfies the following relationship:

\[
\frac{\partial p_D}{\partial y_D} \bigg|_{y_D=0} = -\frac{\pi}{2} q_{\text{Mov}}
\]

The cracks are closed at both ends and the conditions are met:

\[
\frac{\partial p_D}{\partial x_D} \bigg|_{x_D=1} = \frac{\partial p_D}{\partial x_D} \bigg|_{x_D=-1} = 0
\]

Combining the Green's function, the Laplace transformations of the Eq.7-Eq.9 are solved and we can obtain the Laplasse spatial pressure distribution of artificial fracture:

\[
\tilde{p}_D(x_D, s) = \tilde{p}_{\text{avg}}(s) + \frac{\pi}{C_{\text{Mov}}} \int_{-1}^{1} G(x_D, \nu) \tilde{p}_{\nu}(s) d\nu - \frac{2\pi}{s C_{\text{Mov}}} G(x_D, x_{wD})
\]

In Equation 10:

\[
G(x_D, \nu) = \begin{cases} 
-\frac{1}{4} \left( \nu + 1 \right)^2 + \left( x_D - 1 \right)^2 - \frac{4}{3} & -1 \leq \nu < x_D \\
-\frac{1}{4} \left( \nu - 1 \right)^2 + \left( x_D + 1 \right)^2 - \frac{4}{3} & x_D \leq \nu < 1
\end{cases}
\]

MODEL OF MATRIX SYSTEM

Anbarci[25] gave the solution of pressure propagation in coalbed methane reservoirs in 1990. In this paper, the isothermal Lagrangian theory[26] is used to describe the unsteady adsorption desorption process.

Dimensionless definition:

\[
\omega = \frac{\phi \mu c_p}{\sigma} ; \quad r_D = \frac{r}{x_t} ; \quad t_D = \frac{\alpha k t}{\sigma x_t^3} ; \quad V_D = V - V^* ; \quad \sigma = \frac{6 p_s T \mu Z}{\rho_{\text{sc}} q_D p_i^*} ;
\]
\[
\tau = \frac{R^2}{D}; \quad q_D = \frac{\beta T \mu Z}{k h p_i^{\ast}} \cdot q_{sc}; \quad \lambda = \frac{\alpha k \tau}{\omega x_i^{\ast}}; \quad p_{D} = \frac{p_i^{2} - p^2}{p_i^{2} q_D}
\]

Based on dimensionless treatment, the radially elementary volume equation of material balance, the initial formation pressure, the inner boundary and the closed outer boundary condition can be written as:

\[
\frac{1}{r_D} \frac{\partial}{\partial r_D} \left( r_D \frac{\partial p_D}{\partial r_D} \right) = \omega \frac{\partial p_D}{\partial t_D} - \frac{(1 - \omega)}{\lambda} \frac{\partial V_D}{\partial r_D} \bigg|_{r_0=1}
\]

(6)

\[
p_D(r_D,0) = 0
\]

(7)

\[
r_D \frac{\partial p_D}{\partial r_D} \bigg|_{r_0 \to 0} = -1
\]

(8)

\[
\frac{\partial p_D(r_{D},t)}{\partial r_D} = 0
\]

(9)

Combined with Laplace transformations, we can obtain the dimensionless expression of the diffusion equation of coal matrix is:

\[
\frac{\partial \tilde{V}_D}{\partial r_D} \bigg|_{\eta=1} = -\tilde{p}_D \left( \sqrt{\lambda s} \coth \sqrt{\lambda s} - 1 \right)
\]

(10)

Submit Eq.10 into Eq.6, and combining Eqs.7-9, we can get the Laplacean space solution:

\[
\tilde{p}_D = K_0 (r_D \Delta) + I_0 (r_D \Delta) K_1 \left( r_D \sqrt{s} \right) / I_1 \left( r_D \sqrt{s} \right)
\]

(11)

In Equation 11:

\[
\Delta = \sqrt{\frac{\omega s + (1 - \omega)}{\lambda}} \tilde{\xi} \left( \sqrt{\lambda s} \coth \sqrt{\lambda s} - 1 \right)
\]

Since the matrix system of fractured surface and the pressure of fracture are continuous, through integral of Eq.11, we can obtain the general solution of pressure expression for the artificial fracture:
\[ \tilde{p}_{ib}(x_D, s) = \frac{1}{2} \int_{-1}^{1} \tilde{q}_{ib}(v, s) \left[ K_0 \left( \sqrt{(x_D - v)^2 + \Delta^2} \right) + \frac{K_i(r_d \sqrt{s})}{I_1(r_d \sqrt{s})} I_0 \left( \sqrt{(x_D - v)^2 + \Delta^2} \right) \right] dv \]  

