DOA Estimation of Noncircular Signals with Combined ESPRIT for Coprime Linear Array

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Abstract. Direction of arrival (DOA) estimation of noncircular signals for coprime linear array is investigated, and a combined estimation of signal parameters via rotational invariance technique (ESPRIT) based algorithm is proposed. The coprime linear array can be decomposed into two uniform linear arrays, so ESPRIT is firstly used to obtain ambiguous DOA estimations, then unique DOA estimation is achieved by finding the coincide results from the two decomposed subarrays based on the coprimeness. The proposed algorithm increase the degree of freedom, enhance the performance of angle estimation, and identifies more targets than conventional methods. Numerical simulations are conducted to demonstrate the improvement of the proposed algorithm.

Introduction

Direction of arrival (DOA) estimation problem has been extensively studied and used in the field of array signal processing. Most of existing investigations on DOA estimation are mainly exploiting the conventional arrays that the inter-elements spacing is limited to the typical half-wavelength.

Recently, the coprime linear array (CLA) has been proposed and the basic features of one-dimensional and two-dimensional coprime arrays have been thoroughly exploited [1-3]. The CLA makes it possible to sample the spatial signals in a sparse way, and consequently various useful properties can be obtained. The most distinguishing property is autocorrelation of signals can be estimated in a much denser spacing other than the physically sparse sampling spacing.

Actually, in communication systems, noncircular signals have been widely used, such as amplitude modulation (AM) and binary phase shift keying (BPSK) signals [4-5]. Noncircular signals have aroused attentions in the field of array signal processing and the noncircular feature has been widely used to enhance the DOA estimation performance.

In this paper, we combine the noncircular property and CLA to increase the DOF and enhance the DOA estimation performance. We investigate the problem of noncircular DOA estimation for CLA by the proposed "NC-ESPRIT". The proposed algorithm increase the degree of freedom, enhance the performance of angle estimation, achieves higher angle resolution and identifies more targets than conventional methods.

Notations: Lowercase (capital) bold symbols denote vector (matrix). (·)∗ , (·)T denote complex conjugation and transpose, while (·)H, (·)−1 denote conjugate-transpose, inverse matrix, respectively. diag{v} stands for a diagonal matrix whose diagonal is a vector v. ln(·), Re(·) denote the logarithm, and the real operator. E{·} presents the statistical expectation. det{·} stands for the determinant of matrix. I M stands for an M×M identity matrix and 0 M×N is a zero matrix with M×N. angle(·) means to get the phase.
Consider a CLA consists of two uniform linear subarrays with $M$ and $N$ sensors, respectively, where $M$ and $N$ are coprime integers. As the two subarrays share the first sensor at the zero-th position, the total number of sensors is $M + N - 1$. The subarray with $M$ sensors (subarray 1) has the inter-element spacing $\frac{\lambda}{N}$, while the subarray with $N$ sensors (subarray 2) has the inter-element spacing $\frac{\lambda}{M}$.

Figure 1 gives an example of a CLA.

Assume that there are $K$ far-field, uncorrelated narrow-band signals impinging on a CLA from different angles $\theta = [\theta_1, \theta_2, \ldots, \theta_K]$ as shown in Figure 1, where $\theta_k$ is the elevation angle of the $k$-th source, $1 \leq k \leq K$, and $K < \min \{M, N\}$. The noise is additive independent identically distributed Gaussian with zero mean and variance $\sigma^2$, independent of signals. The steering vector corresponding to the $k$-th source for subarray 1 and subarray 2 can be expressed by vector $a_1(\theta_k)$ and $a_2(\theta_k)$,

$$
a_1(\theta_k) = \begin{bmatrix} e^{j\sin\theta_1} & \cdots & e^{j\sin(M-1)\theta_1} \\
1 & \cdots & 1 \end{bmatrix}^T
$$

$$
a_2(\theta_k) = \begin{bmatrix} e^{j\sin\theta_1} & \cdots & e^{j\sin(N-1)\theta_1} \\
1 & \cdots & 1 \end{bmatrix}^T
$$

We just consider the signal of maximum noncircular rate in this paper, as for the definition of noncircular signals [5], the vector of strictly second-order noncircular signals can be expressed as

$$
\mathbf{s}(t) = \mathbf{\Psi}\mathbf{s}_0(t) \quad t = 1, \ldots, L
$$

where $\mathbf{s}_0(t) \in \mathbb{C}^{K \times 1}$, $\mathbf{\Psi} = \text{diag}\{e^{j\phi_1}, e^{j\phi_2}, \ldots, e^{j\phi_K}\}$ is a diagonal matrix with $\phi_k$ being the noncircular phase of the $k$-th signal, and $L$ is the number of snapshots. Then, the received signal vector of subarray 1 and subarray 2 can be defined as

