Ranking Allocation Scheme for Cost Sharing in Logistics

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Abstract. By establishing collaborative relationships at the same level in the supply chain, companies can reduce logistics costs and become more competitive. This paper investigates the synergies between participants in logistics horizontal supply chain collaboration. In particular, carriers exploit synergies by using joint delivery mechanisms among themselves in order to reduce empty lanes and thus increase transportation savings. A key question is how the costs should be fairly distributed among the collaborative partners in order to ensure significant cooperation. Although there are studies on cost sharing using the game theory approach and other decision making techniques separately, according to the authors’ knowledge there is no work that integrates these aspects in a common framework. This paper proposes an integrated method to solve such problems using the cooperating game theory with a view on spreading the cost among freight carriers. A decision support system is presented including the ranking procedure leading to the selection of the cost allocation rule advantageous for all partners. The effectiveness of the method is illustrated on a real-case example.

Introduction

The paper deals with cooperation problems and especially cost sharing in the case of horizontal logistics collaboration. Through close cooperation, especially by planning joint routes companies can reduce operating costs [10], decrease the costs of their distribution activities [5], and improve offered service.

The following example (see Figure 1) demonstrates the potential drivers of horizontal collaboration between two carriers, using joint tours as examples. This particular form of horizontal cooperation assumes a centralized decision making such that the overall routing costs are minimized [17] by organizing optimal vehicle capacity and moving a fully loaded truck from the origin to destination points in a single trip. Let us consider a network of four cities and two carriers: A and B. In the supply chain, cargo is transported by carriers in several lines. A single line deliver from an origin to a destination point with a full truckload. The cost of covering a line is assumed to be the same for all carriers and there is no difference in cost between traveling with an empty or loaded truck. The lines of each carrier and the optimal way to cover them are shown in Figure 1, where a dashed line represents a repositioning. The corresponding stand-alone cost is 550 for carrier A and 400 for carrier B - as it is shown in Figure 1.
collaboration is to minimize transportation costs by organizing joint deliveries. If the carriers collaborate, their total costs will be reduced from 950 to 750.

After a new path for the collaborative plan is decided, the carriers resolve the issue of how to share the costs. On this view, the concept of cooperative game theory (CGT) is a natural choice for solving the problem of sharing the costs of collaboration \[10, 6\]. On the other hand, the selection of an appropriate cost allocation rule involves combined evaluation of various preferences of each member of the coalition. The methods that have been used include mathematical programming \[18\], analytic hierarchy processes \[22\], and fuzzy sets \[24\] based assessment methods. At present, there are still no cost-sharing mechanism that have been generally accepted. Therefore, it is a challenge to design a mechanism that is fair, reasonable and easy to implement.

Although both approaches, games theory and multi-criteria decision-making methods have been successfully applied in logistics \[23, 9\], a systematic integration of both approaches into cost sharing in logistics cooperation problems has not been discussed yet. In this paper, we study collaborative approaches aimed at reducing transportation costs through joint deliveries. A research question now is how the costs should be fairly distributed among the collaborative partners in order to ensure significant cooperation. The purpose is to develop an appropriate mechanism to realize the cost-saving potential in the case of logistics collaboration. This paper proposes an integrated method to solve such problems using the CGT with a view on spreading the cost among freight carriers. We adopt some well-known CGT methods such as equal allocation, proportional allocation, Shapley value, Nucleolus, and Gately point to address the research question. A decision support system, named Ranking Allocation Scheme (RAS), is presented including the ranking procedure leading to the selection of the cost allocation rule advantageous for all partners. The methodology is illustrated with a numerical example.

The remainder of this paper is structured as follows. Section 2 presents the necessary theoretical background of cooperative game theory. Section 3 includes the formulation of the cooperative problem in the case of the vehicle routing problem with time windows. The problem is exemplified in Section 4. Section 5 describes our framework, the proposed method is illustrated by numerical examples taken from logistics. Finally, Section 6 concludes the paper, indicates the limitations of the proposed approach and indicates areas for further research.
Allocation Rules

A cooperative game is a pair \((N, \nu)\), where \(N = \{1, 2, ..., n\}\) is a set of players and \(\nu : 2^N \to \mathbb{R}\) is a characteristic with \(\nu(\emptyset) = 0\). When coalition \(S\) cooperates, the characteristic function defines total benefits obtained by the cooperating players:

\[
\nu(S) = \sum_{i \in S} C(\{i\}) - C(S), \quad \text{for all } S \subseteq N, \tag{1}
\]

where \(C(S)\) is the total cost generated in coalition \(S\).

