Three-Dimensional Adjustment of Precision Control Network Based on Independent Coordinate System

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Abstract. The control network measurement system based on Railway fixed pile can be used in the maintenance of ballasted track. However, how to evaluate the impact of the external observation precision of the control network on the relative position precision and the position precision of the control point does not have a final conclusion. This paper combines line field conditions of Railway line. We have proposed a net-shaped and analysis of field data collected on the basis of three-dimensional control point adjustment calculation. And then, we can get the relation among field observation precision of control point and relative positional precision and positional precision of the control point. The result shows that if the azimuth measurement accuracy is less than ±1″, the accuracy of the vertical measurement and the distance measurement are less than ±1″ and ±(2+2×10⁻⁶)mm, respectively. And then the relative position accuracy of control network will be less than ±4mm. The conclusion plays an important role in the ballasted track measurement standard and work.

Introduction

In order to efficiently and accurately maintain the ballasted track, the Ministry of Railways introduced a set of large-scale maintenance systems and technologies from overseas, which consist of measuring, track lining, track raising and tamping. The EM-SAT120 measuring truck relies on railway fixed piles system. However, there is no complete and unified track maintenance control system in the construction of the ballasted track in the early period of China. Although the newly built ballasted track has established the control network, it cannot be directly applied to the large-scale maintenance system.

In this case, the relevant departments to learn from the experience of foreign fixed-pile foundation system, put forward in China has a ballasted track construction of fixed pile foundation system. With the fixed pile as the benchmark, the measured line with the standard line for comparison analysis, we can calculate the large-scale maintenance systems required data. The relevant departments proposed three methods, namely vector method, coordinate method, CP3 parameter conversion method. Based on the coordinate method, this paper presents a more reasonable way of network construction, and carries on the data calculation and the precision analysis of the data have been produced, obtained the relations between observation accuracy and position accuracy.

The Establishment of Control Point Network

According to site survey, in most sections, the fixed pile within the wall is suitable for establish the network on both side of the line. While in some special sections (such as the station, etc.), the pile may be outside the wall, that cannot lay control points, only one side is available. So the layout of control point is divided into bilateral control network and unilateral control network. For the requirements and laid methods of fixed pile, see reference [4].
The Bilateral Control Network

Bilateral control network is the zigzag network, also known as the standard network, its net form and measurement methods shown in Figure 1.

(1) The zigzag network points are evenly distributed on both sides of the line, the distance between the adjacent points is about 100m;
(2) The observation use free station method, each observation take 5 points, and Electronic Total Station set up near the intermediate point of the 5 points. And the station should be in the center position of track;
(3) The distance of each moving station is about 100m;
(4) At least 3 points are overlapped between the stations. After the measurement, there are 3 sets of observation data at the point near the station, and the other points have 2 sets of observation data;

![Figure 1. Bilateral control network (unit: m).](image)

The Unilateral Control Network

Unilateral control network, also known as improved network, its net form and measurement methods shown in Figure 2.

(1) The unilateral points are evenly distributed in one side of the line, the distance between the adjacent points is about 50m;
(2) The same as bilateral control network, unilateral control network also take 5 points in each observation, and Electronic Total Station set up near the intermediate point of the 5 points. Station should be in the center position of track;
(3) The distance of each moving station is about 100m;
(4) At least 3 points are overlapped between the stations. After completing the measurements, there are 3 sets of observation data at the point near the station. There are 2 sets of observation data at other points.

![Figure 2. Unilateral control network (unit: m).](image)

Adjustment Mathematical Model

The Calculation of Approximate Coordinates

First, suppose the coordinate of the first station is \((0, 0, 0)\), and let the observation direction of first point as the starting direction. And then can calculate the coordinate values of the first control point as its vertical measurement value and distance measurement value is known. With the starting calculation data is the coordinate of first station and the coordinate of first control point, can calculate other control points three-dimensional approximate coordinates of current station. The method is calculated according to the polar coordinate calculation principle:

\[
\begin{align*}
X_i^0 &= X_s^0 + S_\theta \sin A_\theta \cos T_i \\
Y_i^0 &= Y_s^0 + S_\theta \sin A_\theta \sin T_i \\
Z_i^0 &= Z_s^0 + S_\theta \cos A_\theta
\end{align*}
\]  

(1)
Above formula, \( X_s^0, Y_s^0, Z_s^0 \) is station approximate coordinate which has been calculated, \( S_{si}, A_{si} \) is distance measurement value and vertical measurement value, \( T_{si} \) is azimuth measurement value of this point and current station.

