An Overview of Numerical Methods for Reynolds Equation

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Abstract. The development and prospect of numerical calculation method of Reynolds equation in fluid lubrication are introduced. As a fundamental equation of fluid dynamics, Reynolds equation in the form of relatively is simple, and a large number of results have been researched. Most of these results were appeared in the early 21st century. With the continuous improvement of computer hardware and software technology, it is necessary to summarize these results, and combine with new technologies to future researchers for reference. This paper mainly discusses the existing methods from the application scope, the precision and efficiency of the solution, and discusses the process of solving the Reynolds equation by the latest Isogeometric Geometric analysis method. Finally, it is the prospect of the combination of widely used multi-grid method and Isogeometric Geometric analysis method.

Introduction

Fluid lubrication can effectively reduce wear and meet design requirements. The purpose is to separate the friction surface and form a boundary protection film to reduce the loss caused by friction[1]. Reynolds equation is the basic formula of lubrication theory and can be calculated by analytical method[2]. As people the deepening understanding of the complexity of Reynolds equation, the analytic solution has been unable to meet demand and the need to some transformation to solve the equation, then the Reynolds equation of discrete selection of a suitable numerical algorithms. In different models, the calculation method is different, because the equation is non-linear, and the accuracy of the overloading condition is generally difficult to meet the requirements. Because of solving the Reynolds equation is a process of constant iterative calculation, they need to solve for many times. In order to better adapt to the demanding model Reynolds equation of lubrication, the domestic and foreign scholars also used the different methods, in order to better meet the requirements on accuracy and efficiency. Therefore, the study and discussion of different numerical calculation methods provide reference and reference for further research.

Reynolds Equation

The Form of Reynolds Equation

Through the study of fluid dynamic pressure effect, Reynold first proposed and export Reynolds equation for fluid motion differential equations[3], the lubrication film pressure and the approximation of the relationship between the thickness under the isothermal condition is Eq. 1:

\[
\frac{\partial}{\partial x} \left( \frac{\rho \frac{\partial p}{\partial y}}{12 \eta} \right) + \frac{\partial}{\partial y} \left( \frac{\rho \frac{\partial p}{\partial x}}{12 \eta} \right) = \frac{\partial}{\partial x} \left( \frac{b(\eta_1 + \eta_2) \frac{\partial p}{\partial x}}{\frac{\partial p}{\partial y}} \right) + \frac{\partial}{\partial y} \left( \frac{b(\eta_1 + \eta_2) \frac{\partial p}{\partial y}}{\frac{\partial p}{\partial x}} \right)
\]

(1)

Considering the surface roughness and the effect of lubrication, Patir and Cheng[4] put the pressure and shear flow factor in to average flow Reynolds equation, to calculate the pressure of the
fluid field distribution, for the piston cylinder liner, a modified Reynolds equation of the form could be written as Eq. 2:

$$\frac{\partial}{\partial x} \left( \frac{\rho h^3}{12 \eta} \frac{\partial p}{\partial x} \right) + \frac{\partial}{\partial y} \left( \frac{\rho h^3}{12 \eta} \frac{\partial p}{\partial y} \right) = 6U_p \left( \frac{\partial (\rho h)}{\partial y} + \sigma \frac{\partial (\rho h)}{\partial y} + \frac{12}{\partial t} \left( \frac{\partial (\rho h)}{\partial t} \right) \right).$$  (2)

The Normalization of Reynolds Equation

Making some corresponding substitution and normalization, the Reynolds Eq. 3 is:

$$\frac{\partial}{\partial x} \left( K_1 \frac{\partial p}{\partial x} \right) + \frac{\partial}{\partial y} \left( K_2 \frac{\partial p}{\partial y} \right) = f \text{ or } \nabla (k \nabla p) = f$$  (3)

Numerical Algorithm of Reynolds Equation

As a partial differential equation, Reynolds equation is directly solved for solving difficulties. Many scholars abroad are trying to solve this domain. With the mature of numerical calculation method, academia is basic numerical calculation method, the Reynolds equation is solved by using common solution: the FDM (finite difference method), FEM (finite element method), and a new Isogeometric analysis(IGA), etc.