(12)

Combining Eq.4 and Eq.12, we can obtain:

\[ \tilde{p}_{avg}(s) + \frac{\pi}{C_{ib}} \int_{-1}^{1} G(x_D, v) \tilde{q}_{ib}(v, s) dv - \frac{2\pi}{sC_{ib}} G(x_D, x_{wd}) \]

\[ = \frac{1}{2} \int_{-1}^{1} \tilde{q}_{ib}(v, s) \left[ K_0 \left( \sqrt{(x_D - v)^2 + \Delta^2} \right) + \frac{K_i(r_d \sqrt{s})}{I_1(r_d \sqrt{s})} I_0 \left( \sqrt{(x_D - v)^2 + \Delta^2} \right) \right] dv \]

(13)

In order to solve the Eq.13, we need to discretize the numerical value of fractures to solve the problem, so the fractures are evenly divided into 2N segments, and Eq.13 can be rewritten as:

\[ \tilde{p}_{avg}(s) + \frac{\pi}{C_{ib}} \sum_{i=1}^{2N} K_0 \left( \sqrt{(x_{D_i} - v)^2 + \Delta^2} \right) dv - \frac{2\pi}{sC_{ib}} G(x_D, x_{wd}) \]

\[ = \frac{1}{2} \sum_{i=N+1}^{2N} \tilde{q}_{ib}(v, s) K_0 \left( \sqrt{(x_{D_i} - v)^2 + \Delta^2} \right) dv + \frac{1}{2} \sum_{i=1}^{N} \tilde{q}_{ib}(v, s) K_0 \left( \sqrt{(x_{D_i} - v)^2 + \Delta^2} \right) dv \]

\[ + \frac{K_i(r_d \sqrt{s})}{2I_1(r_d \sqrt{s})} \sum_{i=N+1}^{2N} K_0 \left( \sqrt{(x_{D_i} - v)^2 + \Delta^2} \right) dv \]

\[ + \frac{K_i(r_d \sqrt{s})}{2I_1(r_d \sqrt{s})} \sum_{i=1}^{N} K_0 \left( \sqrt{(x_{D_i} - v)^2 + \Delta^2} \right) dv \]

(14)

For any moment, all segmented productions are met:

\[ \frac{1}{2N} \sum_{i=1}^{2N} \tilde{q}_{ib}(v, s) = \frac{1}{s} \]

(15)

Solve Eq.14 and Eq. 15, there are 2N+1 equation set in all, we can obtain the \( \tilde{q}_{ib} \) corresponding to each segment of the fracture and \( \tilde{p}_{avg}(s) \) (the average pressure of the fracture). Then, substituting Eq.13, let \( x_{D_i} = x_{wd} \), we can solve the pressure value of the bottom of well.

If we will take wellbore storage effect and skin effect into consideration, so the expression of bottom-hole pressure in Laplace space should be amended as:
\[
\tilde{p}_{wD} = \frac{1}{s^2 C_D + s \left[ \delta \tilde{p}_{wf} (x_{wD}, s) + S_k \right]}
\] (16)

Based on Eq.16, the relationship between dimensionless bottom-hole pressure, dimensionless pressure derivative and dimensionless time can be obtained by using Stehfest numerical inversion[27]. Figure 2 shows that the relationship between the bottom-hole pressure and the derivative of pressure with time when the dimensionless artificial fracture conductivity \( C_{fD} = 1 \), the wellbore storage coefficient \( C_D = 10^{-9} \), the asymmetry factor \( x_{wD} = 0.2 \), the storativity ratio \( \omega = 0.004 \), the channel flow factor \( \lambda = 0.005 \), the dimensionless rate \( \xi = 1 \), the dimensionless drainage boundary \( r_{eD} = 6 \) (In order to characterize and describe the dynamic condition of pressure propagation better, so we make the value of dimensionless drainage boundary \( r_{eD} \) larger). From Figure 2 (log-log plot), we can see that the whole flow process consists of six stages: (a) Artificial fracture flow stage with the effect of wellbore storage effect, the pressure curve coincides with differential of pressure curve, and it is shown as a slope of 1 curve in log-log plot.; (b) In transition flow stage, the influence of wellbore storage effect is weakened, and the pressure curve and pressure derivative curve are separated; (c) In the linear flow period of the matrix-artificial fracture, with the decrease of the artificial fracture pressure, the fluid in the matrix near the fracture flows perpendicularly to the artificial fracture under the pressure difference, pressure curve and pressure derivative curve are parallel to each other, and the slope of the curve is 1/2 in log-log plot; (d) The cross-flow stage between matrix and fracture in double medium fractures, under the action of pressure difference, the fluid flow between the matrix and the fracture in the...
binary medium causes the cross-flow, the curve characteristic is the concave down of the pressure derivative curve; (e) The pseudoradial flow stage in the entire flow system, the pressure derivative curve is regressed into a horizontal line of a slope of 0.5; (f) In the closed boundary flow stage, the pressure propagation reaches the closed boundary, and the dimensionless pressure and dimensionless pressure derivative curve rise to an angle of 45 degrees.

