High-Dimensional Data Analytics for Sparse Recovery of Guided-Waves Dispersion Curves Using B-Splines

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ABSTRACT

This research presents a technique to recover the dispersion curves of guided-waves by utilizing the inherent sparsity of these signals in the frequency-wavenumber domain. The proposed methodology is a data-driven approach that combines physics-based knowledge with high-dimensional analysis to obtain the dispersion curves of the medium from experimental signals. Initially, a sparse two-dimensional dispersion matrix is constructed using sparse wavenumber analysis. Then, B-splines are fitted to non-zero elements of this matrix to establish an initial estimate of the dispersion curve parameters. These parameters are further optimized using the quasi-Newton algorithm to improve the accuracy of signal prediction. The results demonstrate that this method can significantly reduce the dimensions of Lamb waves signals to approximately 0.05%, which avoids overfitting. The retrieved signals in the frequency-distance domain exhibit a correlation of approximately 50% with the original signals. In comparison with sparse wavenumber analysis, this technique requires two orders of magnitude fewer parameters to represent the medium's dispersion curves.

INTRODUCTION

Dispersion curves are the building blocks for guided-ultrasonic-based structural health monitoring [1-2]. Recently, there has been a growing interest in physics-based machine learning methods for finding dispersion curves based on experimental wave propagation data. In this area, Harley and Moura proposed Sparse Wavenumber Analysis (SWA) which is a data-driven method to recover dispersion modes [3]. This method considers the waveform equations and eliminates the effect of geometric attenuation. It leads to converting signals from the frequency-distance domain into the frequency-wavenumber domain. A sparse complex matrix represented data in this

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domain in which the position of non-zero elements shows wavenumbers of different modes to corresponding frequencies. The recovered sparse matrix in SWA can show the dispersion curves of the medium based on the observed experimental results. Harley and Moura showed that this sparse matrix contains enough information to synthetize guided-wave signals. Signals synthesized in this approach can be used as a basis for damage localization [4].

The sparsity notation can be extended not only through wavenumber but also could be considered for the frequency domain. In other words, instead of using sparse matrices, the dispersive modes characteristics of guided waves are modeled as sparse dispersion curves. To this aim, first, SWA is applied to the experiment signals, and a sparse matrix is derived. B-splines are fitted to the index and values of non-zero elements of this matrix to have an initial guess. Then, using an optimization scheme, an optimal combination of coefficients is obtained to represent the dispersive mode characteristics of guided waves in the form of B-splines.

METHODOLOGY

Advancement of technology in the field of Structural Health Monitoring (SHM) and Non-Destructive Evaluation (NDE) has provided tools to accurately monitor the behavior of elements and structures. Numerous high-resolution equipment is employed in the SHM/NDE field. An example in the field of guided-wave NDE is transducers which observe waves with a high-frequency sampling rate in the order of 10 MHz. Furthermore, numerous inexpensive sensors are used to capture the behavior of guided waves through different locations on the medium under study. This level of access to a massive amount of data can be inferred as a blessing that enables consideration of all potential aspects of a phenomenon. However, on the other side, it creates high-dimensional data, resulting in analysis challenges and computational costs [5].

High Dimensionality

High-dimensional data is a type of data in which the number of dimensions (characteristics, features) is high. This contrasts with the classic statistical setting in which most statistical methodologies have been defined. High-dimensionality can potentially result in analytical and computational challenges [6]. In this setting, a data-driven model cannot be developed using classic statistical methods, or even employing those methods can lead to wrong interpretation [7]. Furthermore, it is a possible scenario that the number of features of a data is larger than the number of accessible data items (observations). A case of high-dimensional data in the field of SHM/NDE is a signal. Any waveform can be considered as one data item that has numerous dimensions. A common strategy to overcome the challenge of high-dimensionality is utilizing pre-knowledge about the intrinsic of data. Data sparsity is an example of this pre-knowledge.