The Finite Difference Method

The finite difference method is proposed by British scholar Southwell[5], and then the linear algebraic equations are solved by combining the initial and boundary conditions[6]. The finite difference method basic idea is: solving domain to draw within the grid nodes instead of continuous solving domain, derivative of equation can be solved through the grid nodes difference quotient instead of a discrete function value, through proper converted into differential algebraic equation[7]. For example, the piston cylinder model can be changed as Eq.4:

$$\frac{\partial \Phi_x}{\partial x} \frac{\rho h^3}{\mu} \frac{\partial p}{\partial x} + \Phi_x \frac{\rho h^2}{\mu} \frac{\partial p}{\partial x} \frac{\partial h}{\partial x} + \Phi_x \frac{\rho h^3}{\mu} \frac{\partial^2 p}{\partial x^2} + \left( \frac{\partial \Phi_y}{\partial y} \frac{\rho h^2}{\mu} \frac{\partial p}{\partial y} \frac{\partial h}{\partial y} + \Phi_y \frac{\rho h^2}{\mu} \frac{\partial^2 p}{\partial x^2} \right) \right)$$

$$= 6U_p \frac{\partial p}{\partial x} \frac{\partial h}{\partial x} + 6U_p \frac{\partial h}{\partial y} \frac{\partial p}{\partial y} + 12 \rho \Phi_{p, y} \frac{\partial h}{\partial y}$$

Substituting the derivative form into the equation can be obtained Eq.5:

$$p_{i,j} = \left( p_{i+1,j} - p_{i-1,j} \right) \left( A + B \right) + \left( p_{i,j+1} - p_{i,j-1} \right) \left( C + D \right) - E \left( 2B + D \right)$$  (5)

A, B, C, D, E is known.

According to the different methods of difference method of boundary, reynolds equation can be transform into a variety of forms: general forward one order difference, first-order backward difference and central difference time back space, the five-point difference method, etc. Under isothermal condition, Zhang Yongliang[8] consider the connecting rod bushing non-newtonian lubricants, because simple differential expression eventually converges fast, also meet the precision requirement, so he use the difference discrete solving Reynolds equation for five. Considering the difference of the grid has important influence on the result of calculation, in order to improve the differential calculation accuracy, Wang Yu[9] inserted between the grid unit and the node number of nodes, but the number of mesh on the calculation efficiency could be compromised. For the non-woven bearing, Yu Rufe[10] give the grid independent analysis of Reynolds equation is carried out with the finite difference method, and it is concluded that the number of meshes have little effect on the low precision, but increases the computation.

How to use less grid to achieve ideal accuracy of the calculation results and the iterative calculation. If you need to reduce the iteration time, you can join the best relaxation factor. Under the condition of constant boundary, Wei Siyuan[11] give iteration factor estimation. But most
scholars are trial by input Numbers for this iterative algorithm of the relaxation factor, according to the experience value, if using adaptive iterative convergence relaxation factor to meet the different boundary conditions, the result can greatly reduce our workload and improve efficiency.

The Finite Element Method

Due to the finite difference method is not suitable for processing the engineering problems with complex boundary conditions, because the grid are used to maintain and coordinate axis in the same direction, for complex to solve regional adaptability is poorer, precision is not high, so in some model of finite difference method is not suitable for to solve Reynolds equation, such as complicated geometry model of bearing, porous model, etc. The finite element method can solve these problems well.

The finite element method (fem) is proposed by the British scholar Zienkiewicz[12], the basic idea is through discrete, the solution domain is divided into multiple cells, each unit can be found to solve the function of the interpolation points, by galerkin variational principle or weighted residual method, we can get the differential equation of the discrete solution[13]. Different discrete forms of Reynolds equation can be used to construct various finite element methods.

Integrate on the domain \(D = [0, 1] \times [0, 1]\) to obtain the weak form Eq.6 of Reynolds equation:

\[
- \int_D \left[ k_1 \frac{\partial p}{\partial x} + k_2 \frac{\partial v}{\partial y} \right] x dY \int_D f dX = \int_D f dX dY
\]

Then, the equation stiffness matrix is derived, and the solution in the unit is approximated by the basis function of the unit[14], and the element stiffness matrix is obtained as: \(K_e P_e = f_e\)

Through the node equilibrium equation, the composite element matrix is Eq.7:

\[
KP = F
\]