THE PLATE OF PRESSURE PROPAGATION AND THE SENSITIVITY ANALYSIS OF PARAMETER

Using the control variable method, based on the original basic parameters, the relationship curves between dimensionless bottom hole pressure and dimensionless pressure derivative and dimensionless time corresponding to different variables are plotted by changing the single variable, and the influence of these parameters on the dynamic propagation of pressure is analyzed, such as artificial fracture conductivity, asymmetry factor, storativity ratio, cross-flow factor, dimensionless drainage radius and so forth.

Artificial Fracture Conductivity $C_{fd}$

![Figure 3. Effect of artificial fracture conductivity to transient pressure.](image)

The non-dimensional artificial fracture conductivity is the embodiment of the interaction effect of artificial fracture permeability, artificial fracture width, artificial
fracture half length and matrix permeability. Figure 3 shows the relationship between the non-dimensional pressure and the pressure derivative and the dimensionless time under different artificial fracture conductivity ($C_fD=1, 10, 100$). As can be seen from the figure: the artificial fracture conductivity has a great influence on the productivity of the whole production cycle. With the increase of the artificial fracture conductivity, the productivity increase is very significant, especially the production period after the influence of the wellbore storage effect is weakened. From the point of view of the process of pressure propagation, under the influence of wellbore storage effect, the greater the conductivity of the artificial fracture is, the shorter the duration of the three flow stage (including the artificial fracture flow stage, the linear flow period of the matrix-artificial fracture and the pseudoradial flow stage) is. From the sensitivity analysis, the artificial fracture conductivity varies from 1 to 10, and the parameter is more sensitive than the change from 10 to 100, so that the artificial fracture conductivity is not as bigger as better, but there is an optimal value.

**The Asymmetry Factor**

The asymmetry factor reflects the degree of asymmetric fracture of artificial fracture, Figure 4 shows the relationship between dimensionless pressure and dimensionless pressure derivative and dimensionless time under different asymmetry factors ($X_{WD}=0.2, 1$), the curve can be seen: the asymmetry factor has a great influence on the whole production cycle, especially the linear flow period of the matrix-artificial fracture and the cross-flow stage between matrix and fracture in double medium fractures; the bigger the asymmetry factor is, the lower the

![Figure 4. Effect of asymmetry factor to transient pressure.](image-url)
productivity is. Therefore, it is necessary to ensure symmetrical fracturing when fracturing, which is beneficial to the improvement of production of single well, and when severe asymmetric fracturing occurs, we need to establish an effective evaluation model for considering the effects of asymmetric fractures.

**Storativity Ratio**

Figure 5 shows the relationship between dimensionless pressure and dimensionless pressure derivative and dimensionless time at different storativity ratios ($\omega=0.001, 0.004, 0.008$), it can be clearly seen from the log-log plot, similar to other double medium models, the storativity ratio influences the degree of the occurrence time of cross-flow between the matrix and the fracture in the matrix system. The storativity ratio has influence on the production of the transition flow stage, the linear flow.

![Figure 5. Effect of storativity ratio to transient pressure](image)

Figure 5. Effect of storativity ratio to transient pressure

period of the matrix-artificial fracture and the cross-flow stage between matrix and fracture in double medium fractures, the smaller storativity ratio is, the more obvious the cross-flow phenomenon is, and the concave down of the pressure derivative curve deeper is in log-log plot.
Cross-flow Coefficient

Figure 6 shows the relationship between dimensionless pressure and dimensionless pressure derivative and dimensionless time at different cross-flow coefficients ($\lambda=0.001, 0.004, 0.008$), which can be obtained from the curve: the cross-flow coefficient affects the time of occurrence of cross-flow between the matrix and the fracture in the matrix system, the cross-flow coefficient only affects the cross-flow stage between matrix and fracture in double medium fractures, and the smaller the cross-flow coefficient is, the earlier the cross-flow occurs.

