2017 International Conference on Mathematics, Modelling and Simulation Technologies and Applications (MMSTA 2017)
ISBN: 978-1-60595-530-8

A Genetic Algorithm for an Integrated Production Policy Under Time-vary Demand and Unit Price

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Keywords: Integrated production batch policy, Time-vary demand, Time-vary unit cost, genetic algorithm, Inventory.

Abstract. Under time-varying demand and raw material purchase cost, this study investigates a genetic algorithm for an integrated production batch policy that considers the batch size of raw materials and finished goods simultaneously over a finite time horizon. A genetic algorithm (GA) with the chromosome of real number type to solve this problem is presented. Although, standard GA operators are used to generate new populations, the particular of this research is that we select one differentiate equation to develop a proposed production scheme. Then, calculate the total cost with this production scheme as the fitness function to evaluate the populations. In this paper, an explicit procedure to obtain an approximating solution is provided and numerical examples to illustrate the proposed model are shown as well.

Introduction

The traditional economic order quantity (EOQ) and economic production quantity (EPQ) are determined respectively in a production system. Sarker et al [1] and Sarker and Khan [2] found out that considering both EOQ and EPQ simultaneously in a production system would derive a desirable performance. However, those contributions on this aspect assumed a constant demand and a fixed unit price, which is no longer suitable for today's time-based competition. Actually, the demand and unit cost may vary with time. During 2005-2007, emerging market economies had a comparatively higher growth than rest of the world, caused the price of commodity increasing very steeply. In beginning 2008, the subprime mortgage problem in US triggered a worldwide financial crisis. The price of most commodities fell dramatically on expectations of diminished demand in a global recession. According to International Monetary Fund (IMF) [3] database for the price of commodity, the price of copper, a critical component of modern industry, was from US$ 3241.9 to US$ 8059.19 per ton raised 162% during 2005M05-2006M05 due to growth from emerging market economies. Later, it was from US$ 8714.18 to US$ 3770.88 per ton dropped 57% during 2008M04-2009M03 owing to the US subprime crisis. Due to advanced technology innovation, rapidly declining on price of components or products and short product life cycle are two of important characters of high-tech products, including computers, communication devices, and consumer electronic products. Consequently, high-tech. industries frequently experienced swinging both demand and price of commodity in recent years. Lee et al. [4] demonstrated that the cost of a personal computer decreased almost linearly with time. Wang [5] found that some components of electronic equipment drop 10-15% annually due to advances in technology and competition. Burruss and Kuettner [6] developed a short product life cycle model with a steep decreasing trend in demand for forecasting of the HP's products. Therefore, the traditional EPQ or EOQ model is no longer suitable for today's time-based competition and high-tech. industries need to advance the inventory policy when demand and price of commodity are looked forward to time varying trend.

For the phenomenon of product life cycle or boom-and-bust seasons, the demand pattern has been assumed a time-vary function for production or inventory problems in many contributions. Donaldson

[7] introduced an inventory replenishment problem with a linear (increasing) trend in demand. Henery [8] was the first to select a recursive algorithm for determining the optimal replenishment schedule under the condition of fixed replenishment lots. Considering both increasing and decreasing demand, Hariga's work [9] was based on replenishment period to deal with the same problem. Hill [10] was the first author who studied economic production quantity under a linearly increasing demand. Rau and Ouyang [11] demonstrated a general equation for EOQ, EPQ and an integrated production-replenishment policy for linear trend in demand. Khouja and Park [12], while considering products with continuous decrease in price for technology innovation, assumed that price decreased exponentially and developed a closed-form approximate solution for the EOQ model with price-dependent holding cost and periodic policy over a finite horizon. Later, Khouja and Goyal [13] assumed that the unit price was a linear or exponential function for developing an EOQ model with a non-periodic policy under a finite time horizon. Ouyang and Rau [14] extended the above work [13] to an EPQ model. Teng and Yang [15] assumed that both demand and unit cost were linear or exponential functions and presented an EOQ model for deteriorating items with shortages, but they assumed that the holding cost of their model is a constant. Based on the same assumption, Teng et al. [16-17] presented two studies on inventory and production models with time-varying demand and unit costs. However, from the above contributions, there are two kinds of assumptions regarding cost issue. Teng et al. [15-17] assumed that the holding cost per unit is a constant, and Khouja et al. [12-13] and Ouyang and Rau [14] assumed the holding cost is a fraction of unit purchase cost per unit time. The latter looks more reasonable and more accurate to reflect the real situation. In addition, Peterson and Silver [18] and Chopra and Meindl [19] deemed that capital cost is often the most important component of the holding cost. Thus, for reflecting on the real situation, each replenishment or production cycle should have not only different unit purchase cost but also different unit holding cost from the capital cost view of point.

