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Key words: Value-at-Risk, Historical simulation, Genetic Algorithm, Portfolio optimization.

Abstract. Value at Risk is an important measure of market risk. It gives the maximum amount of loss in a portfolio of assets. Conditional Value-at-Risk covers the amount of losses exceeding Value-at-Risk and does not suffer from the functional problems (Non-Convexity and non-additivity) of VaR. See Rockafellar and Uryasev (2000)) of VaR. Here VaR is calculated using Historical Simulation and portfolio optimization is done with the help of Genetic Algorithm. This methodology is applied to energy stocks of different countries. Optimization is done to maximize returns and minimize risk separately as well as for combined multi-objective optimization.

Introduction

Value at Risk is a widely used standard risk measure to measure the extent to which portfolio of financial assets is subject to inherent risks. It measures the maximum losses would be in a portfolio. VaR has been used as a regulatory measure like The Basle Committee (Committee set up by the Bank of International Settlements and based in Basle) which has sanctioned institutions to model internal VaR in their capital requirements. Historical Simulation: It reorganizes historical returns in an increasing performance order and assume the future behavior of the markets will follow the same pattern. CVaR is the amount of losses exceeding VaR.

Literature Review

Pirvu (2007) in “Portfolio optimization under the Value-at-Risk Constraint” finds the optimal portfolio model for a given VaR constraint in an incomplete market and containing risky as well as non-risky assets and allowing for intertemporal consumption. Wozabal (2010) in “Value-at-Risk constrained optimization using the Difference of Convex Algorithm” maximizes the expected returns under a risk constraint of VaR. It solves a Value at Risk constrained Markowitz style portfolio selection problem when the return distributions are in many scenario sets. Hull and White (1998) in “Value at Risk when daily changes in market variables are not normally distributed”.

Objectives and Methodology

The objectives of this research are:
1. Historical VaR computation.
   1. Maximize returns
   2. Minimize Risk

Historical VaR calculations are done under the assumption of normal distribution of returns and hoping that future returns will follow the same distribution. The Value at Risk at a level \( \alpha \in (0, 1] \) of a risk \( X \) is given by \( \text{VaR}_\alpha X = -q^+ (1 - \alpha) \) where \( q^+ (1 - \alpha) \) is the upper \((1 - \alpha)\)-quantile of \( X \). For the historical simulation method of calculating VaR, the formula changes to:

Objective 1:
\[
VaR = z(y)\sigma - \mu
\]
Where z(\(\gamma\)) is the z-score for .05 level of significance or 95% confidence based on the standard normal distribution. For the assumption of normal distribution

The expected return of the individual assets \(i\) is presented as a polynomial of first degree:

\[ E(w_i) = w_i \cdot r_i \]

Where \(w_i\) denotes the weight of the individual asset \(i\).

\(r_i\) denotes the expected return of asset \(i\).

Thus the total expected return of portfolio \(P\) can be written as: \(F = \sum_{i=1}^{n} E(w_i)\) and the objective function of the portfolio return to be maximized can be written as follows:

Objective 2.1:

\[ \max \{F = \sum_{i=1}^{n} E(w_i)\} \]

where \(n\) is the number of assets.

Risk can be modelled with weight allocation as:

\[ \text{VaR}(w_i) = z(\(\gamma\))\sigma(E(w_i)) - E(w_i) \]

Total VaR of the portfolio can be written as \(G = \text{VaR}(w_i)\)

Objective 2.2:

\[ \min \{G = \text{VaR}(w_i)\} \]

And the multi objective function to optimize is illustrated as:

Objective 3:

\[ H(w_i) = F(w_i) - \text{VaR}(w_i) \]

Under the following constraints:

\[ \sum_{i}^{n} w_i = 1 \quad \text{and} \quad \sum_{i=1}^{n} r_i w_i \geq 0 \quad \text{and} \quad 0 \leq w_i \leq 1 \]

These constraints are for summation of weights to unity, positive returns and the condition of no short sales respectively.

In a GA application, evaluation is performed by means of the fitness function which depends on the specific problem and the optimization objective of the GA (Petridis et al, 1998). The aim is to select weights of the portfolio invested in each asset in order to maximize the portfolio return and minimize the portfolio risk. The crossover procedure in this regard, plays the role of exchanging weights of the securities of two chosen parents in such a manner that the offspring produced by the crossover represents (Lin and Gen, 2007).

The selection of the best operators for crossover, mutation and selection can be done on the basis of whichever set of operators has the least value on the fitness (objective) function. This, hence, gives us the optimal set of portfolio weights. Since working with mutation and selection operator is tricky, we change crossover operators and compare the results.

Results

0.5% VaR for weekly historical returns is used which can be extrapolated to other time periods through the multi period formula (Kleindorfer and Li 2005). Major Energy companies from India, China and UK are selected for portfolio construction. The data is taken for weekly returns of 7 stocks of Sinopec Corp. (SHA), Petro China (PTR), British Gas (BG), Scottish and Southern Energy plc. (SSE), Reliance (RELIANCE), Bharat Petroleum Corporation Limited (BPCL) and National Thermal Power Corporation (NTPC). The historical data is taken weekly from April 2005 to March 2010. Weekly stock returns are taken averaged over the 5 year period. The time period is selected based on the occurrence of Global Recession and feasibility of data. Data is taken from yahoo finance and the websites of stock exchanges and individual stock companies.

These securities are selected on the basis of the most influential energy sector companies in the respective countries. Cross country disparities in risk structure is neglected. Weekly Returns are compared in order to bring stock values of different currencies to a common denomination.