Dimensionless Drainage Radius

Figure 7 shows the relationship between dimensionless pressure and dimensionless pressure derivative and dimensionless time at different dimensionless drainage radius ($reD=1.2, 3, 5$), which can be seen from the log-log plot: the dimensionless drainage radius has only an effect on the later production, and the smaller the dimensionless drainage radius is, the sooner it enters the closed boundary flow stage.

CONCLUSIONS

(1) Based on Green function and Laplace transformation, a pressure transient

![Figure 6. Effect of cross-flow factor to transient pressure.](image URL)
(2) The artificial fracture conductivity has a great influence on the productivity of the whole production cycle. With the increase of the artificial fracture conductivity, the productivity increase is very significant, especially the production period after the influence of the wellbore storage effect is weakened. The bigger the conductivity of artificial fracture is, the weaker the sensitivity of the parameter is. Therefore, the artificial fracture conductivity has an optimal value.

(3) The asymmetry factor has a great influence on the whole production cycle, especially the linear flow period of the matrix-artificial fracture and the cross-flow stage between matrix and fracture in double medium fractures; the bigger the asymmetry factor is, the lower the productivity is. Therefore, we need to establish an effective evaluation model for considering the effects of asymmetric fractures when severe asymmetric fracturing occurs.

(4) Storativity ratio and cross-flow factor influence the degree and occurrence time of cross-flow between matrix and fractures in the matrix system respectively. The Storativity ratio has influence on the production of the transition flow stage, the linear flow period of the matrix-artificial fracture and the cross-flow stage between matrix and fracture in double medium fractures, the smaller storativity ratio is, the more obvious the cross-flow phenomenon is, and the concave down of the pressure derivative curve deeper is in log-log plot. The smaller the cross-flow coefficient is, the earlier the cross-flow occurs.

(5) The dimensionless drainage radius has only an effect on the later production, the smaller dimensionless drainage radius is, The earlier the closed boundary flow will happen.
NOMENCLATURE

Field Variables

\( k_f \) - Artificial fracture permeability, \( \mu m^2 \);  
\( r_f \) - Artificial fracture half length, m;

\( r \) - Radius, m;  \( k \) - Permeability, \( \mu m^2 \);  
\( \phi \) - Porosity, dimensionless;

\( Z \) - Gas compression factor, dimensionless;  
\( T \) - Thermodynamic temperature, K;

\( \mu \) - Gas viscosity, mPa·s;  
\( w \) - Width of artificial fracture, m;

\( h \) - Reservoir thickness, m;  
\( x_w \) - Coordinates of Wellbore, m;

\( p_i \) - Initial formation pressure, MPa;  
\( q \) - Volume flow of production well, m\(^3\)/d;

\( q_{sc} \) - Gas production under standard condition, m\(^3\)/d;  
\( t \) - Production time, d;

\( r_e \) - Outer boundary of model, m;  
\( V \) - Matrix gas concentration, m\(^3\)/m\(^3\);

\( V_i \) - Initial matrix gas concentration, m\(^3\)/m\(^3\);  
\( D \) - Diffusion coefficient, m\(^3\)/d;

\( R_m \) - Matrix granule radius, m;  
\( \omega \) - Storativity ratio, dimensionless;

\( \lambda \) - Cross-flow factor, dimensionless;  
\( C_d \) - Dimensionless fracture conductivity;

\( s \) - Transformed value of time in the Laplacian space, dimensionless;

\( \tilde{p} \) - Transformed value of pressure corresponding to Laplacian space;

\( \tilde{p}_{wD} \) - Transformed value of dimensionless bottomhole pressure corresponding to Laplacian space;

\( \tilde{q} \) - Transformed value of volume flow corresponding to Laplacian space;

\( \tilde{p}_{avg}(s) \) - The average pressure of fracture at a given moment corresponding to Laplacian space;

\( C_D \) - Wellbore storage factor, dimensionless;  
\( S_k \) - Skin factor, dimensionless;

Special Functions

\( \delta(x) \) — Dirac function;

\( I_0(x) \) — Modified Bessel function (1st kind, zero order);

\( K_0(x) \) — Modified Bessel function (2nd kind, zero order);

\( I_1(x) \) — Modified Bessel function (1st kind, first order);

\( K_1(x) \) — Modified Bessel function (2nd kind, first order);

\( G(x, y) \) — Green function

Special Subscripts

\( f \) — Artificial fracture parameter;  
\( m \) — Matrix parameter in double medium;

\( D \) — Dimensionless parameter;  
\( SC \) — Parameters under standard condition.
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