Genetic algorithms (GAs) are one of most common search techniques to mimic the process of natural selection and natural genetics for optimization and search problems. A genetic algorithm to induce learning technique was first introduced by Holland [20]. Including production control, facility layout, line balancing, production planning, supply chain management and design, and other, many contributions have applied to production and inventory problems. However, most of studies focused on production control, facility layout and supply chain and few concerned with inventory problems. For characteristics of the chromosome in genetic algorithms, many contributions selected it to solve those inventory problems with discrete demand, such as economic lot size problem (ELSP) and dynamic lot size problem. The formulation is ideally suited for using GAs, so Khouja et al. [21] proposed a genetic algorithm to derive this ELSP solution. Gaafar [22] selected genetic algorithms for the deterministic time-varying lot-sizing problem with batch ordering and backorders. References on these problems are not listed, as they have no direct bearing on this field considered here. Genetic algorithms only need a computable objective function with no requirements of mathematical theory proof such as convexity. Recently, Bera et al. [23] studied GAs applying to a realistic inventory model with continuous demand under finite horizon. Considering deteriorating, inflation, budget constraints, shortages and finite planning time horizon, Jana et al. [24] also selected GAs for inventory models with continuous demand.

Assumptions and Notation

To develop our proposed model, the following assumptions and notation are used.

Assumptions:

- A finite foreseeable time horizon is considered.
- The purchase price is predetermined at the beginning of each production cycle.
- Both the demand and unit purchase cost are linear or exponential functions.
- Demand and unit purchase cost are greater than zero at the end of the time horizon.

- Production capacity is always greater than demand.
- Shortages, delivery time and cost are negligible.
- Single product made from single raw material.
- No stock is held at the beginning and the end of the time horizon.

Notation:

H planning horizon.
n number of production cycles.
$TC(n)$ total cost for n cycles in the integrated production system.
$f_d(t)$ demand at time t, $f_d(t) = a + bt$ or $f_d(t) = ae^{bt}$.
$f_u(t)$ unit raw material cost at time t, $f_u(t) = c + dt$ or $f_u(t) = ce^{dt}$.
P finite production capacity $P > f_d(H)$ for $f_d'(t) > 0$ or $P > f_d(0)$ for $f_d'(t) < 0$.
t_i terminating time of the ith production cycle.
T_i time interval of the ith production cycle, ($T_i = t_i - t_{i-1}$).
c_m ordering cost for raw material.
c_p production set up cost.
m_f manufacturing fee for each finished goods.
h_p holding cost for the finished product.
h_m holding cost for raw material.
Q_i the production quantity at the ith cycle.

The Mathematical Model

The proposed model is an unconstrained nonlinear problem, where we have to determine the replenishment schedule t_i and the number of production cycles n in the planning horizon H. The Fig. 1 shows the inventory level for an increasing demand case and Fig. 2 shows the inventory level for a decreasing demand.

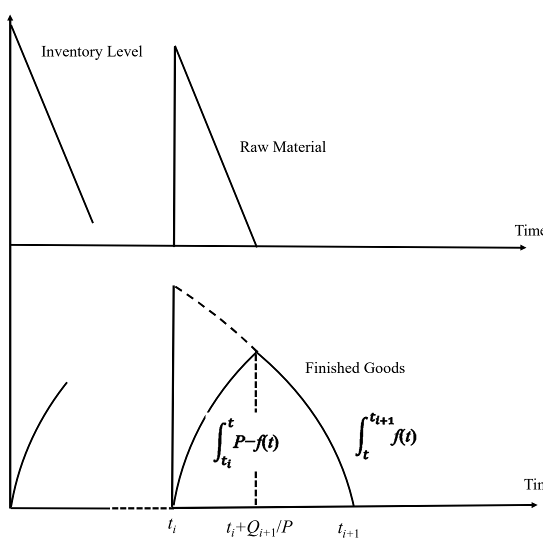

Figure 1. The inventory level for increasing demand.