**Definition 1.** The game \((N, \nu)\) is superadditive if the value \(\nu\) satisfies the following equation

\[
\nu(S \cup T) \geq \nu(S) + \nu(T) \quad \text{for all coalitions } S, T \subset N \text{ and } S \cap T = \emptyset. \tag{2}
\]

Let vector \(\phi = (\phi_1(\nu), ..., \phi_n(\nu)) \in \mathbb{R}^n\) denote payoffs of the players. In a *budget balanced* cost allocation, the total costs allocated to the players is equal to the total costs incurred by the grand coalition. It results in respective allocation of cooperation benefits (cost savings) among players \(\phi_i(\nu), \ i = 1, ..., n \ (\sum_{i \in N} \phi_i(\nu) = \nu(N))\). In a *stable* cost allocation, the total cost allocated to a subset of players should be less than or equal to the total cost incurred by that subset \(\nu(S)\). The set of cost allocations that are budget balanced and stable is called the *core* of a collaborative game.

**Definition 2.** The *core* of coalitional game \((N, \nu)\) is defined as

\[
\text{Core} = \left\{ \phi = (\phi_1(\nu), ..., \phi_n(\nu)) : \sum_{i \in N} \phi_i(\nu) = \nu(N) \land \sum_{i \in S} \phi_i(\nu) \geq \nu(S), \text{ for all } S \subseteq N \right\}.
\]

The core is the most attractive solution concept in cooperative game theory. It is the set of valid, efficient allocations that can not be improved upon upon any subcoalition of \(N\). For a coalitional game, the core may be empty, i.e. a budget balanced and stable cost allocation may not exist.

The equal allocation rule is introduced first. This gives an equal portion of benefits to each player and is defined by the equation \(\phi_i(\nu) = \frac{\nu(N)}{n}\). Another way of sharing savings is to distribute them in proportion to the initial contributions of each partner. This is expressed as \(\phi_i(\nu) = \frac{C(i)}{\sum_{j \in N} C(j)} \nu(N)\). This proportional method is generally easy to implement, as it does not consider the effect of synergies among the players.

The Shapley value is a well-known cost allocation rule. It is defined in terms of marginal contribution.

**Definition 3.** The marginal contribution of player \(i\) to a coalition \(S\) is

\[
\nu(S \cup \{i\}) - \nu(S).
\]

**Definition 4.** The Shapley value is defined by the formula

\[
\phi_i(\nu) = \sum_{S \subseteq N \setminus \{i\}} \frac{|S|!(n - |S| - 1)!}{n!} (\nu(S \cup \{i\}) - \nu(S)), \quad \text{for all } i \in S
\]
The Shapley value uses the marginal contribution of players in the grand coalition as a guiding factor. The Shapley value may lie outside the core even when the core is non-empty.

The Nucleolus is another solution concept that is studied in cooperative game theory \[21\]. When we compute the Nucleolus of a game, we lexicographically maximize the minimal gain, and the difference between the standalone cost of a subset and the total cost allocated to that subset, over all the subsets of the collaboration.

Consider a payoff vector \( x \in (N, \nu) \). For each \( x \) and each \( S \in 2^N \setminus \{N, \emptyset\} \), the excess of coalition \( S \) at \( x \) is defined as \( e_S(x) = x(S) - \nu(S) \). Define the vector \( e(x) = (e(x))_{S \in s^N \setminus \{N, \emptyset\}} \). This is the vector of all the excesses of the different coalitions at \( x \).

**Definition 5.** Let \( r(x) \) be a vector arranged in order of decreasing magnitude. for any pair of payoff vectors \( x \) and \( y \), for the first entry \( z \) in which they differ, \( x \) is smaller than \( y \) in the lexicographical order. Let the lexicographical order be denoted by \( \succ_{lex} \).

**Definition 6.** The Nucleolus of the game \((N, \nu)\) is

\[
nc(N, \nu) = \{x \in X(N, \nu) : \text{there does not exists } y \in X(N, \nu), \ e(y) \succ_{lex} e(x)\}.
\]

The Gately point is another interesting solution concept for coalitional games; see more \[7\].