In this way, the approximate coordinates of all control points in the first station can be calculated. In the field survey, there are at least two common observation points between adjacent stations, so when computing the coordinate of the next station, we can use the coordinates of the common points to get the result by resection principle. Then according to Eq. 1 to obtain the coordinates of other control points in current station.

The above method is repeated until the approximate coordinates of all the stations and the control points are calculated.

**The Establishment of Error Equation**

Suppose that \( L_{ij} \) is the azimuth value between station and a certain control point and \( v_{ij} \) is its correction value. According to the above method, the approximate coordinates of the points can be determined as \( X_i, Y_i, Z_i, X_j, Y_j, Z_j \), and their correction values is \( \delta x_i, \delta y_i, \delta z_i, \delta x_j, \delta y_j, \delta z_j \). The azimuth is \( T_{ij} \). So the relationship is as follows:

\[
T_{ij} = \arctan \frac{Y_j + \delta y_j - (Y_i + \delta y_i)}{X_j + \delta x_j - (X_i + \delta x_i)} = (L_{ij} + v_{ij}) + w_i
\]  

(2)

Above formula, \( w_i \) is the orientation uncertain of station i.

Expanding the Eq. 2 according to Taylor series, take first degree, and merge the same item, you can get the vertical error equation:

\[
\begin{align*}
v_{ij} &= -\delta w_i + \rho \frac{\Delta Y_j - \Delta Z_{ij}}{D^2_{ij}} \delta X_i - \rho \frac{\Delta X_j - \Delta Z_{ij}}{D^2_{ij}} \delta Y_i - \rho \frac{\Delta X_j - \Delta Z_{ij}}{D^2_{ij}} \delta Z_i - l_{ij} \\
l_{ij} &= L_{ij} + w^0_{ij} - T^0_{ij}
\end{align*}
\]  

(3)

Above formula, \( \rho = 206265'' \), \( D^0_{ij} \) is the approximate value of edge \( ij \), \( \Delta X_{ij}, \Delta Y_{ij} \) is the coordinate increment on \( X,Y \) of edge \( ij \). \( w^0_{ij}, \delta w_i \) is the orientation angle approximation value and correction, \( T^0_{ij} \) is the azimuth angle approximation value of edge \( ij \);

\[
D^0_{ij} = \sqrt{(X_j - X_i)^2 + (Y_j - Y_i)^2}
\]

\[
T^0_{ij} = \arctan \frac{Y_j - Y_i}{X_j - X_i}
\]

\[
w^0_{ij} = \frac{\sum_{j=1}^{n} (T^0_{ij} - L_{ij})}{n}
\]

Above formula, \( n \) is the direction number of certain station.

In the same way, the error equation of distance in control network can be established:

\[
\begin{align*}
v_{ij} &= -\frac{\Delta X_j}{S^0_{ij}} \delta x_i - \frac{\Delta Y_j}{S^0_{ij}} \delta y_i - \frac{\Delta Z_j}{S^0_{ij}} \delta z_i + \frac{\Delta X_j}{S^0_{ij}} \delta x_j + \frac{\Delta Y_j}{S^0_{ij}} \delta y_j + \frac{\Delta Z_j}{S^0_{ij}} \delta z_j - l_{ij} \\
l_{ij} &= S_j - S^0_{ij}
\end{align*}
\]  

(4)