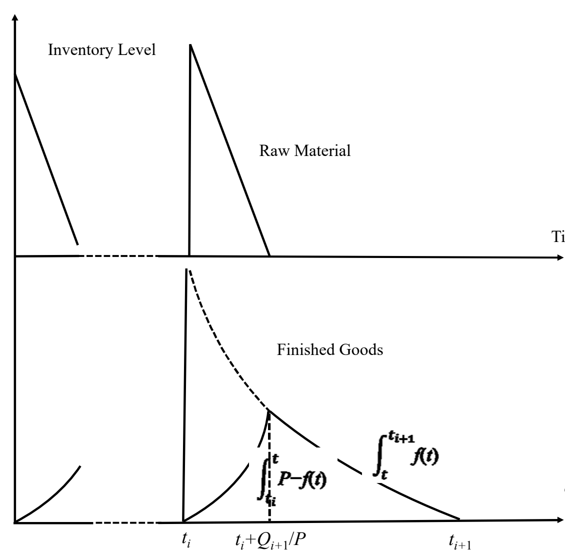

Figure 2. The inventory level for increasing demand.

From the above figures, it is obvious that there are two subsystems, including a replenishment subsystem for raw material and a production subsystem for finished product, to consider. The total cost for the material in replenishment subsystem, including purchase cost, order cost and holding cost can be expressed as follows:

$$TC_m(n) = nc_m + \sum_{i=1}^{n} f_u(t_{i-1}) \int_{t_{i-1}}^{t_i} f_d(\mathrm{t_i})dt + h_m f_u(t_{i-1}) \sum_{i=1}^{n} \frac{\left(\int_{t_{i-1}}^{t_i} f_d(\mathrm{t_i})dt\right)^2}{2P} \quad \text{for} \quad i = 1,2.....,n. \qquad (1)$$

The total cost for finished goods in production subsystem, including set up cost, manufacturing fee and holding cost, can be expressed

$$TC_p(n) = nc_p + \sum_{i=1}^{n} m_f \int_{t_{i-1}}^{t_i} f_d(t_i)dt + h_p\left(m_f + f_u(t_{i-1})\right) \sum_{i=1}^{n} \left[\int_{t_{i-1}}^{t_i} \int_{t}^{t_i} f(u)dudt - \frac{\left(\int_{t_{i-1}}^{t_i} f(t)dt\right)^2}{2P} \right]. \qquad (2)$$

Therefore, the total relevant cost for this integration model for both raw material and finished product is

$$TC(n) = n\left(c_m + c_p\right) + \sum_{i=1}^{n}\left(m_f + f_u(t_{i-1})\right) \int_{t_{i-1}}^{t_i} f_d(\mathrm{t_i})dt + h_p \sum_{n=1}^{n} \int_{t_{i-1}}^{t_i} \int_{t}^{t_i} f(u)dudt$$
$$+\left[\left(h_m - h_p\right) f_u(t_{i-1}) - h_p m_f\right] \sum_{i=1}^{n} \left[\frac{\left(\int_{t_{i-1}}^{t_i} f(t)dt\right)^2}{2P} \right] . \qquad (3)$$

Differentiating Eq. (3) with respect to t_i, i =1, 2….n-1, we obtain Eq. (4). TC (n) has a stationary point at t_i if Eq. (4) equals zero, which is also a necessary condition to reach the optimal solution.

$$\frac{\partial TC(n)}{\partial t_i} = \left(f_u(t_{i-1}) - f_u(t_i)\right) f_d(t_i) + h_p\left[f_u(t_{i-1}) \int_{t_{i-1}}^{t_i} f_d(t_i)dt - f_c(t_i) f_d(t_i)\left(t_{i+1} - t_i\right) \right]$$
$$+ f_u^{'}(t_i)\left(\int_{t_i}^{t_{i+1}} f_d(t_i)dt + h_p + \int_{t_i}^{t_{i+1}} \int_{t}^{t_{i+1}} f(u)dudt \right) + \frac{\left(h_m - h_p\right)}{2P} f_u^{'}(t_i) \left(\int_{t_i}^{t_{i+1}} f_d(t_i)dt \right)^2 . \qquad (4)$$
$$+ \frac{f_u(t_i)}{P}\left[\left(h_m - h_p\right) f_u(t_{i-1}) - h_p\right] \int_{t_{i-1}}^{t_i} f_d(t_i)dt - \frac{f_u(t_i)}{P}\left[\left(h_m - h_p\right) f_u(t_i) - h_p\right] \int_{t_i}^{t_{i+1}} f_d(t_i)dt$$

In this problem, we have to determine the number of production cycles n and production schedule t_i which is a complex nonlinear problem. Thus, we propose a genetic algorithm with the chromosome of real number type for seeking to an approximately optimal solution. According to Bellman's principle of optimization [25], once we find the correct t_1 and the remaining production schedule t_2 to t_n can be determined by Eq. (4). Thus, we only have to search t_1 and neglect n.