**Definition 7.** The Gately point is defined by the formula

\[
G_i(\nu) = \frac{\nu(N) - \nu(N \setminus \{i\})}{\sum_{j \in N} (\nu(N) - \nu(N \setminus \{j\}))} \nu(N)
\]

Each of the presented solution concepts has different properties and leads to different allocation of benefits among players. The players can have different preferences, which of the solutions is the best. If the players want to cooperate, however, then it is important that they decide on how to share the costs equitably.

**Problem Definition and Model Assumptions**

The classical vehicle routing problem (VRP) is faced each day by thousands of companies and organizations \[4\]. In an extended version called as the vehicle routing problem with time windows (VRPTW) the service to each carrier must start within a certain time window and the vehicle must remain at the customer’s location throughout service. As it was presented in Section \[1\] the owners of depots (carriers) cooperatively manage the VRPTW applying join deliver can save their costs. The model used for the proposed approach assumes a set of denotes all representing firm in the collaborative transportation system in which all carriers share the network and transportation resources. It is assumed that they consider possible cooperation by joint delivers to reduce the travel time, the unloaded distance, the number of required vehicles and finally the total transportation cost.

While cooperative coalitions usually promise benefits to the cooperating partners, the question arises how the total extra benefits (or cost saving) should be fairly distributed among coalition partners \[1\]. Despite the fact that previously researchers have suggested various approaches to answer the question including: mathematical programming \[15\], genetic algorithm \[8\] and game theory \[10\], in general, the question is still open. In this paper, we try to answer the question of proposing a formal cost allocation model. Our study is motivated by \[3\], presenting the review of different ranking methods to solve complex logistics decision-making problem. The proposed
framework includes two main elements: a comparison of different allocation rules exemplified in Section 2 which can be used to share the cooperation benefits (expressed in operational cost savings and extra revenues) among the coalition partners and application of the preference ranking method (Ranking Allocation Scheme) for selecting the best allocation rule for a given logistics coalition. On the one hand, we use a game theoretical analysis to compute the potential savings.

For the convenience of description, let us establish a mathematical model for VRPTW in which each carrier (player) cooperate with each other [14]. Let \( G = (V, A) \) be a digraph generated from problem data, \( A \) is the arc set of \( G \). \( V = P \cup \{v_0\} \) is the node set where \( P = \{v_i \in V, i = 1, 2, ..., n\} \) represents the customers and node \( v_0 \) denotes the depot. The location of each customer is known and his demands are deterministic. Each node \( v_i \in V \) has an associated customer demand, a service time and a service-time window. For each pair of nodes \( < v_i, v_j >, (i \neq j, i, j = 0, 1, ..., n) \), a nonnegative distance \( d_{ij} \) and a nonnegative travel time \( t_{ij} \) are known. The demand of each customer should be fulfilled. Each vehicle has a capacity that is known, so overloading the vehicle is not allowed. We assume also that the players are rational. Considering the VRPTW problem with cooperation of different carriers (owners of depots) we assume that all customers are serviced without violating vehicle capacity, time window, precedence and coupling constraints with a minimum number of vehicles and for the same number of routes with the minimum total cost, as a result of cooperation (see [14]).

We formulate the collaborative transportation problem as a cooperative game \((N, \nu)\). The characteristic function \( \nu(S) \) assigns to each possible coalition of carriers \( S (S \subseteq N) \) a cost savings realized by the carriers in coalition \( S \). \( \phi_i(\nu) \) is the cost savings allocated to carrier \( i \). The allocation results should be profitable for the coalition members. The following notation is used in the description of the real case example illustrating the problem presented in the next Section, the notations used in the model building are listed in Table 1.

**Computational Example**

To answer on our research question and gain in-depth understanding in problem, we generated artificial data (Table 2) on a scale comparable with real cases based on existing cost like in [13] function, namely

\[
C(S) = \alpha Veh(S) + \beta Dist(S) + \gamma ST(S) + \lambda WT(S) \quad \text{where} \quad \alpha \gg \beta \gg \gamma \gg \lambda.
\]

We selected this type of function because it is focused on a practical application of VRP problem at logistics and transportation industry.