Sparsity

A key solution to address the challenge of high-dimensionality is taking advantage of the sparsity notion. Data sparsity means there is a specific space that can show vibration with a combination of a few modes. In the area of SHM/NDE, a tangible example is the mode shapes, showing that we can represent the vibration signals with a combination of a few modes. This sparse space also makes it feasible to implement a model with a lower number of coefficients. In this sparse space, it is possible to fully observe a data specimen with a few sample points. This is the key notion behind a well-known method called Compressive Sampling [7], which is widely used in SHM/NDE [8]. Using sparsity is a solution to address challenges where the number of dimensions is larger than the number of observed data items. It is a common case in SHM/NDE since high-resolution techniques are used, but just a handful of unique observation data like a damaged situation are available. However, this blessing requires a conversion from the initial space to a sparse space.

In this context, it is crucial to understand how to find the sparse domain that corresponds to the specific data. In this regard, the physics behind the phenomena may provide insight into determining the sparse domain. In this research, the sparse space for guided-waves is the frequency-wavenumber domain. The details of this sparse space are explained in the next section.

$$\alpha = arg\min_{\alpha} \parallel \boldsymbol{\Phi}\alpha\Psi - x \parallel_2^2 + \tau \parallel \alpha \parallel_1 \quad (1)$$

Waveform Equations

Elastic wave propagation in semi-finite medium-like plates is formalized with displacement in radius and perpendicular directions. These displacements are the summation of corresponding guided wave modes. With the assumption of contributing m modes, displacements are:

$$U_r(r,z,\omega) = \sum_m B_m(z,\omega) H_1^{(1)}\left(k_m(\omega) r\right) \quad (2)$$

$$U_z(r,z,\omega) = \sum_m C_m(z,\omega) H_0^{(1)}\left(k_m(\omega) r\right) \quad (3)$$

Where *r* and *z* are the coordinates in the cylindrical system from the origin of wave emission. $k_m(\omega)$ is the wavenumber of mode m for corresponded frequency, ω. $H_0^{(1)}$ and $H_1^{(1)}$ are zeroth and first order Hankel function of the first kind [9]. By the assumption of far-field condition, the observed Lamb waves at receiver sensors are a linear combination of waves in *r* and *z* directions. Then, Lamb waves' displacement is approximately represented as:

$$U(r,\omega)=\sum_m T_m(\omega)\sqrt{\frac{1}{k_m(\omega)r}}e^{jk_m(\omega)r}$$
$$T_m(\omega)=\mathrm{Re}_m(\omega)+j\,\mathrm{Im}_m(\omega) \tag{4}$$

where T as a complex function of frequency compensates all effects such as coefficient of linear combination and effects of transmitting and receiving transducers as the same sensors are used for any observation.

Sparse Wavenumber Analysis

Sparse Wavenumber Analysis (SWA) considers the sparsity in the frequency-wavenumber domain in terms of m modes for each frequency. In this regard, SWA considers all signals with a specific frequency. It determines the best sparse vector in wavenumber. This procedure repeats for all frequencies in the interval of interest and results in a sparse matrix.

$$\boldsymbol{v}(\omega)=\underset{\boldsymbol{v}(\omega)}{arg\,min}\parallel \boldsymbol{\Phi}\boldsymbol{v}(\omega)-U(r,\omega)\parallel_2^2+\tau\parallel \boldsymbol{v}(\omega)\parallel_1 \tag{5}$$

where $\mathbf{v}(\omega)$ is the sparse vector and $\mathbf{\Phi}$ is the conversion matrix that transforms signals from the frequency-wavenumber to the frequency-distance domain. τ is the regularization factor set a balance how the model fitted based on the maximum likelihood or number of coefficients. It is worth mentioning that, in the data-driven scheme, methods are not distinguished between different types of modes like symmetric or asymmetric ones.

Sparse Dispersive Mode Analysis

In the data-driven scheme, functions k and T are estimated with B-splines [10]. Then, to obtain an acceptable representation of sparse modes, the problem converts to optimization with the combination of maximum likelihood and sparsity of B-spline coefficients in the shape of a matrix.

$$\theta=\underset{\theta}{arg\,min}\parallel \widehat{U}(r,w;\theta)-U(r,\omega)\parallel_2^2+\tau\parallel\theta\parallel_1 \tag{6}$$

where θ contains all the coefficients for Splines to estimate functions k and T (see Equation 4). Consequently, the displacement is calculated.