$$
\mathbf{x}_1(t) = \mathbf{A}_1\mathbf{\Psi}\mathbf{s}_0(t) + \mathbf{n}(t) \quad t = 1, \ldots, L
$$

$$
\mathbf{x}_2(t) = \mathbf{A}_2\mathbf{\Psi}\mathbf{s}_0(t) + \mathbf{n}(t) \quad t = 1, \ldots, L
$$

where $\mathbf{A}_1 \in \mathbb{C}^{M \times K}$ and $\mathbf{A}_2 \in \mathbb{C}^{N \times K}$ are the steering matrixes of subarray 1 and subarray 2 defined as $\mathbf{A}_1 = [a_1(\theta_1), \ldots, a_1(\theta_K)]$ and $\mathbf{A}_2 = [a_2(\theta_1), \ldots, a_2(\theta_K)]$. $\mathbf{s}(t) \in \mathbb{C}^{K \times 1}$ is the narrow-band noncircular signal vector, $\mathbf{n}(t) \in \mathbb{C}^{K \times 1}$ denotes the additive white Gaussian noise and $\mathbb{E}[\mathbf{n}(t)\mathbf{n}^H(t)] = \sigma^2\mathbf{I}_M$.

**DOA Estimation Based on Combined ESPRIT**

**NC-ESPRIT Algorithm for DOA Estimation with Subarrays**

When the noncircular signals impinge on a subarray (subarray 1 e.g.) of CLA, we use the array output and its conjugation to extend the subarray output as
\[
y_i(t) = \begin{bmatrix} x_i(t) \\ J_i x_i(t) \end{bmatrix} = \begin{bmatrix} A_i \Psi \\ J_i A_i \Psi^* \end{bmatrix} s_0(t) + \begin{bmatrix} n(t) \\ J_i n(t) \end{bmatrix} = B_i s_0(t) + n_o(t)
\]

(6)

where \( J_i = \begin{bmatrix} 0 & & \cdots & 1 \\ \vdots & & \ddots & \vdots \\ 1 & & \cdots & 0 \end{bmatrix} \) \( M \times M \) and \( B_i = \begin{bmatrix} A_i \Psi \\ J_i A_i \Psi^* \end{bmatrix} \), denotes the extended direction matrix of subarray 1. Let

\[
B_i = \begin{bmatrix} b_i(\theta_1, \phi_1) & b_i(\theta_2, \phi_2) & \cdots & b_i(\theta_K, \phi_K) \end{bmatrix},
\]

where

\[
b_i(\theta, \phi) = \begin{bmatrix} a_i(\theta) e^{-j\mu_k} \\ a_i(\theta) e^{j\mu_k} \end{bmatrix}
\]

(7)

The covariance matrix of the extended data model can be expressed as

\[
R_i = B_i R_s B_i^H + \sigma_r^2 I_{2M}
\]

(8)

where \( R_s = \mathbb{E}[s(t)s^H(t)] \) is the source covariance matrix. Applying eigen-decomposition to the covariance matrices (8) yields

\[
R_i = U_{s1} A_{s1} U_{s1}^H + U_{s2} A_{s2} U_{s2}^H
\]

(9)

Since the extended direction matrix \( B_i \) and signal subspace span the same space, there exist nonsingular matrices \( T \), makes

\[
U_{s1} = B_i T
\]

(10)

Define \( J_x = \begin{bmatrix} 1 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & 1 \end{bmatrix} \) \( (M-1) \times M \) and \( J_y = \begin{bmatrix} 0 & \cdots & 0 \\ 0 & \cdots & 1 \end{bmatrix} \) \( (M-1) \times M \).

Define two selection matrix \( J_a = \begin{bmatrix} J_x & 0 \\ 0 & J_x \end{bmatrix} \) and \( J_b = \begin{bmatrix} J_y & 0 \\ 0 & J_y \end{bmatrix} \).

Partition the extended steering matrix \( B_i \) as

\[
B_i = \begin{bmatrix} B_{i1} \\ B_{i2} \end{bmatrix} \in \mathbb{C}^{2M \times K}
\]

(11)

Then \( J_a B_i \) means to get the front \( M-1 \) rows of \( B_{i1} \) and \( B_{i2} \), \( J_b B_i \) means to get the last \( M-1 \) rows of \( B_{i1} \) and \( B_{i2} \).

From [5-6], the direction matrix satisfies rotational invariance

\[
J_a B_i \Phi_0 = J_b B_i
\]

(12)

where \( \Phi_0 = \begin{bmatrix} e^{-j\mu_1} & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & e^{-j\mu_k} \end{bmatrix} \) is a diagonal matrix which contain DOA information with \( \mu_k = \pi N \sin \theta_k \).