This example includes the coalition networks of four carriers (labeled A - D). Table 2 presents the instances used in our test and related calculations, in particular the total cost of transportation for each potential coalition. Note that the \( CS \) function here is monotonic and superadditive. The monotonicity property implies that the value of each coalition does not decrease when a newcomer joins. In this case, superadditivity essentially means that any two disjoint coalitions, \( S \) and \( T \), can achieve as much together as they could have acting separately. It is therefore profitable to form
Table 1. Notations

Sets
- \( I \): the set of all players (i.e. owners of depots)
- \( K \): the set of all allocation rule (i.e. Shapley value)
- \( N \): grand coalition
- \( S,T(S,T \subseteq N) \): a subcoalition

Indices
- \( i,j \): the index of individual players
- \( k \): the index of allocation rules

Parameters
- \( \nu \): characteristic function
- \( \phi_i(\nu) \): cost saving allocated to carrier \( i \)
- \( C(S) \): total cost of subcoalition \( S \)
- \( CS(S) \): cost saving of subcoalition \( S \)
- \( SR(S) \): saving ratio of subcoalition \( S \)
- \( Veh(S) \): total number of vehicles
- \( Dist(S) \): total travel distance of subcoalition \( S \)
- \( ST(S) \): total schedule time of subcoalition \( S \)
- \( WT(S) \): total waiting time of subcoalition \( S \)
- \( \alpha, \beta, \gamma, \lambda \): penalty factor for \( Veh(S), Dist(S), ST(S), WT(S) \) respectively
- \( a_k \): allocation rule
- \( c_j \): Saving Ratio of \( j\) – players
- \( r_j(a_k) \): Besson rank of an allocation rule \( a_k \) with respect to \( j\) – carrier
- \( rc_j \): Besson rank of \( j\) – carrier

Table 2. Test instances and related calculations.

<table>
<thead>
<tr>
<th>Coalition</th>
<th>( Veh(S) )</th>
<th>( Dist(S) )</th>
<th>( ST(S) )</th>
<th>( WT(S) )</th>
<th>( C(S) )</th>
<th>( CS(S) )</th>
<th>( SR(S) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>{A}</td>
<td>20</td>
<td>150</td>
<td>3</td>
<td>2</td>
<td>1521.3</td>
<td>0.0</td>
<td>0.0%</td>
</tr>
<tr>
<td>{B}</td>
<td>30</td>
<td>170</td>
<td>3.5</td>
<td>3</td>
<td>2127.6</td>
<td>0.0</td>
<td>0.0%</td>
</tr>
<tr>
<td>{C}</td>
<td>32</td>
<td>120</td>
<td>2.5</td>
<td>2</td>
<td>2097.1</td>
<td>0.0</td>
<td>0.0%</td>
</tr>
<tr>
<td>{D}</td>
<td>40</td>
<td>100</td>
<td>2</td>
<td>1</td>
<td>2480.9</td>
<td>0.0</td>
<td>0.0%</td>
</tr>
<tr>
<td>{A, B}</td>
<td>45</td>
<td>300</td>
<td>6</td>
<td>4.5</td>
<td>3317.6</td>
<td>331.23</td>
<td>10.0%</td>
</tr>
<tr>
<td>{A, C}</td>
<td>45</td>
<td>250</td>
<td>5</td>
<td>3.5</td>
<td>3177.2</td>
<td>441.22</td>
<td>13.9%</td>
</tr>
<tr>
<td>{A, D}</td>
<td>54</td>
<td>220</td>
<td>4.5</td>
<td>2.5</td>
<td>3587.9</td>
<td>414.23</td>
<td>11.5%</td>
</tr>
<tr>
<td>{B, C}</td>
<td>55</td>
<td>260</td>
<td>5.5</td>
<td>4.5</td>
<td>3755.4</td>
<td>469.23</td>
<td>12.5%</td>
</tr>
<tr>
<td>{B, D}</td>
<td>63</td>
<td>210</td>
<td>5</td>
<td>3.5</td>
<td>4055.2</td>
<td>553.23</td>
<td>13.6%</td>
</tr>
<tr>
<td>{C, D}</td>
<td>64</td>
<td>200</td>
<td>4</td>
<td>2.5</td>
<td>4081.7</td>
<td>496.23</td>
<td>12.2%</td>
</tr>
<tr>
<td>{A, B, C}</td>
<td>69</td>
<td>390</td>
<td>7</td>
<td>6</td>
<td>4890.1</td>
<td>855.85</td>
<td>17.5%</td>
</tr>
<tr>
<td>{A, B, D}</td>
<td>76</td>
<td>380</td>
<td>6.5</td>
<td>4</td>
<td>5246.8</td>
<td>882.90</td>
<td>16.8%</td>
</tr>
<tr>
<td>{A, C, D}</td>
<td>78</td>
<td>380</td>
<td>6</td>
<td>3.5</td>
<td>5216.6</td>
<td>882.68</td>
<td>16.9%</td>
</tr>
<tr>
<td>{B, C, D}</td>
<td>86</td>
<td>340</td>
<td>5.5</td>
<td>4.5</td>
<td>5684.4</td>
<td>1021.08</td>
<td>18.0%</td>
</tr>
<tr>
<td>{A, B, C, D}</td>
<td>97</td>
<td>480</td>
<td>8</td>
<td>5</td>
<td>6682.5</td>
<td>1544.35</td>
<td>23.1%</td>
</tr>
</tbody>
</table>

