Efficient Computation and Uncertainty Analysis of Underwater Acoustic Propagation based on Kriging Surrogate Model

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ABSTRACT

In this paper, a Kriging surrogate model is introduced to simplify the computation of underwater acoustic propagation and quantify the related uncertainty. By using a Kriging model to replace the single frequency and multi-frequency FOR3D acoustic model, the computational speed is increased by dozens of times while ensuring that the maximum mean square error is confined within 5%. In this way, the balance between efficiency and accuracy of the underwater acoustic propagation is achieved. On the basis of this Kriging surrogate model, the Monte Carlo simulation is also used to analyze the uncertainty of acoustic propagation loss, in terms of the posterior probability density distribution. Finally, the confidence intervals corresponding to different confidence levels are calculated in order to quantify the uncertainty of transmission loss.

INTRODUCTION

The actual underwater acoustic environment is greatly influenced by temperature and salinity. The sound velocity profile also shows a strong uncertainty with the dynamic changes of ocean movement. Uncertainty of various ocean parameters, such as sound velocity, horizontal distance, vertical depth, angle and frequency, lead to the uncertainty in sound field calculation. We have used the Tianhe II supercomputer [1] to do a series of parallel acceleration [2, 3] experiments of the FOR3D underwater acoustic model in order to quantify the uncertainties. However, we found that the calculation time of the traditional serial version of the FOR3D underwater acoustic is still very long. Even after parallel acceleration, the computational cost is still too large for the Monte Carlo method to quantify the uncertainty, which is consistent with the study of some other researchers [4, 5]. Under the premise of ensuring the accuracy, this study aims to use a Kriging surrogate model to simplify the underwater acoustic models and increases the calculation efficiency. In the meanwhile, the uncertainty of the acoustic model can be conveniently quantified based on the Kriging technique in terms of the confidence intervals. In addition, Kriging method can make the best of the testing data to construct a more accurate model, which is helpful to further improve the prediction accuracy. In contrast, the classical underwater acoustic models such as FOR3D do not have the interface to take testing data into consideration.

The study of the influence of the model parameters on the underwater acoustic propagation has a long history. Domestic and foreign scholars have also done a series of related research. Prof. Robinson [6] of Harvard university couples the acoustic model
with the Harvard marine forecasting system, which realized the dynamic prediction of the underwater acoustic environment. At the same time, the data assimilation technique is applied to the prediction of underwater acoustic environment to quantify the corresponding uncertainty. Cai et al. [7] of Duke University use Bayesian to infer the substitution of the RAM underwater acoustic model. Prof. Da Liang long [8] of the Naval Submarine Institute couples the ocean numerical model (POM model) with the underwater acoustic propagation model (FOR3D model) to establish the marine-acoustic coupling numerical model while the uncertainty is quantitatively estimated at the same time. In this paper, a Kriging surrogate model is used to improve the computational efficiency. Meanwhile the Monte Carlo method is used to quantify the related uncertainty.

METHOD

Kriging Model

The Kriging model is a statistical model that contains correlation (i.e., statistical relationships between measurement points) [9, 10]. This model is widely used in geo-statistics [11] and computational physics [12, 13]. The whole model runs through a two-step process. Firstly, we create a mutation function and a covariance function to estimate the statistical dependency (called spatial correlation) values that depend on the correlation model (fitting model). Secondly, unknown values are predicted (predictions). The fitting model is a polynomial fit while the covariance model is based on semi-variogram and covariance function. The semi-variogram is a function of geostatistical analysis [14], which is:

$$\gamma(x, h) = \frac{1}{2} \text{Var}[Z(x) - Z(x + h)]$$  \hspace{1cm} (1)

$Z(x)$ is the measured value at position $x$ and $Z(x + h)$ is the measured value at position $x + h$. At the same time, $Z(x)$ satisfies the stationary hypothesis, so:

$$\gamma(x, h) = \frac{1}{2} E[Z(x) - Z(x + h)]^2$$  \hspace{1cm} (2)

The covariance function is defined as:

$$\text{Cov}(X, Y) = E[(X - E(Y))(Y - E(Y))]$$  \hspace{1cm} (3)

In the kriging method, a number of functions for fitting the semi-variogram model are provided, such as exponential function (EXP), linear function (LIN), Gaussian function (GAUSS), cubic spline, etc. for different applications.