Genetic Algorithm

An explicit procedure to derive an approximate solution by this proposed genetic algorithm shows in the following:

Coding scheme:

We assume that the planning horizon is always less than one year. Consequently, randomly select a chromosome consisting of 20-bits for the first production cycle time t_1 under a reasonable range. For example, t_1 = 0.625 is represented by the bit string 1010,0000,0000,0000,0000.

Fitness function:

With a trial values (chromosome) for the starting point t_1 in Eq. (4) to get obtain t_2. Repeatedly solve t_3, t_4,….., t_{n-1}, t_n until $t_{n-1} < H$ and $t_n \geq H$. Then, let $t_n = H$ and compute $TC(n, \{t_i\})$ and let $t_{n-1} = H$ and compute $TC(n-1, \{t_i\})$from Eq. (3). Finally, compare with these two costs and select the lower one. We obtain the fitness function as follows:

$$\text{Fitness function:}\begin{cases} TC\left(n,\{t_i\}\right) & \text{if} \quad TC\left(n,\{t_i\}\right) < TC\left(n-1,\{t_i\}\right) \\ TC\left(n-1,\{t_i\}\right) & \text{if} \quad TC\left(n,\{t_i\}\right) > TC\left(n-1,\{t_i\}\right) \end{cases} \tag{5}$$

Genetic operators:

Three standard genetic operators are used, namely, reproduction, mutation and crossover.

Input parameters:

Parameters of genetic algorithm are population size = 100, probability of mutation = 0.01, probability of crossover = 0.7, generation size = 100, initial rate of crossover operation = 0.7, initial rate of mutation operation = 0.05, initial rate of reproduction operation = 0.1 and stop condition.

Output:

The local optimal production schedule and the total cost.

Step 1: Let $g = 0$, where g is the generation count.

Step 2: Generate initial population P(g) randomly.

Step 3: Evaluate $P(g)$ to determine the best fitness value (best_fitness) and initialize the rates of operations. Repeat.

Step 4: Generate new population $P(g+1)$ using GA operators' rates from above offsprings.

Step 5: Evaluate $P(g+1)$ and determine number of survivors. Save the new best_fitness.

Step 6: Set $P(g) = P(g+1)$. Until a stopping criterion is reached.

Step 7: Compute the output based on the results.

Numerical Examples

A program written in Visual Basic executes the following examples. To illustrate this proposed model, we provide four examples with fluctuating demand and unit price as follows: $f_d(t)$ = 16000-8000t (for Examples 1 and 3), $f_d(t)$ = 16000+8000t (for Examples 2 and 4), $f_u(t)$ = 16-8t (for Examples 1 and2) and $f_u(t)$ = 16-8t (for Examples 3 and 4) on purpose. Other parameters are c_m = 200, c_p =100, H = 0.5, h_m = 8% and h_p = 10%. Applied to the proposed solution procedures, the production cycles for these examples (1-4) are 7, 8, 1 and 1 and the corresponding total costs are 159041, 202291, 170718 and 218916 respectively. Production schedules and replenishment quantities for these examples are shown in Table I.

Table 1. Production schedules and replenishment quantities for numerical examples.

Ex.	I	1	2	3	4	5	6	7	8
1	t	0.067	0.135	0.205	0.276	0.349	0.423	0.500	
	Q	1034	981	925	865	801	734	661	
2	t	0.065	0.129	0.193	0.255	0.318	0.379	0.440	0.500
	Q	1075	1128	1179	1229	1278	1325	1371	1416
3	t	0.500							
	Q	6000							
4	t	0.500							
	Q	10000							

Summary

Genetic algorithms (GAs) are one of common search approaches applying to production and inventory problems. Foreseeing a time-varying demand and unit cost under a finite time horizon, this study presents a genetic algorithm to solve an integrated inventory model in a manufacturing system. Without mathematical theory proof of convexity, we proposed a genetic algorithm with the chromosome of real number type for seeking to an approximately optimal solution. The particular of our proposed genetic algorithm is that we select Eq. (4), differentiating the total cost function with respect to initial time of each production cycle, to extend the production scheme for the fitness function. Thus, we only have to search t_1 and neglect n. This algorithm has the potential applied to other complex problems. Thus, the further research direction will focus on various issues for a supply chain or production system.

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