a grand coalition. The saving ratio of subcoalition \( SR(S) \) is as high as 23.1% in the case of the grand coalition, what means that the carriers acting together can reduce their total transportation costs significantly.
Analysis with Game Theory Approach

It should be noted that the non-empty core implies that all the carriers in the coalition are willing to collaborate, as the profit distribution is stable. Moreover, the imputation suggested by the allocation rules presented above is in the core, as, for every coalition \( S \subset N \), condition \( \sum_{i \in S} \phi_i(\nu) \geq \nu(S) \) holds.

<table>
<thead>
<tr>
<th>Carriers in coalitions</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
</tr>
</thead>
<tbody>
<tr>
<td>( C({i}) )</td>
<td>1521.3</td>
<td>2127.6</td>
<td>2097.1</td>
<td>2480.9</td>
</tr>
<tr>
<td>Equal allocation</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cost saving</td>
<td>386.1</td>
<td>386.1</td>
<td>386.1</td>
<td>386.1</td>
</tr>
<tr>
<td>Net cost</td>
<td>1135.2</td>
<td>1741.5</td>
<td>1711.0</td>
<td>2094.8</td>
</tr>
<tr>
<td>Savings ratio</td>
<td>25.4 %</td>
<td>18.1 %</td>
<td>18.4 %</td>
<td>15.6 %</td>
</tr>
<tr>
<td>Proportional allocation</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cost saving</td>
<td>285.6</td>
<td>399.6</td>
<td>393.7</td>
<td>465.7</td>
</tr>
<tr>
<td>Net cost</td>
<td>1235.7</td>
<td>1728.2</td>
<td>1703.4</td>
<td>2015.1</td>
</tr>
<tr>
<td>Savings ratio</td>
<td>18.8 %</td>
<td>18.8 %</td>
<td>18.8 %</td>
<td>18.8 %</td>
</tr>
<tr>
<td>Shapley value</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cost saving</td>
<td>321.6</td>
<td>395.6</td>
<td>404.3</td>
<td>422.8</td>
</tr>
<tr>
<td>Net cost</td>
<td>1199.7</td>
<td>1732.0</td>
<td>1692.8</td>
<td>2058.0</td>
</tr>
<tr>
<td>Savings ratio</td>
<td>21.1 %</td>
<td>18.6 %</td>
<td>19.3 %</td>
<td>17.0 %</td>
</tr>
<tr>
<td>Nucleolus</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cost saving</td>
<td>275.6</td>
<td>414.0</td>
<td>413.8</td>
<td>440.9</td>
</tr>
<tr>
<td>Net cost</td>
<td>1245.7</td>
<td>1713.5</td>
<td>1683.3</td>
<td>2040.0</td>
</tr>
<tr>
<td>Savings ratio</td>
<td>18.1 %</td>
<td>19.5 %</td>
<td>19.7 %</td>
<td>17.8 %</td>
</tr>
<tr>
<td>Gately point</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cost saving</td>
<td>318.8</td>
<td>403.1</td>
<td>403.0</td>
<td>419.5</td>
</tr>
<tr>
<td>Net cost</td>
<td>1202.5</td>
<td>1724.4</td>
<td>1694.1</td>
<td>2061.4</td>
</tr>
<tr>
<td>Savings ratio</td>
<td>21.0 %</td>
<td>18.9 %</td>
<td>19.2 %</td>
<td>16.9 %</td>
</tr>
</tbody>
</table>

Subsequently, all allocation rules according to the solution concepts presented in Section 2 are applied to shippers the collaboration network. Each allocation is computed through a simple data set as is given in Table 2. For convenience of comparison, the results are listed in Table 3 especially the allocations based on the different solutions concepts: equal and proportional, Shapley value, Nucleolus, and Gately point. The rows represent the corresponding costs allocated to the customers for each solution concept. The cost savings are calculated using formula (1). The Savings Ratio of a coalition \( S \) is defined in term of

\[
SR(S) = \frac{CS(S)}{C(S)}
\]

as a percentage of the corresponding Stand-alone cost. This ratio reflects the individual satisfaction among carriers via coalition formation. Net cost equals to Stand-alone cost minus Cost savings.