In the process of replacing the underwater acoustic model, the input of the Kriging model are the environmental parameters, the meshing and the results of the sound field calculation [15]. The output $P$ is composed of the polynomial $F$ and the random distribution $Z$, which are developed by the marine environmental parameters [14]

$$P(\varepsilon) = F(\beta, \varepsilon) + Z(\varepsilon) = f^T(\varepsilon)\beta + Z(\varepsilon)$$  \hspace{1cm} (4)

where $F(\beta, \varepsilon)$ is a regression equation, which is used to describe the global approximation of the output i.e. the sound field. $\beta$ is the regression coefficient, $f(\varepsilon)$ is a polynomial containing $\varepsilon$, which can be 0-order, first-order, second-order polynomial. $Z(\varepsilon)$ is the approximation error, which reflects the randomness of the sound field. The related physical quantity has the following statistical characteristics:

$$E(Z(\varepsilon)) = 0$$  \hspace{1cm} (5)

$$\text{Var}(Z(\varepsilon)) = \delta^2$$  \hspace{1cm} (6)
\[ Cov[Z(\varepsilon_i), Z(\varepsilon_j)] = \delta z^2 [R_{ij}(\theta, \varepsilon_i, \varepsilon_j)] \] (7)

\( R_{ij}(\theta, \varepsilon_i, \varepsilon_j) \) is a correlation function, that is, to interpolate from the random deviations of the polynomial fit, i.e. \( Z(\varepsilon) \). In general, the Kriging formula may be derived based on the framework of best linear unbiased prediction (BLUP) [16, 11], specifically through the unbiasedness and minimization of the mean squared error (MSE). where ‘best’ means the Kriging predictor has a minimum mean square error; ‘linear’ indicates the Kriging predictor can be realized in linear mixed models for the estimation of random effects; ‘unbiased’ requires the expectation of the Kriging predicted values is equal to the true value under certain assumptions made, such as normality of the Gaussian process. After comparison, the GAUSS correlation function and the second order polynomial was used to establish the semi-variogram model in this paper.

**Monte Carlo Method**

As an important approach in the field of computational science, Monte Carlo method can be used to solve the actual problem from a statistical perspective using a large number of statistical tests [17]. In the underwater acoustic calculation, if the acoustic model is directly used as a forward model under the Monte Carlo framework, it will require a lot of calculation cost, which is not desirable in practical applications. Kriging surrogate model can be used as a substitute for the input - output response of the simulation model, which can obtain the input - output relationship efficiently with a small amount of computation. Therefore, the Monte Carlo simulation integrated with Kriging meta model [18,19] can greatly reduce the computational load and it is practically feasible.

**EXPERIMENTS**

The whole experiment consists of three steps. We run the FOR3D underwater acoustic model in the beginning, getting the acoustic propagation loss. Then, we build a Kriging surrogate model and optimize the accuracy of the Kriging model to meet the requirements. And we analyze the uncertainty of sound propagation loss in the end.

**Model Substitution**

![Figure 1. FOR3D acoustic model.](image1.png)

![Figure 2. Munk sound velocity profile.](image2.png)
The FOR3D underwater acoustic model is a parabolic equation model which used to solve the three-dimensional wide-angle fluctuation equation to predict the acoustic propagation loss in the three-dimensional marine environment [20]. The actual modeling scene is shown in Figure 1. In the experiment, Tianhe II supercomputer [21] is used to speed up the calculation through the multi-threaded parallel program OPENMP. The calculation scale is $4000 \times 2000 \times 40$ (horizontal distance (m) $\times$ depth (m) $\times$ angle). When using 24 threads in parallel calculation, the final cost time is 42.357s.

