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The Dynamic Optimization Model of Individual Income Tax

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Abstract. Individual income tax is the General term of legal norms which adjust the social relations that happened in the collection or management personal income tax between the tax authorities and the natural person. A dynamic optimization model was proposed to analysis the personal income tax from macro view, which took the personal income tax as model parameter, optimized the investment and consumption, and aimed at increasing the effectiveness of consumption. The solution of the model suggested that high earners should pay more taxes in order to increase their happiness index, while low-income individuals should pay less, even pay no tax.

Introduction

Individual income tax is a process which is used by government to adjust the natural person's income of different level, and to redistribute the part of the income. It was listed early in the 'Guideline of Tax Administration' enacted by China in 1950, and then, foreigners were required taxes by the first legal rules of 'Individual Income Tax Law of the People's Republic of China' in 1980.Since then, the Individual Income Tax Law was revised several times, and the tax exemption of individual income tax was determined as 1,600 yuan in 2006, and then was increased in 2007, 2011 and 2018, and now it was 5000 yuan[1]. Furthermore, some innovations were made the levy individual income tax more scientific.

The obvious difference among the several adjustments is that the tax amount of those with lower incomes became much less than those with higher incomes. Why do high-earning individuals have to pay more taxes than low-earning ones? After searching on many rules, documents and statement announced by government, the reason is that high-earning individuals have higher income, so they have a duty to pay more taxes so that government could more easily adjust the income level. By these reasons, one could find it was hardly to persuade the high-earning individuals, because they think it was the more efforts and hard-working which have given their high income, while some low-earning ones just sat idle and enjoyed the fruits of others' work. This viewpoint rooted in someone's brain has resulted in more and more debates on network.

The researching papers also did not give a satisfying answer. There are three types of paper about individual income tax, the first types focus on the debates of which items should include in or exclude from the tax[2,3], and the second mainly analysis the method of collect tax and the levied objects[4,5], the last one study the range and rate of tax[6,7,8]. No paper gives the real reason, especially through economic or mathematical view. By studying the macro-economy and dynamic

optimization method, a dynamic optimization model about individual income tax was proposed. The model aimed to increase individuals' happiness index by optimizing one's investment and consumption path. Solving the model by mathematical method, the solution indicated that, in order to increase one's happiness, high-earning individuals should pay more tax than low-earning ones.

The model proposed in this paper gives a novel analysis of mathematical view, and the result also is a supplementary and interpretation of the tax policy.

Construction of Dynamic Optimization Model

As a single social individual, individuals obtain labor remuneration by participating in social activities, consumption of remuneration can satisfy their basic living needs and higher level of enjoyment. Generally speaking, the amount they consume affects their happiness. On the other hand, the individual's activities inevitably influenced by the level of the whole social development, therefore, infrastructure and social services provided by the government, also affect each one's well-being. Conversely, funds to provide infrastructure and improve the service level come from tax, so the improvement of individual income tax may lead to two aspects of contradiction: tax may decrease the amount of consumption, so as to reduce personal well-being; on the contrary, tax enables the government to have more funds improving individual happiness.

The utility function of income tax g and individual consumption $C = C(t, g)$ is

$$U = U(C(t, g), g)$$

The relations between U and C or U and g is illustrated in Figure 1 and Figure 2. Obviously, we have

$$\frac{\partial U}{\partial C} > 0, \frac{\partial^2 U}{\partial C^2} < 0, \frac{\partial U}{\partial g} > 0$$

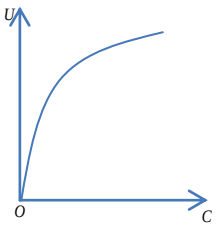

Figure 1. Relation of U and C.

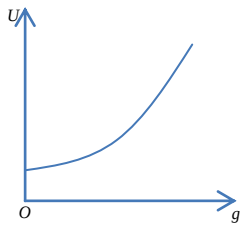

Figure 2. Relation of U and g.

