The Application and Analysis of Nonlinear Filtering Used in GPS Dynamic Positioning

Xiang-Lei WANG\textsuperscript{a, *}, Shuo DING

Beijing Satellite Navigation Center, Beijing, China

\textsuperscript{a}chxywxl@163.com

*Corresponding author

Keywords: Nonlinear Filtering, GPS, Dynamic Positioning, EKF, UKF.

Abstract. For several kinds of nonlinear filtering method is commonly used in GPS dynamic positioning (EKF, UKF, CKF, PF) has carried on the detailed research, this paper analyzed the characteristics of different nonlinear filtering algorithm, and the assessment criteria of filtering performance is analyzed, finally the GPS dynamic positioning of the linearization of the model error and noise types on the result of calculation has carried on the detailed discussion. Different nonlinear filtering algorithm is analyzed by an example in the application of GPS dynamic positioning, by comparing the result shows that the CKF and UKF integrated performance is excellent.

Introduction

Determination of GPS dynamic positioning, is to use the GPS signals, relative to the user state of the antenna parameters of motion of the earth, the state parameter generally includes 3 d coordinate seven, three-dimensional velocity and time, etc. Dynamic positioning is relative to the static positioning, its characteristic is the determination of motion carrier real-time location, redundant observation less (or even no redundant observation), so the kalman filter is a kind of very good computing tools.

For a linear system when the state noise and measurement noise is gaussian white noise, can be obtained by kalman filtering recursive state parameter of the optimal solution. For non-gaussian and nonlinear dynamic system, if you want to get the optimal solution, to need an infinite dimensional integral operation, can only adopt approximate method to get the optimal solution, many scholars have put forward a lot of nonlinear filtering algorithm.

When using GPS dynamic positioning, common observation is carrier phase observation value, code pseudorange observation or their linear combination, the observation itself is nonlinear, and the observation noise is gaussian white noise, so the traditional linear kalman filtering methods cannot be applied to the GPS dynamic positioning, to take advantage of nonlinear filtering algorithm, the nonlinear filtering algorithm commonly used include: extended kalman filtering (EKF), no trace kalman filter (UKF), kalman filtering (CKF) volume and particle filter (PF), such as nonlinear filtering method is different from a different perspective to solve the nonlinear problem, has its own characteristics.

GPS Dynamic Positioning State Model

Dynamic positioning of the discrete state space model can be expressed as detailed derivation (direct result is given here, and the detailed process see the literature):

\[ X_k = f(X_{k-1}) + W_k. \]  

\[ L_k = h(X_k) + e_k. \]
Here, \( X_k \), \( L_k \) are state vector and observation vector respectively. 
\( f(\cdot), h(\cdot) \) are state equation and observation equation, and are nonlinear in GPS dynamic positioning.
\( W_k, e_k \) are state noise and observation noise respectively, covariance matrix are \( \Sigma_w \) and \( \Sigma_e \), assumptions are independent random variables.

**Commonly Used Nonlinear Filtering in GPS Dynamic Positioning**

**Extended Kalman Filtering (EKF)**

EKF is Taylor expansion near the estimate value for the nonlinear system, take one order truncated as state equation and observation equation of approximation to realize linearization, and then after linearization of the system using kalman filtering for state estimation.

EKF based on statistics model and observation model, provide the system state minimum variance estimate. State equation and measurement equation in its previous estimates for linear processing:

\[
F_{k-1} = \frac{\partial f}{\partial X_{k-1}}_{X_{k-1}=X_k},
H_k = \frac{\partial h}{\partial X_k}_{X_{k-1}=X_k}.
\] (3)

To nonlinear system is transformed into discrete linear systems, linear kalman filter to estimate. The algorithm implementation is no longer here, see the related literature.

**Unscented Kalman Filter (UKF)**

As we can see from the above description of EKF EKF some drawbacks: first, because the first order Taylor expansion is taken, the precision of the mean and covariance can only to the first-order accuracy; Second is the first order Taylor expansion in a single point for nonlinear equation, without considering prior uncertainty of random variable; Third, EKF involves the calculation of Jacobi matrix, for more complex nonlinear system, the calculation is complicated.