In this study, the calculation results of FOR3D are used as the theoretical results. On this basis, a small amount of theoretical data is used to train this Kriging model, and the calculation of underwater acoustic propagation is carried out with high accuracy and efficiency. The input parameters and design space of the Kriging model is shown in Table I. In addition, we also add the propagation loss value of the corresponding point to the input data. In the model substitution, the Munk sound velocity profile is selected for calculation, which is a deep sea profile, reflecting the sound velocity with the depth of the change. The minimum speed of sound in the sound velocity profile exists at 1500 meters, that is, 1500 m/s as shown in Figure 2.

<table>
<thead>
<tr>
<th>Frequency/Hz</th>
<th>horizontal distance/m</th>
<th>Depth/m</th>
<th>Angle/°</th>
<th>Sound velocity/(m/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>25</td>
<td>400-3600</td>
<td>500-1400</td>
<td>0-15</td>
<td>Munk</td>
</tr>
<tr>
<td>25-125</td>
<td>1800-3600</td>
<td>500-1100</td>
<td>0-15</td>
<td>Munk</td>
</tr>
</tbody>
</table>

We artificially initialize the sample points uniformly but necessarily including the ends of the bounds due to a fact that Kriging method is good at interpolating rather than extrapolating. The single-frequency FOR3D model is replaced with the input values including the coordinate values of the boundary points, the center point of the calculation area and the corresponding sound propagation loss value. The calculated acoustic propagation loss is calculated and the results are compared. Kriging will generate the model estimation error MSE value in the actual calculation process. Herein the MSE $\psi_i$ actually can represent the difference between the sound-field results $P_i$ predicted by Kriging meta model and the counterpart results $W_i$ (as theoretical values) calculated by FOR3D algorithm:

$$\psi_i = Var(P_i - W_i)$$  \hspace{1cm} (8)

where the subscript $i$ denotes the number of training sample points which are taken into consideration. The estimation error will be gradually reduced by adding the maximum estimation error point to the training sample until the MSE value and the calculation error is small and stable. The calculation process is shown in Figure 3.

In the experiment, the actual running time of Kriging model is 0.842s, which is much shorter than the running time of the FOR3D model and achieves a 50x speed-up as show in Table II. The whole region error of this Kriging model is shown in Figure 4. The distribution of relative errors are most in the vicinity of 1% in Figure 5, which shows Kriging model is accurate enough. Figure 6 illustrates the evolution process of the estimated MSE error.
Kriging surrogate model can be further extended by considering the multi-frequency underwater acoustic propagation model. In view of the fact that the FOR3D model can only calculate the initial field of a fixed frequency at one time, the efficiency of the multi-frequency Kriging surrogate model is much better. In the experiment, five uniformly-spaced frequencies within the range of 25 Hz-125 Hz were selected. And the actual running time of the kriging model is 11.334s, achieving a 20x speed-up as show in TABLE II. First, the corresponding values of acoustic propagation was obtained by FOR3D model, and the multi-frequency Kriging model was used instead. It can be seen from the Figure 7 that most of the relative errors are concentrated within 1%, which shows a high accuracy. In order to visualize the results of the calculation, Figure 8 shows the calculated values of both Kriging model and FOR3D in the case of 50Hz frequency. Figure 9 and Figure 10 show that the error standard deviation of the Kriging surrogate model is gradually reduced and stabilize at a smaller value in the calculation process. This also shows that the Kriging model has achieved a high accuracy.
TABLE II. THE EFFICIENCY AND ACCURACY OF KRIGING MODEL.