EXPERIMENTAL RESULTS

In this paper, the results of a guided-wave experiment on an aluminum plate [4], which is publicly available, are used. In this experiment, 17 PZT transducers were mounted randomly on the top surface of an aluminum plate with a size of 122×122×0.28 cm. First, a 10 µs chirp signal from 0 to 2 MHz is transmitted from one of the PZTs to serve as transducers, and the remaining PZTs served as receivers to observe Lamb waves. This procedure repeats for all PZTs and results in 272 different signals across 136 unique distances. Figure 1 shows the location of transmitter/receiver sensors.

This experiment resulted in data including 272 signals observed for 100 milliseconds with 10,000 sampling points for each signal (100 kHz sampling frequency over time). These signals are preprocessed to remove early and late arrival to remove noises and multipath interferences, respectively. Then, signals are converted to the frequency-distance domain. Figure 2 shows three observed signals at different distances in time and frequency domains.

In this research, signals are limited to frequencies from 150 to 750 kHz. To implement the proposed method, first, data is converted to the frequency-wave number method using SWA. Three dispersive modes are assumed to be contributed. Considering the maximum possible wavenumber of 2000 m^{-1} results in a sparse matrix with a size of 600×2000. SWA determines the non-zero elements of this sparse matrix. Note that the extracted information contains the value of wavenumbers as the index of non-zero elements and their complex values. Figure 3(a) shows this sparse matrix. As shown in this figure, the overall shape of dispersive modes is visible. However, these modes are not perfectly separated. To separate points from different modes, RANSAC [11] is employed to fit polynomials of order two to extract points related to each mode. An interval of ±100 kHz is set to cover points top and bottom of fitted lines (see Figure 3(b)). This procedure is repeated three times to extract points for three modes.

These indices and their corresponding values are employed to fit B-splines as initial estimations for function k and T. B-splines are in the order of two, and 13 knots are considered, which are distributed uniformly in the interval of frequencies of interest. This results in ten coefficients for each function that will be used as an initial guess for SDMA.

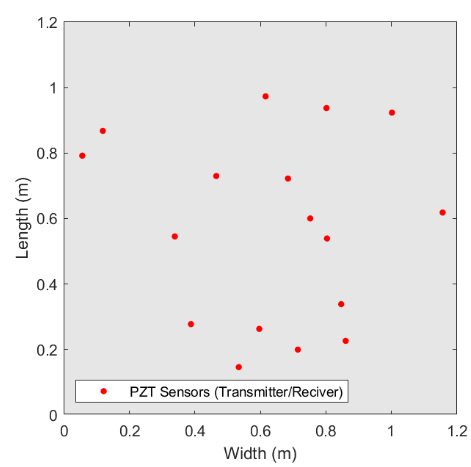
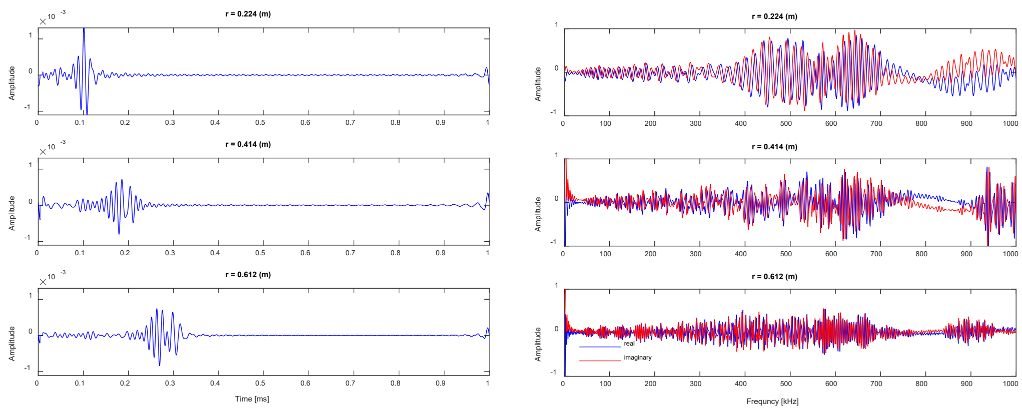