Above formula, \( \Delta Z_{ij} \) is the coordinate increment on \( Z \) of edge \( ij \), \( S_j \) is the distance observation value, \( S^0_{ij} \) is the distance approximation value;
\[ S_{ij}^{0} = \sqrt{(X_j - X_i)^2 + (Y_j - Y_i)^2 + (Z_j - Z_i)^2} \]

The vertical error equation is as follows:

\[ v_i = -\rho \frac{\Delta X_i \Delta Z_i}{S_{ij}^{0} D_{ij}^{0}} \delta x_i - \rho \frac{\Delta Y_i \Delta Z_i}{S_{ij}^{0} D_{ij}^{0}} \delta y_i + \rho \frac{\Delta Y_i \Delta Z_i}{S_{ij}^{0} D_{ij}^{0}} \delta z_i + \rho \frac{\Delta X_i \Delta Z_i}{S_{ij}^{0} D_{ij}^{0}} \delta x_i + \rho \frac{\Delta Y_i \Delta Z_i}{S_{ij}^{0} D_{ij}^{0}} \delta y_i - \rho \frac{\Delta Y_i \Delta Z_i}{S_{ij}^{0} D_{ij}^{0}} \delta z_i - l_i \]

\[ l_i = A_i - A_{ij}^{0} \]  \hspace{1cm} (5)

Above formula, \( A_i \) is the vertical observation value, \( A_{ij}^{0} \) is the vertical approximation value;

\[ A_{ij}^{0} = \arccos \left( \frac{Z_j - Z_i}{S_{ij}^{0}} \right) \]

The Determination of Weight Value

In this three-dimensional network, there are three different types of observation value. In this paper, the weight-ratio of azimuth, vertical and distance is determined according to empirical weighting method.

First of all, suppose that the observation square errors about distance, azimuth and vertical are \( m_s, m_\alpha, m_\beta \), and the square error of the azimuth regard as unit weight variance, denoted as \( m_\alpha = \sigma_0 \), then:

\[ \begin{align*}
    P_s &= \frac{\sigma_0}{m_s} = \frac{m_\alpha}{(a + bS_{ij}^{0})^2} \\
    P_\alpha &= \frac{\sigma_0}{m_\alpha} \\
    P_\beta &= \frac{\sigma_0}{m_\beta}
\end{align*} \]  \hspace{1cm} (6)

Above formula, \( a \) is fixed range error, \( b \) is proportional error. They can be obtained from the nominal accuracy of total station.

The Calculation of Relative Position Accuracy

According to error Eq. 3, Eq. 4, and Eq. 5, can get the coefficient matrix \( B \) of the error equation which is using in the adjustment. According to error Eq. 6, can obtain the weight matrix \( P \) of the observation value. Then according to the following Eq. 7, can get \( Q_X \) that the covariance matrix of coordinates:

\[ Q_X = (B^T PB)^{-1} \]  \hspace{1cm} (7)

The relative position relationship between two points in the control network is established by the coordinate difference of two points. Suppose that the coordinates of any two neighboring points are \( A(X_i, Y_i, Z_i) \) and \( B(X_j, Y_j, Z_j) \), then:

\[ \begin{align*}
    \Delta X_{ij} &= X_j - X_i \\
    \Delta Y_{ij} &= Y_j - Y_i \\
    \Delta Z_{ij} &= Z_j - Z_i
\end{align*} \]  \hspace{1cm} (8)

According to Eq. 8 and law of error propagation, we can get:
\[
\begin{align*}
Q_{\Delta x_i} &= Q_{\Delta x_i} + Q_{\Delta x_j} - 2Q_{\Delta x_i,\Delta x_j} \\
Q_{\Delta y_i} &= Q_{\Delta y_i} + Q_{\Delta y_j} - 2Q_{\Delta y_i,\Delta y_j} \\
Q_{\Delta z_i} &= Q_{\Delta z_i} + Q_{\Delta z_j} - 2Q_{\Delta z_i,\Delta z_j}
\end{align*}
\] (9)

Above formula, \( Q_{\Delta x_i}, Q_{\Delta y_i}, Q_{\Delta x_j}, Q_{\Delta y_j}, Q_{\Delta z_i}, Q_{\Delta y_j}, Q_{\Delta z_i}, Q_{\Delta z_j} \) can be obtained according to Eq. 7.