Shortcomings of the UKF come up with a good customer service EKF (Julier and Uhlmann, 1997), UKF is a deterministic sampling method, it will by UT transformation state probability distribution in choice of sigma point set, then these points spread through the nonlinear system, state the mean and covariance information respectively from the point of the weighted and preserved in form, for nonlinear gauss system, UKF can achieve the third order accuracy (Taylor series expansion); For the nonlinear non-gaussian systems, UKF can reach the precision of the second order. UKF algorithm detailed calculation process can be found in the literature (Wan and Merwe, 2000).

**Cubature Kalman Filter (CKF)**

CKF and UKF are also determined by a set of weighted samples to approximate distribution function of random variable, by means of nonlinear transformation of this group of sample point, capturing the statistic characteristics of random variables after nonlinear transformation. But they are used to determine the sample point is not the same as the criterion, the UKF USES UT changes the basic sigma points, CKF the rule of volume produce basic volume points (Cubature points), experimental proof CKF than UKF in the higher dimensional case, has better numerical stability (Arasaratnam and Haykin, 2009).

Using the third order volume principle to obtain the basic points and corresponding weight of volume:
\[ \xi^{(i)} = \sqrt{n_e} \begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix}, \quad i = 1, \ldots, 2n_e, \]
\[ w^{(i)} = 1/2n_e, \quad i = 1, \ldots, 2n_e. \]  

\( n_e \) is the dimensions of the state parameter, so the volume is 2 times of state parameter dimension. Symbol \( [1] \) is \( n_e \) dimensions unit vector elements \( e = [1, 0, \cdots, 0] \) for all arrangement and change the element symbol of point set, called a complete full symmetric point set, \( [1] \), is said the \( [1] \) focus of the first point.

\[ [1] = \begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \\ 0 \\ 0 \\ \vdots \\ 0 \\ 1 \\ 0 \end{bmatrix}, \cdots , \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \\ 0 \end{bmatrix}, \cdots , \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 0 \\ -1 \end{bmatrix}. \]  

CKF algorithm basic process can be found in the literature (Arasaratnam and Haykin, 2009).

**Particle Filter (PF)**

Particle filter algorithm is recursive filtering problem of discrete time an approximation of the bayesian solution (Legland and Oudjane 2000). The basic idea is to use a set of weighted random samples (particles) \( \{x'_i, w'_i\}_{i=1}^N \) to approximate the posterior probability density function of time.

\[ p(x_k | z_k) \approx \sum_{i=1}^N w'_i \delta(x_k - x'_i). \]  

\( \{x'_i, i = 1, \ldots, N\} \) is sample set, and the weight is \( \{w'_i, i = 1, \ldots, N\}, \sum_{i=1}^N w'_i = 1 \), said and there was a real posterior \( p(x_k | z_k) \) density of the discrete weighted approximate representation, and the complex calculation of mathematical expectation (usually with complex integral operation) can be simplified as and operation, such as:

\[ E(g(x_k)) = \int g(x_k) p(x_k | z_k) \, dx_k. \]  

Can be approximate to:

\[ E \left( g(x_k) \right) = \sum_{i=1}^N w'_i g(x'_i). \]  

Standard PF concrete implementation step reference related literature, there is no longer here.

**Example Analysis**

Here using the measured data, the sampling interval of 1 second, satellite as the 15 degree Angle. Precise ephemeris and satellite clock difference using IGS 15 min interval data, 9 order and 5 days (elm) were used respectively to get, with commercial software GrafNav7.80 processing carrier difference results for comparison the true value of reference, positioning model for single frequency precision and dynamic positioning, calculate a total of 4200 data of epoch. Section 3 introduces four kinds of nonlinear filtering is used to calculate.
Figure 1. EKF, UKF position estimate error.

