Error Analysis of Sound Source Localization System for Small Microphone Based on Time Delay Estimation

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Abstract. Based on time delay, Sound source localization method has a smaller amount of calculation compared with the sound source localization method of controllable beam synthesizer and high resolution spectral estimation, and can achieve a higher positioning accuracy after improved, so it is the most suitable method for real-time positioning, widely used in the field of passive acoustic localization. This paper tries to spread the factors that affect the positioning accuracy of time delay method, and provides the direction for improving the effect of time delay sound source localization.

Introduction

Sound source localization method has a long history. From the military to the people's livelihood, a considerable part of the field has an urgent need for sound source localization. Especially in military field, in the background of information warfare development, threats from the search radar electronic jamming, low altitude mutation, stealth technology and anti-radiation missile have become a national concern, the technology on passive sensor is therefore drawing attention. Nowadays, passive acoustic source localization is mainly used in the research of intelligent mine bombs, aircraft detection and battlefield environment investigation. It is worth mentioning that the Iraq war, American soldiers equipped with sound progenitor acoustic measurement system can definite the position by measuring the muzzle shock wave received and projectile shock waves generated by flight, the positioning accuracy is 1.2 degree on bearing, horizontal 3 degrees, which detect 90% shooting. On the battlefield, the accuracy of the positioning system once increases by one percentage point, and the casualty rate of soldiers will be greatly decreased. From this point of view, technology is life. How to reduce the error of the positioning system and improve the positioning accuracy is the key research object of the military scientific research personnel. This paper takes the small microphone positioning system as an example, analyzes the positioning error of passive sound source positioning, and provides reference for the researchers.

Microphone Array Model of Four Element Cross Array

Estimation on the received signal pickup is the key to time delay location technology. Take the four element cross array as an example.
Establish a Cartesian coordinate system as the figure shows, set the orthogonal four pickups, A, B, C, D, the distance of the four pickups to the center is D, the source as S, the Cartesian coordinates \((x, y, z)\), spherical coordinate \((r, \varphi, \theta)\). According to the transformation relationship between the rectangular coordinate and the spherical coordinate, there is

\[
\begin{align*}
  x &= r \sin \varphi \cos \theta \\
  y &= r \sin \varphi \sin \theta \\
  z &= r \cos \theta
\end{align*}
\]  

\(-90^\circ \leq \varphi \leq 90^\circ, \ 0^\circ \leq \theta \leq 360^\circ\)

The time difference between the arrival time of the sound element B, C, D is \(\tau_B, \tau_C, \tau_D\) respectively the distance from B, C, D to A is \(d_B, d_C, d_D\), then there is

\[
\begin{align*}
  d_B &= c \cdot \tau_B \\
  d_C &= c \cdot \tau_C \\
  d_D &= c \cdot \tau_D
\end{align*}
\]

\(C=3400\text{m/s}\). C is the sound speed that sound travels in the air. According to the principle that sound is transmitted by spherical wave, the element is on the sphere with S as center, we have:

\[
\begin{align*}
  (x - D)^2 + y^2 + z^2 &= r_A^2 \\
  x^2 + (y - D)^2 + z^2 &= (r_A + d_B)^2 \\
  (x + D)^2 + y^2 + z^2 &= (r_A + d_C)^2 \\
  x^2 + (y + D)^2 + z^2 &= (r_A + d_D)^2
\end{align*}
\]  

\(r_A\) is the distance between the sound source and the element A. From solution of formula (2):

\[
\begin{align*}
  r_A &= \frac{d_B^2 + d_C^2 - d_D^2}{2(d_C - d_B - d_D)} \\
  x &= \frac{2d_A r_A + d_C^2}{4D} \\
  y &= \frac{2(d_B - d_D) r_A + (d_C^2 - d_B^2)}{4D}
\end{align*}
\]