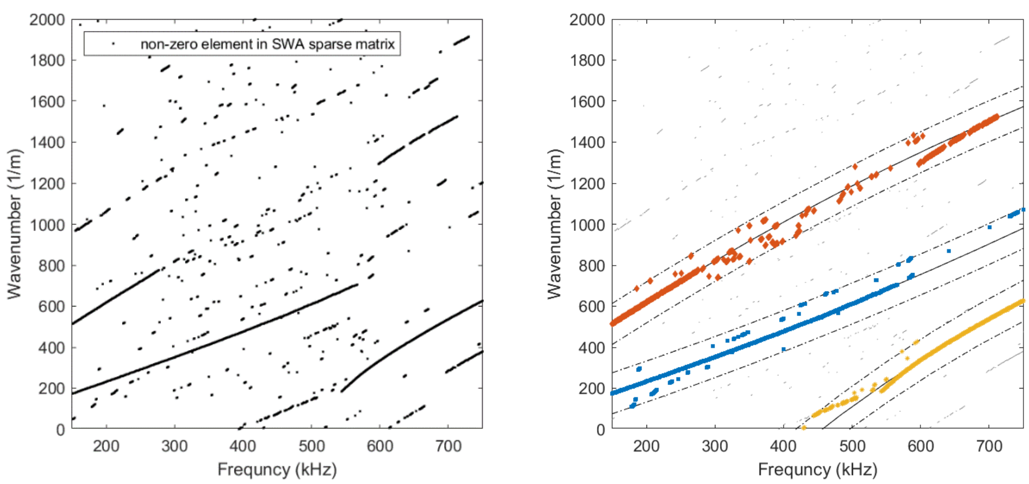
Figure 1. Arrangement of transmitter/receiver sensors on the surface of the aluminum plate under study.

SDMA is employed to find the best coefficients of B-splines with the maximum likelihood. The objective function here is the correlation between predicted and original signals in the frequency-distance domain. To this aim, a quasi-Newton optimization algorithm is used. The correlation value for initial estimation is 11% increased to approximately 48%. Figure 4 shows how B-spline functions are changed from the initial estimation and optimized one. Figure 5 compare observed and predicted signals at three different distances in time domain.



(a) (b)

Figure 2. Observed signals at three different distances; (a) time domain and (b) frequency domain.



(a) (b)

Figure 3. (a) sparse matrix resulted from SWA; considering the behavior of medium in the interval of 150 to 750 kHz and limited to wavenumbers less than 2000 m^{-1}; (b) separated three modes from elements of SWA matrix