The total cost savings, 15443.35 are divided among the four carriers according to the proposed allocation rules, as summarized in Table 3. These results show clearly that it is worth to cooperate and each participant can gain more profits comparing with stand-alone operation. The cost savings
are in the range from 16.6% to 25.4%. The equal cost allocation rule treats all four players equally. However, the equal allocation cannot reveal the contributions of the partners and might generate serious instability in the collaboration, or even reduce the incentive to collaborate.

The Proposed Approach – RAS Method

Algorithm

Although all allocation rules presented in Section 4 lies in the core, they also leave the fundamental question as to which allocation rule should be chosen to guarantee the stability of the coalition. On the one hand, in practice, suppliers mainly use intuition to select allocation rules. On the other hand, the literature has a context-dependent list of desirable conditions that a sharing mechanism needs to satisfy and the list of methods to enable decision-makers to make the right choice for their decisions. These methods can be grouped into two groups [2]: methods of the unique approach of synthesis such as TOPSIS, SMART, WEIGHTED SUM, MAUT, MAVT, UTA, AHP, ANP and the outranking methods of synthesis as PROMETHEE, ELECTRE, and ORESTE. However, the application of decision theory to real-world problems in practice is by far not trivial and could be one of the hardest parts of the decision-making process [19]. Moreover, there is no single MCDA method that can claim to be a superior method for all decision-making problems. Generally, selecting an appropriate MCDA method depends on the decision problem at hand, preferably, the simpler method should be chosen for the decision analysis [19].

Although there are several preference ranking methods [3], this paper inspired by ORESTE method [16], introduces the Ranking Allocation Scheme (RAS) which has the potential to be acceptable for choosing an appropriate allocation rule in the VRP. Concerning the evaluation and ranking of actions, we choose the method of weighted sum thanks to the simplicity of its mathematical model, which will be easily integrated with allocation rules results. In addition, the ranking method has been used to avoid the uncertainty of the decision-makers appreciations when evaluating the criteria.

The proposed decision-aiding framework is the problem structuring method which provides guidance for cooperative carriers in VRPTW problem. The RAS method deals with the situation where allocation rules $a_k (k = 1, 2, ..., m)$ are ranked according to players (carriers) Saving Ratio $c_j (j = 1, 2, ..., n)$, and the main objective is to find a global preferences structure on a set of allocation rules. Finally, the steps of the algorithm (RAS) which combines CGT with the ranking method is presented as follows. INPUT:

- set $a_k (k = 1, 2, ..., m)$ of allocations rules, consists of two or more rules,
- set of carriers consists of two or more players,
- total cost $C(S)$ of subcoalition $S$,
- cost saving $CS(S)$ of subcoalition $S$.

Step 1 Building the decision making committee.

The committee (represented by carriers) is formed to identify the allocations rules and makes a preliminary if necessary. One extra person (not carrier) plays a role of analyst. The analyst is familiar with the RAS procedure. The analyst provides methodological guidelines and advise in different phases of the decision process.

Step 2 Initial setting.

The input data are collected. To assure the effectiveness of the proposed methodology, these data must be collected accurately.
Step 3 Calculate the Savings Ratio.
Based on formula (3), the Saving Ratio is calculated by the analyst for each allocation rule \( a_k \).

Step 4 Calculate Besson rankings of the carriers and allocations rules.
The Besson \[20\] rank is derived by analyst for each carrier based on its weak order among all allocation rules with respect to Saving ratio of each carrier (as a result of Step 3). It could happen that allocation rules or carriers have the same significance (if their Saving ratios are equal). In this case, Besson’s ranking is determined by taking into account the sums of integers ranks. Thus, if there are two allocation rules, which are ranked by the carrier in the fifth position, then they will obtain Besson’s ranking \( \frac{5+6}{2} = 5.5 \). The Besson rank of an allocation rule \( a_i \) with respect to a carrier \( j \) is denoted by \( r_j(a_i) \) and the Besson rank of carriers \( j \) is denoted by \( rc_j \). For more details about Besson’s rank, please refer to \[20\].