Considering the formulas, we have

\[
\begin{align*}
  \tan \varphi &= \frac{y}{x} = \frac{(d_B - d_D)}{d_C} \cdot \frac{2r_A + d_D + d_B}{2r_A + d_C} \\
  \sin \theta &= \sqrt{\frac{(x^2 + y^2)}{r^2}} \\
  &= \frac{c}{2D} \sqrt{(\tau_B - \tau_D)^2 + \tau_C^2}
\end{align*}
\]
\[ r = \sqrt{L_A^2 + d_c r_A + d_c^2 / 2 - D^2} \]
\[ = \sqrt{(r_A + d_c/2)^2 + \left(\frac{d_c}{2}\right)^2 - D^2} \]  
(6)

When \( r_A \not\parallel d_B, d_c, d_D, \) there is
\[
\frac{2r_A + d_B + d_D}{2r_A + d_c} \approx 1, \left(\frac{d_c}{2}\right)^2 - D^2 \approx 0
\]

To simplify the formula above:
\[
\tan \varphi \approx \frac{d_B - d_D}{d_c} = \frac{\tau_D - \tau_B}{\tau_c}
\]
\[
r \approx r_A + \frac{d_c}{2} \approx r_A
\]

Therefore the solution is
\[
\varphi \approx \arctan \frac{\tau_D - \tau_B}{\tau_c}
\]  
(7)

\[
\theta = \arcsin \left( C \sqrt{\frac{(\tau_B - \tau_D)^2 + \tau_c^2}{2D}} \right)
\]  
(8)

\[
r \approx \frac{C}{2} \cdot \frac{\tau_B^2 + \tau_D^2 - \tau_c^2}{\tau_c - \tau_B - \tau_D}
\]  
(9)

The estimation of acoustic source position by time delay estimation is as follows:
\[
\begin{align*}
\varphi &= \arctan \frac{\tau_D - \tau_B}{\tau_c} \\
\theta &= \arcsin \left( C \sqrt{\frac{(\tau_B - \tau_D)^2 + \tau_c^2}{2D}} \right) \\
r &= \frac{C}{2} \cdot \frac{\tau_B^2 + \tau_D^2 - \tau_c^2}{\tau_c - \tau_B - \tau_D}
\end{align*}
\]  
(10)

**Error Analysis**

**Azimuth Error Analysis**

Statistical error characteristics of time delay \( \tau_B, \tau_D, \tau_C \) are the same, the azimuth error caused by time delay is
\[
\sigma_\varphi = \sqrt{\left( \frac{\partial \varphi}{\partial \tau_B} \cdot \sigma_{\tau_B} \right)^2 + \left( \frac{\partial \varphi}{\partial \tau_C} \cdot \sigma_{\tau_C} \right)^2 + \left( \frac{\partial \varphi}{\partial \tau_D} \cdot \sigma_{\tau_D} \right)^2}
\]  
(11)

Because that
\[ \varphi \approx \arctan \frac{\tau_\theta - \tau_B}{\tau_c} \]  

Let’s take the derivatives:

\[ \frac{\partial \varphi}{\partial \tau_B} \approx \frac{1}{1 + \tan^2 \varphi} \left( -\frac{1}{\tau_c} \right) \]

\[ \frac{\partial \varphi}{\partial \tau_c} \approx \frac{1}{1 + \tan^2 \varphi} \left( -\frac{\tau_B - \tau_B}{\tau_c} \right) \]  

\[ \frac{\partial \varphi}{\partial \tau_B} \approx \frac{1}{1 + \tan^2 \varphi} \cdot \frac{1}{\tau_c} \]  

Plug(13)into (11)

\[ \sigma_\varphi \approx \frac{c \sqrt{1 + \cos^2 \varphi}}{2D \sin \theta} \cdot \sigma_\tau \]  

\[ \sin \theta = \frac{c}{2D} \sqrt{(\tau_B - \tau_B)^2 + \tau_c^2} \]  