<table>
<thead>
<tr>
<th>Model</th>
<th>Run-time</th>
<th>Speed-up ratio</th>
<th>STD max</th>
</tr>
</thead>
<tbody>
<tr>
<td>Kriging/single-freq FOR3D Parallel</td>
<td>0.842s/42.375s</td>
<td>5032.67%</td>
<td>6.055</td>
</tr>
<tr>
<td>Kriging/multi-freq FOR3D Parallel</td>
<td>11.334s/230.556s</td>
<td>2034.20%</td>
<td>4.933</td>
</tr>
</tbody>
</table>

Uncertainty of Transmission Loss

Assuming the probability density function of a random variable is known, multiple sets of random variables can be obtained by random sampling in Monte Carlo simulation. Each set of random variables is obtained through a statistical experiment. Then the random variables are input into the model calculation for a lot of results. In the end we conduct statistical analysis of all the results. The variables used in the experiment and their ranges are showed in TABLE III.

TABLE III. RANGE OF VARIABLES.

<table>
<thead>
<tr>
<th></th>
<th>Single-freq surrogate Model</th>
<th>Multi-freq surrogate Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>horizontal distance</td>
<td>1010m±10m</td>
<td>510m±10m</td>
</tr>
<tr>
<td>depth</td>
<td>1010m±10m</td>
<td>1010m±10m</td>
</tr>
<tr>
<td>angle</td>
<td>11°±1°</td>
<td>11°±1°</td>
</tr>
<tr>
<td>Sound velocity</td>
<td>1502m/s±2m/s</td>
<td>1502m/s±2m/s</td>
</tr>
<tr>
<td>frequency</td>
<td>25HZ</td>
<td>51HZ±1HZ; 101HZ±1HZ</td>
</tr>
</tbody>
</table>

Monte Carlo simulation is performed on Kriging models of single frequency and multi-frequency FOR3D models. Since there is no any priori information, it is assumed that the model variables are uniformly distributed and then randomly sampled. We take 2000 sets of variables to do the calculation and results are showed in Figure 11 and Figure 12.
From the above results, the uncertainty of acoustic propagation loss is estimated. Then we use the sample mean and the standard deviation to calculate the confidence limits to determine the confidence intervals [22]. The obtained 90% confidence interval and 95% confidence interval of the transmission loss are showed in TABLE IV:

<table>
<thead>
<tr>
<th></th>
<th>90% confidence interval</th>
<th>95% confidence interval</th>
</tr>
</thead>
<tbody>
<tr>
<td>Single-freq surrogate Model 25HZ</td>
<td>73.518db-75.704db</td>
<td>73.249db-75.915db</td>
</tr>
<tr>
<td>Multi-freq 51HZ±1HZ</td>
<td>139.746db-139.786db</td>
<td>139.742db-139.790db</td>
</tr>
<tr>
<td>Multi-freq 101HZ±1HZ</td>
<td>139.777db-139.850db</td>
<td>139.770db-139.857db</td>
</tr>
</tbody>
</table>

The fitting probability density curves are shown in Figure 13 and Figure 14:

![Figure 13. Confidence intervals of transmission loss based on single-freq surrogate model.](image1)

![Figure 14. Confidence intervals of transmission loss based on multi-freq surrogate model.](image2)

From the results, it is shown that the acoustic propagation loss is concentrated for different frequencies. The confidence interval is small, which plays a positive guiding role in the application of localization and inversion parameters.

**CONCLUSIONS**

In this study, a Kriging surrogate model is used to calculate the underwater acoustic transmission loss. The computational efficiency of the single frequency FOR3D model is increased by 50 times, and the computational efficiency of the multi-frequency FOR3D model is increased by 20 times. At the same time, the maximum mean square error is confined within 5%. The balance between the calculation efficiency and the calculation precision is achieved. Based on Kriging model, we realize the Monte Carlo
simulation to obtain the probability density curve and confidence intervals of acoustic propagation loss. The uncertainty of propagation loss is finally quantified.

ACKNOWLEDGEMENT

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