Figure 2. CKF, EKPF positioning estimate error.
Each filter position error of estimated true results as shown in figure 1, figure 2, the horizontal axis as the observation epoch, Y-axis is three-dimensional position errors truth value (filtered estimate three-dimensional position and the difference between the true value), the plane error true value RMSE are shown in table 1, according to the 2000 epoch, valuation calculation results after convergence. The filtering computation time and output error statistics are shown in table 1. Each filter NEES estimate as shown in figure 3. Still clearly see two colors, because EKF, UKF and CKF NEES value almost overlapping.

Table 1. Filtering error and calculation time statistics.

<table>
<thead>
<tr>
<th>Plan</th>
<th>Plane position rmse (m)</th>
<th>Calculation time (s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>EKF</td>
<td>0.5907</td>
<td>45</td>
</tr>
<tr>
<td>UKF</td>
<td>0.5623</td>
<td>478</td>
</tr>
<tr>
<td>CKF</td>
<td>0.5623</td>
<td>430</td>
</tr>
<tr>
<td>PF</td>
<td>0.503</td>
<td>2678</td>
</tr>
</tbody>
</table>

From the calculation results can be seen:
(1) From the estimated results can see, in the practical application of GPS dynamic positioning, EKF, UKF, CKF and PF four filtering algorithm results are roughly the same. UKF, CKF and PF the estimated results to a bit better, but not by much.
(2) Can be seen from the figure 3, in terms of conformance testing, the results after convergence, the result of the four types of filter are generally able to meet the requirements of the consistency of the filter. Consistency of EKF, UKF and CKF, three filter estimate state covariance matrix are
optimal. Particle filter results more close to the true posterior probability density, so the consistency of the filter is better than the other three kinds of filtering.

(3) Can be seen from table 1, on the operation time, is still the fastest EKF, the slowest PF, UKF and CKF quite CKF the UKF is slightly faster.

Conclusion

Integrated data processing situation of this example, can draw the following conclusion:

(1) From the GPS real calculation example calculation we can see that the GPS observations for the linearization of nonlinear filtering of nonlinear equation is small, the influence of both EKF based on Taylor series expansion, and USES the deterministic sampling method, the UKF CKF and PF with random sampling estimation results are very close. Because the single-frequency precise point positioning, introduced the pseudorange observation, the error of observation value itself is larger, so that a filter algorithm, filtering convergence is slow.

(2) With single frequency be estimated parameters more precise point positioning (PPP), especially when more visible satellites, this example USES the number of satellite is 5, state parameter reached 13. Higher dimensional case, the numerical stability of filtering algorithm is particularly important, in this example is the question of pathological covariance matrix, cannot be used in the UKF and CKF Cholesky decomposition to solve the square root of the covariance matrix, this kind of situation can use singular value decomposition (SVD) instead.

(3) The same to the problem of filtering of high-dimensional PF more prone to problems, first of all in this example USES the standard particle filter algorithm, the filter has no convergence. The main reason is because the likelihood probability density function is relatively narrow, and focus on the basic importance of the end of the probability density function, make the particle degradation is serious, so as to make the filtering divergence. Therefore USES the particle filter algorithm based on EKF, importance using density function by using EKF, closer to the posterior probability density function of the real. From the point of specific data processing, particle filter is not like EKF, UKF and CKF, it is not a deterministic calculation algorithms, need according to different situations, to choose them. At the same time for engineering application practice, the calculation of particle filter is very time consuming, for the real-time dynamic positioning is almost unbearable. Now dynamic navigation and positioning satellite applications, the more observable satellite, satellite signal receiving equipment sampling rate is becoming more and more high, has higher requirement for filter calculation efficiency. Therefore particle filter is applied to the engineering practice, there are a lot of work to do.

Acknowledgement

This work is supported by Research on Algorithm of Gns Multistations Networking for Atomic Clocks Realtime Comparison (Grant NO. 11403112).

Reference