Pitch Error

\[ \sigma_\theta = \sqrt{\left( \frac{\partial \theta}{\partial \tau_B} \cdot \sigma_\tau \right)^2 + \left( \frac{\partial \theta}{\partial \tau_c} \cdot \sigma_\tau \right)^2 + \left( \frac{\partial \theta}{\partial \tau_B} \cdot \sigma_\tau \right)^2} \]  

Because that

\[ \theta = \arcsin\left( \frac{c}{2D} \sqrt{(\tau_B - \tau_B)^2 + \tau_c^2} \right) \]  

Take partial respect to (17)

\[ \frac{\partial \theta}{\partial \tau_B} = \frac{c^2}{2D^2 \sin 2\theta} (\tau_B - \tau_B) \]

\[ \frac{\partial \theta}{\partial \tau_c} = \frac{c^2}{2D^2 \sin 2\theta} \tau_c \]  

\[ \frac{\partial \theta}{\partial \tau_B} = \frac{c^2}{2D^2 \sin 2\theta} (\tau_B - \tau_B) \]

Plug (18) into (16), we get:

\[ \sigma_\theta = \frac{c^2}{2D \sin 2\theta} \sqrt{2(\tau_B - \tau_B)^2 + \tau_c^2} \]

\[ = \frac{c}{2D \cos \theta} \sqrt{1 + 2 \tan^2 \varphi} \]  

Distance Error

Without considering the influence of effective sound velocity error and array system error, given the upper bound of root mean square error for the azimuth estimation, we can calculate the upper
bounds of root mean square error for time delay estimation. The delay estimation error of four element cross array should satisfy the following formula:

\[
\sigma_t \leq \Delta \varphi \cdot \frac{D \sin \theta}{c \sqrt{1 + \cos^2 \varphi}}
\]  

(20)

Array Mounting Errors

The installation error of the array will cause the great ranging error, because the lift damper can not be strictly arranged in a plane, three side. The four pickups with a little deviation, the positioning result will varies greatly. The research shows that the influence of [2] installation error of the measuring distance increases with the azimuth angle, distance and the influence on the near distance so large target azimuth angle and distance is unfavorable state of passive localization.

Error Caused by Media

Sound waves are mechanical waves, and mechanical vibrations propagate in the medium to produce mechanical waves. The vibration of the source body in the medium is called sound waves. In the medium, there is interaction force between the particles, and the sound wave travels in all directions with the aid of the medium. It is a spherical array wave. The vibration of the particle near the wave source drives the vibration of the particle far away from the source, therefore the vibration transmission from the near to the distant. The transmission is vibration form and energy, each particle vibrates only in their own equilibrium position, and will not move with the wave. The vibration period and frequency of each particle are consistent with the vibration period and frequency of the source. Vibration is the cause of the fluctuation, and the fluctuation is the result of vibration propagation. A single particle generates vibration and the vibration of a large number of particles generates vibration. The higher the temperature is, the faster the speed will be. At 15 degrees, the sound speed is 340m/s in the air and 346m/s at 25 degrees. The relationship between temperature and wave speed can be expressed by the following formula:

\[
C = 331.45 + 0.61T \text{ (m/s)}
\]

The propagation distance of acoustic wave is first concerned with the absorption of atmosphere, followed by temperature, humidity, air pressure and so on. The near field model takes spherical wave as the object of study, and the far field model takes plane wave as the object of study.

Noise

The process of pickup picking sound signal is inevitably affected by environmental noise. Environmental noise includes sound interference from otherbody, and irregular oscillation interference air particle itself, but also consider the reverberation sound field. Even if there is no vocal body, the air particles are always oscillating irregularly. The motion is influenced by many factors, and the most important factor is temperature. This is also the source of "white noise". There is no sound field without white noise in the real environment, so the influence of white noise cannot be ignored. Noise can be divided into two types, stationary noise and non-stationary noise. Stationary noise can be solved by combining single microphone and VAD based on statistics, and the noise superimposed on human voice will be more troublesome. For noise sources with reference signals, echo cancellation algorithms can be used to eliminate them.

References