According to (10) and (12), we obtain

\[
J_a U_{s1} T^{-1} \Phi_0 T = J_b U_{s1}
\]

(13)

define

\[
\Gamma_i = T^{-1} \Phi_0 T
\]

(14)

According to (13)
\[
\Gamma_1 = (J_s U_{st})^H (J_s U_{st}) \quad (15)
\]
and \((J_s U_{st})^+ = \left((J_s U_{st})^H (J_s U_{st})\right)^{-1} (J_s U_{st})^H\), \(\Gamma_1\) has the same eigenvalues with \(\Phi_1\). Perform the EVD of \(\Gamma_1\), which can be written as

\[
\Gamma_1 = U \Lambda U^H 
\]

where \(U = [u_1, \ldots, u_K]\), \(\Lambda = \text{diag}\{\mu_1, \ldots, \mu_K\}\). It is worth noting that the eigenvalues of \(\Gamma_1\) in \(\Lambda\) are corresponding to the diagonal elements of \(\Phi_1\), and the eigenvectors of \(\Gamma_1\) in \(U\) are estimates of the column vectors in matrix \(B_i\).

From (1), the elevation angle of \(k\)-th source can be estimated by

\[
\hat{\theta}_k = \arcsin\left(-\frac{\text{angle}(\mu_k)}{N} - \frac{2k}{N}\right) 
\]

where \(\hat{\theta}_k\) is the estimate of \(\theta_k\), \(k = 0, 1, \ldots, N - 1\), so there are totally \(N\) ambiguous angles of \(k\)-th source on subarray 1.

Similarly, we can obtain \(M\) ambiguous angles of \(k\)-th source on subarray 2 using the same method.

In practice, the covariance matrix \(R_i\) is usually obtained by

\[
\hat{R}_i = \frac{1}{L} \sum_{l=1}^{L} y_i(t) y_i^H(t) \quad (18)
\]

where \(L\) is the snapshot number, therefore, the correct estimations will not be strictly overlapped, and the DOA is estimated by averaging two closest solutions,

\[
\hat{\theta}_k = \frac{\hat{\theta}_{ik} + \hat{\theta}_{2k}}{2}, k = 1, \ldots, K 
\]

where \(\hat{\theta}_{ik}\) and \(\hat{\theta}_{2k}\) denote the corresponding angles of the two closest solutions of the two decomposed subarrays.

Till now, we have presented the combined NC-ESPRIT based DOA estimation algorithm of noncircular signals for coprime linear array. The major steps can be summarized as follows:

1. Construct the extended sample array output matrix \(y\) according to (6).
2. Compute \(\hat{R}_i\) according to (18) and perform the EVD of \(\hat{R}_i\).
3. Construct \(J_s U_{st}\) and \(J_s U_{st}\), compute \(\Gamma_{st}\) according to (15).
4. Perform EVD of \(\Gamma_{st}\) and estimate the ambiguous elevation angles via (17).
5. Select the \(K\) nearest ambiguous angles as the estimate DOAs.

Simulation Results

This section uses Monte Carlo simulations to assess the DOA estimation performance of the proposed method.
Using Noncircular Features to improve DOA Estimation Performance

In Figure 2, we compared the DOA estimation performance of circular and noncircular sources for CLA. Assume that there are $K=2$ noncircular signals impinging on the CLA with elevation angles being $\theta=(10^\circ,30^\circ)$ and the noncircular phases being $\varphi=(5^\circ,15^\circ)$ with $M=5$, $N=7$, $L=300$. From Figure 2 we can see that by introducing noncircular property, the DOA estimation performance of noncircular sources is much better than that of circular sources.

Comparison RMSE Performance Versus Different Array Geometry

In this simulation, we assume that there are $K=2$ noncircular signals impinging on a ULA and a CLA. For fair comparison, the ULA has $M + N - 1$ sensors. Figure 3 indicates the RMSE performance of the NC-ESPRIT for ULA and NC-ESPRIT for CLA. The CRB is plotted as a benchmark. It is illustrated in Figure 3 that the proposed methods has better DOA estimation performance on CLA than on ULA.

Comparison of DOA Estimation RMSE Versus Different Parameters

In Figure 4 we show the variation tendency of DOA estimation performance with a changing element’s number of subarray 1 while the element’s number of subarray 2 is fixed. From Figure 4, we can see that the DOA estimation performance of RI-PM increased with the increase element’s number of subarray 1 in a same SNR condition.
Conclusions

In this paper, we proposed the NC-ESPRIT method for the DOA estimation of noncircular signals. Compared with the conventional ESPRIT method for circular signal, the proposed method has better estimation performance, increase the degree of freedom, and identifies more targets than conventional methods by exploiting the noncircular property. Numerical simulation results verify the effectiveness and improvement of the proposed method.

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