MATLAB™ was used for running the optimization algorithms for both the single objective functions of reward maximization and risk minimization as always the multi objective portfolio optimization through Genetic Algorithm. Optimtool app of MATLAB™ was used for genetic algorithm. The function structure, options and results are shown in respective sections.
This algorithm was run on a machine with 3rd Generation Intel(R) Core(TM) i5-3337U 2.7 GHz processor with 4 GB RAM.

The following graphs show the market fluctuations of the stocks of all the energy companies of China, UK and India taken into consideration to give a simple analysis of raw input data. The sudden fluctuation in 2008 is due to Global Recession. This was one of the main reasons of selecting the time period from 2005-2010.

**Value at Risk**

**Expected Returns of Individual Assets for 5% Historical VaR.** These historical VaR values are calculated using MATLAB™. The historical simulation used here assumes normal distribution of expected returns. This table gives the feasible points of portfolio returns vs risk for individual assets and can be plotted to show the proximity with the efficient portfolio frontier found through various available techniques of portfolio optimization. Comparisons with different time period VaR and Expected Returns can be done on the basis of guidelines provided in Kleindorfer and Li (2005).

**Maximizing Returns** The maximizing of returns function is done for three different crossover operators of single point, two-point and arithmetic to compare and find best results. Roulette wheel selection is used along with elitist strategy to stop the function from being trapped near a local optimum value. Population is taken to be 100. The function tolerance is set to 1e-10. Other option values are set to default. The constraints and function inputs are done as shown in the following graphic:

**Best Fitness and Best Individual Graphs for Single Point Crossover.** The optimal portfolio values for the single point crossover method are [0.36114 0.08233 0.020660.00477 0.524710.001730.00565] for the seven weights in order (up to 5 decimal places). The objective function value is 0.0067 or 0.67% returns after 73 generations.

**Two-Point Crossover**

**Best Fitness and Best Individual Graphs for Two-point Crossover.** The optimal portfolio values for the two-point crossover method are [0.02469 0.07056 0.00806 0.00177 0.880550.009280.00606] for the seven weights in order (up to 5 decimal places). The objective function value is 0.0069 or 0.69% returns after 100 generations.

**Best fitness and Best Individual Graphs for Arithmetic crossover.** The optimal portfolio values for the arithmetic crossover method are [0.10291 0.12711 0.13807 0.175560.107510.179060.17067] for the seven weights in order (up to 5 decimal places). The objective function value is 4.4225e-04 or 0.044225% VaR after 51 generations.

**Minimizing Risk.** The minimizing of risk function is done for three different crossover operators of single point, double point and arithmetic to compare and find best results. Roulette wheel selection is used along with elitist strategy to stop the function from being trapped near a local optimum value. Population is taken to be 100. The function tolerance is set to 1e-10. Other option values are set to default.

**Best fitness and Best Individual graphs for single point crossover.** The optimal portfolio values for the arithmetic crossover method are [0.10920 0.10593 0.12607 0.233390.09767] for the seven weights in order (up to 5 decimal places). The objective function value is 5.8376e-04 or 0.053876% VaR after 51 generations.

**Two-point Crossover**

**Individual Graphs for Two-point Crossover.** The optimal portfolio values for the two-point crossover method are [0.098360.10349 0.14201 0.25614 0.094440.167590.13888] for the seven weights in order (up to 5 decimal places). The objective function value is 5.3876e-04 or 0.053876% VaR after 61 generations.

**Best Fitness and Best Individual Graphs for Arithmetic Crossover.** The optimal portfolio values for the arithmetic crossover method are [0.10920 0.10593 0.12607 0.233390.09767] for the seven weights in order (up to 5 decimal places). The objective function value is 4.4225e-04 or 0.044225% VaR after 51 generations.
0.179640, 14806] for the seven weights in order (up to 5 decimal places). The objective function value is 5.5241e-04 or 0.055241% VaR after 68 generations.

**Multi Objective Optimization.** The optimization of returns function is done for only one point crossover. Roulette wheel selection is used along with elitist strategy to stop the function from being trapped near a local optimum value. Population is taken to be 100. The function tolerance is set to 1e-10. Other option values are set to default. The negative values on the returns axis is due to the minimizing nature of the Optimtool of MATLAB™. The graph has been rotated on a horizontal axis. We will neglect the points which go into the negative risk range. This happens due to certain outliers and function trapping problems, possibly due to the complex nature of VaR function, which are difficult to fix and out of this paper’s scope.

**Conclusion**

This paper presented the results of application of genetic algorithm to single objectives of expected return maximization and of Value at Risk minimization. GA was also used for Multiobjective portfolio optimization where the return-risk tradeoff values were found. Value at Risk being commonly used and being better than many other risk metrics like standard deviation is used in this paper. Other risk methods like CVaR are better than VaR as it is a coherent risks measure. Portfolios are constructed with the help of equities of major energy companies. This paper eventually provides the optimal portfolio weights to be distributed among these stocks.

A comparison of crossover operators for maximizing returns and minimizing risk showed that the arithmetic crossover is better than single point and two-point as it gives the lowest objective function value for minimizing risk and function value greater than single point and equal to two-point but has better weight distribution and takes less computation time. The roulette selection operator along with the elitist strategy gave better results as it prevents the function from getting trapped to a local extremum to a large extent. The default MATLAB™ values for the remaining operators gave acceptable results. The efficient frontier constructed by the solutions of the multiobjective function optimization gave a good approximation of an ideal efficient frontier and portfolio selection problem for an investor is effectively solved with the selection of return-risk tradeoffs closer to the constructed efficient frontier.

**References**