Step 5 Calculate the projection distances.
The projection distances \[20\] correspond to the relative position of the allocation rules with respect to an arbitrary origin \( O \) and can be expressed by the following equation

\[
d_j(O, a_i) = \frac{1}{2}(rc_j + r_j(a_i)).
\]

The smaller projection distance, the better is the position of the allocation rules.

Step 6 Rank the projections (global ranks).
A mean global Besson rank \( r_j(a_i) \) is assigned to all the projection distances from the lowest to the highest ones. Smaller \( r_j(a_i) \) indicates the better position of the particular allocation rule.

Step 7 Calculate the mean ranks
For each allocation rules, a mean rank is computed by the summation of their global Besson ranks over the entire set of carries using the following expression

\[
r(a_i) = \sum_{j=1}^{n} r_j(a_i)
\]

Step 8 Prioritization of allocation rules.
The mean ranks are sorted increasingly to determine the global weak order of the allocation rules. Finally, the ranking of the allocation rules is obtained.

Step 9 The choice phase.
In this step the decision indicating the selected allocation rule to be implemented by the players - carriers is made. The analyst presents the ranking results to the players in the decision-making committee. The players obtain information about the saving costs in the case of cooperation in comparison to the costs when they act independently. The players may agree and approve the allocation rule proposed in the algorithm or no. In the case of disagreements among players, the decision-making committee could be asked to set up a new set of allocation rules and the evaluation procedure is repeated. If disagreements still arise, a new different set of players can be proposed for which the evaluation procedure is performed again. Generally, the decision support tool is iterative and it is up to the analyst decision when the procedure terminates. If the allocation rule proposed in the algorithm is approved by the players it is recommended to be implemented by them in practice.
Experimental Computational Results

This section illustrates the method proposed in Section 5.1 to demonstrate the computation, applicability, and assurance of RAS for selection of allocation rules outlined in Section 2 in the case of grand coalition of four carriers A, B, C, and D.

In this method, at first, the Savings Ratio for each allocation rule and each carrier is determined, as shown in Table 3. Now, based on Step 3 of the algorithm, the Besson ranking of all allocation rules and carriers are determined, as shown in Table 4 respectively. Since in our example all carriers are equal in negotiation, they all receive rank 1. Now, applying formula 4, the projection distance are computed, as shown in Table 5. From this table, the rankings of the global ranks are obtained, as given in Table 6. As the last step of this method, the final ranking the candidate allocation rule is obtained, as shown in Table 7. From Table 7 Shapley value is the best choice, Nucleolus and Gately point is the second choice and Equal is the worst method for coalition. The Shapley value applied as the selected allocation rule results in the Saving costs and Saving ratios obtained by particular players presented in Table 3.

<table>
<thead>
<tr>
<th>Carriers</th>
<th>Equal</th>
<th>Proportional</th>
<th>Shapley value</th>
<th>Nucleolus</th>
<th>Gately point</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>25.4 %</td>
<td>18.8 %</td>
<td>21.1 %</td>
<td>18.1 %</td>
<td>21.0 %</td>
</tr>
<tr>
<td>B</td>
<td>18.1 %</td>
<td>3</td>
<td>18.6 %</td>
<td>19.5 %</td>
<td>18.9 %</td>
</tr>
<tr>
<td>C</td>
<td>18.4 %</td>
<td>18.8 %</td>
<td>19.3 %</td>
<td>19.7 %</td>
<td>19.2 %</td>
</tr>
<tr>
<td>D</td>
<td>15.6 %</td>
<td>18.8 %</td>
<td>17.0 %</td>
<td>17.8 %</td>
<td>16.9 %</td>
</tr>
</tbody>
</table>

Table 4. Besson ranking of allocation rules

Table 5. Projection distance

<table>
<thead>
<tr>
<th>Carriers</th>
<th>Equal</th>
<th>Proportional</th>
<th>Shapley value</th>
<th>Nucleolus</th>
<th>Gately point</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>1</td>
<td>2.5</td>
<td>1.5</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>B</td>
<td>3</td>
<td>2</td>
<td>2.5</td>
<td>1.5</td>
<td>1.5</td>
</tr>
<tr>
<td>C</td>
<td>3</td>
<td>2.5</td>
<td>1.5</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>D</td>
<td>3</td>
<td>1</td>
<td>2</td>
<td>2.5</td>
<td>2.5</td>
</tr>
</tbody>
</table>