Let $[0,T]$ denote one's whole life, so one's aim is to maximize his sum of whole utility by reasonably planning their income, consumption, investment and paying individual income tax, i.e.

$$\max \quad W(C(t, g), g) = \int_0^T U(C(t, g), g) e^{-\beta t} dt \tag{1}$$

Where β is discount rate. Generally speaking, consumption and tax should affect by the investments and profits. Let $K = K(t)$ denotes one's capital stock, and one gets profit $f(K)$ by

investment. In order to obtain a long return, a rational consumer should divide the profit into four parts, including consumption, capital depreciation, tax and additional investment, i.e.

$$\dot{K} = f(K) - C - nK - g$$

Where n is the rate of capital depreciation, $\dot{K} = dK/dt$ is additional investment. Let $K(0) = K_0$ denotes the capital on $t = 0$, and the capital when one is dead should be greater than zero, a dynamic optimization model is

$$\begin{aligned} \max \quad & W(C,g) = \int_0^T U(C,g)e^{-\beta t}dt \\ s.t. \quad & \dot{K} = f(K) - C - nK - g \\ & K(0) = K_0, \ \ K(T) \geq 0 \end{aligned} \tag{2}$$

Model Solving

Based on the theory of dynamic optimization, we have

Theorem 1[9] Let $x^*(t), u^*(t), t \in [t_1, t_2]$ are solutions of optimization model

$$\begin{aligned} \max \quad & \int_{t_1}^{t_2} f(t, x(t), u(t))dt \\ s.t. \quad & \dot{x}(t) = g(t, x(t), u(t)) \\ & x(t_1) = x_1, \ \ x(t_2) \geq 0 \end{aligned} \tag{3}$$

Where f and g are both second-order continuous differentiable functions. Construct Hamilton function

$$H(t, x(t), u(t), \lambda(t)) = f(t, x(t), u(t)) + \lambda(t)g(t, x(t), u(t))$$

Then, there is Hamilton multiplier $\lambda^*(t), t \in [t_1, t_2]$ which satisfies:

(1) Optimality conditions:

$$\frac{\partial f(t, x^*(t), u^*(t))}{\partial u} + \lambda^*(t)\frac{\partial g(t, x^*(t), u^*(t))}{\partial u} = 0$$

(2) Euler equation:

$$\dot{\lambda}^*(t) = -\frac{\partial f(t, x^*(t), u^*(t))}{\partial x} - \lambda^*(t)\frac{\partial g(t, x^*(t), u^*(t))}{\partial x}$$

(3) Feasibility conditions:

$$\dot{x}^*(t) = g(t, x^*(t), u^*(t)), x^*(t_1) = x_1$$

(4) Transversality condition:

$$x^*(t_2) \geq 0, \lambda^*(t_2) \geq 0, x^*(t_2) \cdot \lambda^*(t_2) = 0$$

Based on theorem 1, for the dynamic optimization problem of equation (2), there is a Hamilton multiplier $\lambda^*(t,g), t \in [0,T]$ makes the optimal solution $K^*(t), C^*(t,g), t \in [0,T]$ of equation (2) correspond to the following four conditions:

(1) Optimality conditions:

$$\frac{\partial U(C^*,g)e^{-\beta t}}{\partial C} = \lambda^*$$

(2) Euler equation:

$$\dot{\lambda}^*(t) = \lambda^*(t)[n - f'(K)]$$

(3) Feasibility conditions:

$$\dot{K} = f(K) - C - nK - g, K(0) = K_0$$

(4) Transversality condition:

$$K^*(T) \geq 0, \lambda^*(T) \geq 0, K^*(T) \cdot \lambda^*(T) = 0$$

Since the focus of this paper is the impact of personal income tax on total utility, personal income tax g can be regarded as the parameter of optimization problem. When the investment path and consumption path take the optimal path $K^*(t), C^*(t,g), t \in [0,T]$ respectively, the corresponding optimal utility can be regarded as the function of the parameter g, that is