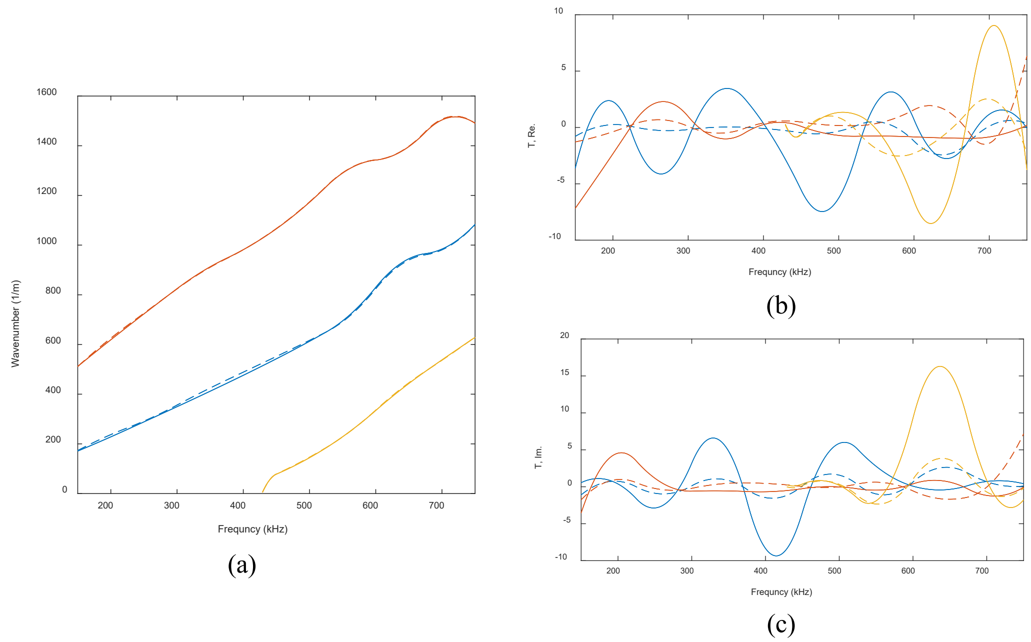


Figure 4. Comparison of initial and optimized B-splines for (a) $k(\omega)$, (b) real, and (c) imaginary parts of $T(\omega)$

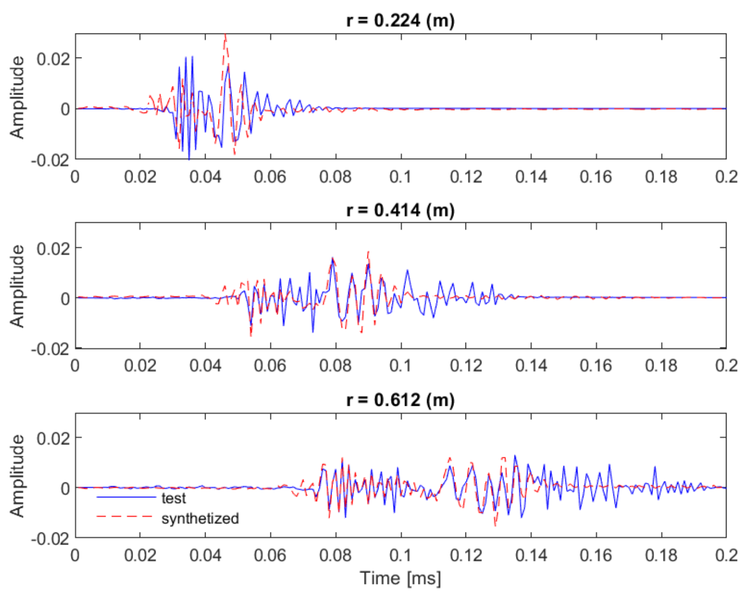


Figure 5. Observed and predicted signals in the time domain at three different distances

CONCLUSION

In this paper, the SDMA method is proposed to convert Guided-wave signals data to a lower space using the knowledge of the sparsity of Guided-wave in the frequency-wavenumber domain. This conversion gives an estimation of the dispersion modes' behavior of the medium. These dispersion modes can be used to synthetize signals in the frequency- or time-distance domain. To measure how this conversion preserves the information in the initial domain, the correlation of observed and predicted signals is calculated. Results show that the proposed method is accurate in the same order as SWA but with a fewer number of dimensions.

In this paper, signals are limited to frequencies from 150 to 750 kHz. Then, raw data has 163,200 (=272×(750-150)) features or dimensions. The SWA method converts this data to a sparse complex matrix. Considering three modes in this interval, each frequency needs three variables: one for location on non-zero elements in the matrix and two variables for real and imaginary parts of that element resulting in 5,400 dimensions. Implementing the proposed method for the same scenario, considering ten coefficients for B-splines results in data with 90 dimensions. This is due to the fact the SWA considers the sparsity in only the wavenumber direction, which is able to reduce the dimensionality by two orders of magnitude (3.33%), and our proposed method considers the sparsity in both directions results in dimension reduction by four orders of magnitude (0.05%).

Two potential factors could be assumed for this deficiency of this model. First, the waveform equations are not accurate enough for the current setup, i.e., it is not sufficient to cover all aspects of the phenomena under study. Second, there is more capacity in the data-driven part to improve the model. For potential future work, other types of guided waves can be used to employ this data-driven approach. Furthermore, obtained characteristics of dispersive modes can be employed as a basis for damage detection purposes.

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