Table 6. Rankings of the projections

<table>
<thead>
<tr>
<th>Carriers</th>
<th>Equal</th>
<th>Proportional</th>
<th>Shapley value</th>
<th>Nucleolus</th>
<th>Gately point</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>1.5</td>
<td>15</td>
<td>4.5</td>
<td>9.5</td>
<td>9.5</td>
</tr>
<tr>
<td>B</td>
<td>19</td>
<td>9.5</td>
<td>15</td>
<td>4.5</td>
<td>4.5</td>
</tr>
<tr>
<td>C</td>
<td>19</td>
<td>15</td>
<td>4.5</td>
<td>9.5</td>
<td>9.5</td>
</tr>
<tr>
<td>D</td>
<td>19</td>
<td>1.5</td>
<td>9.5</td>
<td>15</td>
<td>15</td>
</tr>
</tbody>
</table>
The integration of game theory concepts and the ranking method has several advantages for the decision support that can summarize below: Manage cooperative situation by taking into account all the factors; Improve the uncertainty that covers the evaluation of decision-makers’ judgments during the problem analysis; Can help companies make more effective and informed transportation fleet decisions, especially from the cooperation perspective. Following these considerations, the present study uses a ranking method to answer this cooperative question and ensure the general applicability of the collaboration mechanism chosen.

Conclusions

This paper considers the cost allocation problem of logistics horizontal supply chain collaboration and designs a practical method of choosing efficient cost allocation rules based on game theory. The aim is to develop a scheme to distribute the gains from collaboration in order to make collaboration appealing to shippers. Our contributions in this study can be summarized as follows. First, the paper presents a scheme based on assumed costs. Cost savings and savings ratios were determined for all possible coalitions and next values: Proportional allocation, Equal allocation, Shapley value, Nucleolus, and Gately point. Settlements determined using game theory are free of subjectivity, and a method based on the five methods used to reach such a settlement can be considered fair and transparent. Second, for the logistics coalition, we obtain the optimal cost-sharing mechanism to improve their competitiveness and profitability. Subsequently, we proposed a novel ranking method to determine the optimal game strategy to set up a group of cooperating companies. Third, we develop a ranking method algorithm and illustrate its applicability by using a numerical example. Using the proposed RAS algorithm, we can further investigate the other classical logistics applications for companies, e.g. clearing the costs of: joint production, joint marketing campaigns, expanding into new markets, investing in new technologies (innovation), co-investing, bids associated with very large (strategic) investments, etc.

The presented research can be extended in several ways. The four-person game can be extended to a \( n \)-person \( (n > 4) \) game by adding shippers. A greater number of solution concept of the game and respective allocation rules can be taken into account. In both cases of extensions, the proposed RAS method is still valid. All the input parameters in the model (e.g. \( Dist(S) \), \( ST(S) \), \( WT(S) \)) are assumed to be known to all the players. However, the reality of modern-day business is such that these parameters are usually incomplete and not available to all the players in the supply chain game. In these cases, a model with asymmetric information may be applied, using ideas developed in \[11\] and \[12\]. Another issue is to illustrate the proposed approach with an extensive numerical example in which different cost allocation rule are compared and gain insight into the decision support system in case of a larger number of players. Furthermore, another research direction

<table>
<thead>
<tr>
<th>Solution</th>
<th>Sums rank</th>
<th>Rank</th>
</tr>
</thead>
<tbody>
<tr>
<td>Equal</td>
<td>58.5</td>
<td>5</td>
</tr>
<tr>
<td>Proportional</td>
<td>41</td>
<td>4</td>
</tr>
<tr>
<td>Shapley value</td>
<td>33.5</td>
<td>1</td>
</tr>
<tr>
<td>Nucleolus</td>
<td>38.5</td>
<td>2.5</td>
</tr>
<tr>
<td>Gately point</td>
<td>38.5</td>
<td>2.5</td>
</tr>
</tbody>
</table>
may be concern on investigation of the bargaining powers of each of the coalition members. As it stands, these are not factored into the sharing mechanism. To put it in a nutshell, we are thus confident that, with the indicated modification, our study can serve as a practical and broad preference point for further cost-sharing design in logistics.

References