$$\begin{aligned}
W(g) &= \int_0^T U(C^*,g)e^{-\beta t}dt \\
&= \int_0^T \left\{U(C^*,g)e^{-\beta t} + \lambda^*[f(K) - C - nK - g - \dot{K}]\right\}dt \\
&= \int_0^T \left\{U(C^*,g)e^{-\beta t} + \lambda^*[f(K) - C - nK - g]\right\}dt - \int_0^T \lambda^*\dot{K}dt \\
&= \int_0^T \left\{U(C^*,g)e^{-\beta t} + \lambda^*[f(K) - C - nK - g]\right\}dt - \lambda^*\dot{K}\Big|_0^T + \int_0^T \dot{\lambda}^*Kdt \\
&= \int_0^T \left\{U(C^*,g)e^{-\beta t} + \lambda^*[f(K) - C - nK - g] + \dot{\lambda}^*K\right\}dt \\
&\quad - \lambda(T,g)^*K(T) + \lambda(0,g)^*K(0)
\end{aligned}$$

Using the Envelope Theorem[10], take the derivative of the above equation with respect to the parameter g, we get

$$
\begin{aligned}
W'(g) &= \int_0^T \left\{ \frac{\partial U}{\partial C} \cdot \frac{\partial C^*}{\partial g} e^{-\beta t} + \frac{\partial U}{\partial g} e^{-\beta t} + \frac{\partial \lambda^*}{\partial g} [f(K^*) - C^* - nK^* - g] \right. \\
&\quad \left. - \lambda^* \cdot \frac{\partial C^*}{\partial g} - \lambda^* + \frac{\partial \dot{\lambda}^*}{\partial g} K \right\} dt - \frac{\partial \lambda^*(T,g)}{\partial g} K^*(T) + \frac{\partial \lambda^*(0,g)}{\partial g} K^*(0) \\
&= \int_0^T \left\{ \left[\frac{\partial U}{\partial C} \cdot e^{-\beta t} - \lambda^* \right] \cdot \frac{\partial C^*}{\partial g} \right\} dt + \int_0^T \left\{ \frac{\partial U}{\partial g} e^{-\beta t} - \lambda^* \right\} dt + \int_0^T \frac{\partial \lambda^*}{\partial g} \dot{K} dt \\
&\quad + \int_0^T \frac{\partial \dot{\lambda}^*}{\partial g} K dt - \frac{\partial \lambda^*(T,g)}{\partial g} K^*(T) + \frac{\partial \lambda^*(0,g)}{\partial g} K^*(0)
\end{aligned} \tag{4}
$$

By using the optimality condition and integrating by parts for the third integral, we have

$$
W'(g) = \int_0^T \left\{ \frac{\partial U}{\partial g} e^{-\beta t} - \lambda^* \right\} dt = \int_0^T \left\{ \frac{\partial U}{\partial g} - \frac{\partial U}{\partial C} \right\} e^{-\beta t} dt \tag{5}
$$

Equation (5) states that the impact of personal income tax on happiness depends on the difference between the marginal utility of personal consumption and personal income tax, that is:

When $\partial U/\partial g > \partial U/\partial C$, $W'(g) > 0$, residents' happiness increases with individual income tax increases; When $\partial U/\partial g < \partial U/\partial C$, $W'(g) < 0$, residents' happiness increases with individual income tax decreases.

Conclusion

Considering the influence of the individual income tax g for utility U is performed by the country's public welfare way indirectly, thus it is aftereffect and insensitivity. But on the other hand, consumer utility is more directly, and the marginal utility $\partial U/\partial C$ decreases with consumption C increases. When consumption C is less(low-income people), $\partial U/\partial C$ is larger than $\partial U/\partial g$, government can increase the happiness of this part of the group by reducing individual income tax. On the contrary, when consumption C is high (high-income people), $\partial U/\partial C$ will gradually decrease, and be less than $\partial U/\partial g$, at this time, the way of raising the personal income tax can increase the happiness of this group.

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