Considering the complexity of the surface structure in the numerical simulation of the cylinder sealing performance, Chen Tao[15] chose the finite element method to solve the Reynolds equation. Tom Gustafsson[16] consider the Reynolds equation with uncertain membrane thickness approximation, by random Galerkin finite element method (fem) in the space and the random field of high order discrete numerical, not only calculating the stress field of a journal bearing calculation and illustrating the effectiveness of the method and operability, but also ordering time and improving the efficiency. In order to shorten the momentum equation used in solving time, Guo Le[17] used the three steps of finite element method to discrete the Reynolds equation, and combined the high order with finite element space time format and discrete to get better numerical stability. Considering the influence of centrifugal force and the shape of surface grooves, Meng Qingrui[18] used the adaptive and better triangular element in the solution of finite element method to better approximate the surface of complex shape. Considering the discontinuity of the film thickness when dealing with the sealing parts of spiral grooves, JARRAY M[29] applied the finite element solution of Reynolds lubrication equation and film discontinuity to the spiral groove seal.

The Isogeometric Analysis Method

For the solution of Reynolds equation, the finite element method is generally preferred. It is suitable for dealing with geometric regions of complex shapes, but it is not efficient enough. The isogeometric method is very good at solving the Reynolds equation, such as the IGA technology was introduced by Hughed et al.[20], the analysis of the domain based functions in CAD (such as non-uniform b-spline, NURBS). Compared with traditional finite element, the solution of Reynolds equation requires less freedom to express the pressure and therefore has the potential to improve efficiency.

Basis function of finite element method (fem) is generally Lagrange interpolation functions and Hermite interpolation functions, such as geometric basis functions is used for NURBS basis function, through the control points to control the shape of the surface or entity, points less, the speed is faster, more efficient.
Then, the Non-uniform Rational B-Spline (NURBS) can be introduced by multiplying B-spline basis functions with appropriate weights. Similarly, using the B-spline basis function \( N_{i,p}(\xi) \), \( N_{j,q}(\eta) \) and weight \( w_{i,j} \), the two-dimensional NURBS basis function \( R_{i,j}^{p,q}(\xi,\eta) \), can be obtained as Eq.8:

\[
R_{i,j}^{p,q}(\xi,\eta) = \frac{N_{i,p}(\xi)N_{j,q}(\eta)w_{i,j}}{\sum_{i=1}^{m} \sum_{j=1}^{n} N_{i,p}(\xi)N_{j,q}(\eta)w_{i,j}}
\]

In isogeometric analysis, NURBS is applied to define \( F \) in the way, Eq.9:

\[
F(u,v) = \sum_{i=1}^{m} \sum_{j=1}^{n} R_{i,j}(u,v)w_{i,j}
\]

The stiffness matrix \( A \) can be calculated as follows Eq.10:

\[
A = \sum_{c,p,q} \sum_{j=1}^{n} \sum_{i=1}^{m} w_{c,p,q} \frac{N_{i,p}(\xi)N_{j,q}(\eta)}{\sum_{i=1}^{m} \sum_{j=1}^{n} N_{i,p}(\xi)N_{j,q}(\eta)w_{i,j}} \left( \gamma_{1,c,p,q}U_{c,p,q}U_{c,p,q}^T + c_{22,c,p,q}V_{c,p,q}V_{c,p,q}^T \right)
\]

In solving piston dynamics Reynolds equation, Liu Wei[21] used the geometric method (IGA) to compare with the traditional finite element method (FEM), the precision is the same, but the IGA had faster convergence speed, efficiency was obvious, it was concluded that IGA in solving piston lubrication Reynolds equation is more effective than the traditional FEM results.

Summary and Prospect

Efficiency and precision are the key problems of the Reynolds equation in the solution process, although there have been many numerical solutions to the Reynolds equation to date. The difference method in solving the Reynolds equation has faster convergence speed, because it used to represent the continuous function of discrete nodes, the error is big, but also to solve the domain has a lot of restrictions, mesh for this piece of no good adaptive as finite element method. The finite element method use the interpolation function of a continuous function approximation, the precision of phase compared with an improved finite difference method, while finite element of finite volume method in some ways made up for some defects, but also on the efficiency of solving this piece and improvement. The isogeometric method is based on the finite element method, which is used to replace the node with a small number of control points. In the meantime, the efficiency of the calculation is greatly improved.

From the development of the numerical calculation of fluid lubrication, the future direction is mainly focused on the solution of Reynolds equation, but further research is still needed in some areas. Such as geometric method as a new kind of algorithm for solving Reynolds equation, you can try the geometric method combined with multiple grid method to solve the Reynolds equation. This solution can be used as Reynolds equation algorithm and future research direction. It may greatly improve our accuracy and efficiency.

Reference