Then, according to the following Eq. 10, it is possible to calculate the relative position square error between any adjacent points:

\[
m_{\Delta p} = \hat{\sigma}_0 \sqrt{Q_{\Delta x_i} + Q_{\Delta y_i} + Q_{\Delta z_i}}
\] (10)

Above formula, \( \hat{\sigma}_0 \) is the estimated value of unit weight variance, can be determined according to reference [2].

The Simulation Calculation of Relative Position Accuracy

The Calculation of Bilateral Control Network

As shown in Figure 3, 15 observation points were selected for calculation, free station 6 times to observe, adjacent station distance is about 100m.

![Figure 3. Calculation network of bilateral control network (unit: m).](image)

In the control network, the prior observation accuracy of azimuth, vertical, distance is taken respectively as \( \pm 0.5', \pm 0.5', \pm (1+1\times10^{-4})mm \), \( \pm 1', \pm 1', \pm (1+2\times10^{-4})mm \) and \( \pm 1', \pm 1', \pm (2+2\times10^{-4})mm \). Calculated with the method described in Section 3 above, the position precision and the relative position precision is shown in the following table 1.

The Calculation of Unilateral Control Network

As shown in Figure 4, 9 observation points were selected for calculation, free station 3 times to observe, adjacent station distance is about 100m.

![Figure 4. Calculation network of unilateral control network (unit: m).](image)

It's the same as the bilateral control network, in unilateral control network, the prior observation accuracy of azimuth, vertical, distance is taken respectively as \( \pm 0.5', \pm 0.5', \pm (1+1\times10^{-4})mm \), \( \pm 1', \pm 1', \pm (1+2\times10^{-4})mm \) and \( \pm 1', \pm 1', \pm (2+2\times10^{-4})mm \). Calculated with the method described in Section 3 above, the position precision and the relative position precision is shown in the following table 2.
Table 1. Calculation result of position precision and relative position precision about bilateral control network.

<table>
<thead>
<tr>
<th>Control Point</th>
<th>Priori Accuracy of Observation</th>
<th>( m_p ) (mm)</th>
<th>( m_{\Delta p} ) (mm)</th>
<th>( m_p ) (mm)</th>
<th>( m_{\Delta p} ) (mm)</th>
<th>( m_p ) (mm)</th>
<th>( m_{\Delta p} ) (mm)</th>
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Table 2. Calculation result of position precision and relative position precision about unilateral control network.

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<th>Control Point</th>
<th>Priori Accuracy of Observation</th>
<th>( m_p ) (mm)</th>
<th>( m_{\Delta p} ) (mm)</th>
<th>( m_p ) (mm)</th>
<th>( m_{\Delta p} ) (mm)</th>
<th>( m_p ) (mm)</th>
<th>( m_{\Delta p} ) (mm)</th>
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<td>( \pm 0.5', \pm 0.5' ), ( \pm (1 + 1 \times 10^{-6}) \text{mm} )</td>
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</table>
Conclusions and Recommendations

According to the relevant provisions of the fixed pile system coordinate method, the control network of this paper is put forward, and the following conclusions and recommendations are obtained through the calculation of the data collected at the project and the analysis of the precision of the network:

(1) The accuracy of unilateral network is significantly higher than the bilateral network;
(2) In general, the scene is very complex of measurement, some control points are blocked and can not be laid by bilateral network. At that time, we can use unilateral network. By this method, can guarantee the accuracy of point, but also solve the problem at the scene;
(3) In the control network measurement, the Priori Observation Accuracy of azimuth, vertical and distance within $1\pm1\degree$ and $2\times10^{-6}mm$, then relative position precision of the network can be controlled in $4mm$;
(4) The data used in this calculation are collected around 12:00 to 14:00 due to the limitation of experimental site, and the summer temperature is relatively high, which may have certain influence on some extent;
(5) The method of empirical weighting in this paper needs to be studied further.

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