

## Study on Cooperative Game Model in Financial Regulation

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**Abstract.** Financial regulation is effective to control financial risk and to promote economic development. However, when making decision separately, regulation institutions tend to maximize their own profit and ignore cooperation. Considering the factor of cost and profit, the paper studied the cooperation decision in financial regulation with the method of game theory and discussed cooperation possibility between central bank and regulatory institutions in different situations. The paper studied the situation which the fifth solution to replicator dynamic equation does not exist. We find a situation when cooperation probability of both sides increase as time goes on. In this circumstance, the profit that uncooperative party gains due to free ride is smaller than the profit when both of parties cooperate.

### Introduction

Up to now, many scholars from home and abroad have carried out researches involving financial regulation and gain remarkable results which are listed but not limited as follows.

Saltuk Ozerturk (2017)[1] studied the impact that issuer skin in the game regulation has exerted on CRA's rating accuracy with the consideration of moral hazard problem. Rahim Khanizad, Dr. Gholamali Montazer (2017) [2] studied the game theory in banking system. Based on the comparison of the cooperative and non-cooperative games in a testing environment, it is shown that, instead of competition, the cooperation between banks could provide more benefits.

There are also fruitful contributions made by domestic scholars. Ba Shusong and Sheng Changzheng (2016)[3] believed financial regulatory system depended on financial structure and the traditional separate supervision could not suit the requirement of financial regulation. They argued financial regulatory policies were not unified which made it possible for regulatory arbitrage and even gave rise to regulation competition. Zhang Zhiyuan (2015)[4] studied the cooperation choice in financial regulation with game theory, found the equilibrium solution and discussed the cooperation possibility between central bank and regulatory institutions. However, he ignored the necessary condition when getting the fifth solution, about which this paper is going to study.

### The Study of Financial Regulation Cooperation with the Game Theory Model

Considering cost and profit, this paper studies the cooperation in financial regulation in China with game theory. There are hypotheses in the model: the information is asymmetric when making decision; the decision making is repetitive and continuous; both sides can choose to cooperate or not with a certain probability; cost and profit is perceptible.

Four situations involve where central bank and three regulatory institutions face in model.

**Noncooperation Between Two Parties.** When the two parties do not cooperate, they do not have to pay the cost for cooperation in financial regulation. Assume the revenue central bank gets as  $R_1^N$  from its own regulation field, three regulatory institutions get as  $R_1^M$  from their own regulation field. Therefore, the profit function is  $\pi_{22}^N = R_1^N$  and  $\pi_{22}^M = R_1^M$  respectively.

**Cooperation Between Two Parties.** In this case, they must pay for the cost of financial regulation, thus the revenue resulted from the synergy effect is to be distributed among institutions.

Assume the cost of central bank as  $C_1^N$ , revenue resulted from the synergy effect as  $S_1^N$ . Similarly, assume the cost of three institutions as  $C_1^M$ , revenue resulted from the synergy effect as  $S_1^M$ . Therefore, we get the profit functions:  $\pi_{11}^N = R_1^N + S_1^N - C_1^N$  for central bank and  $\pi_{11}^M = R_1^M + S_1^M - C_1^M$  for three regulatory institutions.

**Central Bank Cooperates While Three Regulatory Institutions Do Not Cooperate.** In this situation, central bank needs to pay the cost  $C_1^N$ . Three institutions get part of revenue for free because of synergy effect, i.e. three institutions get  $\overline{S^M}$  respectively and central bank gets  $S_2^N$ . Therefore, the profit functions are  $\pi_{12}^N = R_1^N + S_2^N - C_1^N$  and  $\pi_{12}^M = R_1^M + \overline{S^M}$ .

**Three Regulatory Institutions Cooperate While Central Bank Does Not Cooperate.** With the same thought as in last situation, we easily deduce profit function:  $\pi_{21}^N = R_1^N + \overline{S^N}$  for central bank, and  $\pi_{21}^M = R_1^M + S_2^M - C_1^M$  for three regulatory institutions.

We set  $x$  meaning the probability central bank cooperates and  $y$  representing the probability three institutions cooperate, both of which vary from 0 to 1. In addition, set  $t$  as time. According to Yu Weisheng(2007)[5], the replicator dynamic equation of the central bank is

$$F(x) = \frac{dx}{dt} = x(1-x) \left[ y(\pi_{11}^N - \pi_{21}^N) + (1-y)(\pi_{12}^N - \pi_{22}^N) \right]. \quad (1)$$

Set  $F(x)$  as 0, there is:

$$x_1^* = 0. \quad (2)$$

$$x_2^* = 1. \quad (3)$$

$$y^* = (\pi_{12}^N - \pi_{22}^N) / \left[ (\pi_{12}^N - \pi_{22}^N) - (\pi_{11}^N - \pi_{21}^N) \right]. \quad (4)$$

We can see that when  $(\pi_{12}^N - \pi_{22}^N) - (\pi_{11}^N - \pi_{21}^N) = 0$ ,  $y$  approaches infinity.

Similarly, we get replicator dynamic equation of three regulatory institutions as shown in Eq.5. And when  $(\pi_{21}^M - \pi_{22}^M) - (\pi_{11}^M - \pi_{12}^M) = 0$ ,  $x$  approaches infinity.

$$F(y) = \frac{dy}{dt} = y(1-y) \left[ x(\pi_{11}^M - \pi_{12}^M) + (1-x)(\pi_{21}^M - \pi_{22}^M) \right]. \quad (5)$$

The paper will discuss the situation in detail when  $(\pi_{12}^N - \pi_{22}^N) - (\pi_{11}^N - \pi_{21}^N) = 0$  and  $(\pi_{21}^M - \pi_{22}^M) - (\pi_{11}^M - \pi_{12}^M) = 0$ . There are several propositions after induction.

*Proposition 1:* Central bank:  $\overline{S^N} > S_1^N - C_1^N; S_2^N - C_1^N > 0$  or  $\overline{S^N} < S_1^N - C_1^N; S_2^N - C_1^N < 0$ ; Regulation institutions:  $\overline{S^M} > S_1^M - C_1^M; S_2^M - C_1^M > 0$  or  $\overline{S^M} < S_1^M - C_1^M; S_2^M - C_1^M < 0$ .

In this case,  $\pi_{12}^N - \pi_{22}^N = S_2^N - C_1^N$  is positive(or negative),  $\pi_{11}^N - \pi_{21}^N = S_1^N - C_1^N - \overline{S^N}$  is negative(or positive). Thus,  $\pi_{12}^N - \pi_{22}^N$  will never equal to  $\pi_{11}^N - \pi_{21}^N$ . Similarly,  $\pi_{21}^M - \pi_{22}^M$  does not equal to  $\pi_{11}^M - \pi_{12}^M$ .

*Proposition 2:* Central bank:  $\overline{S^N} > S_1^N - C_1^N; S_2^N - C_1^N < 0$  or  $\overline{S^N} < S_1^N - C_1^N; S_2^N - C_1^N > 0$ , Regulation institutions:  $\overline{S^M} > S_1^M - C_1^M; S_2^M - C_1^M > 0$  or  $\overline{S^M} < S_1^M - C_1^M; S_2^M - C_1^M < 0$ .

When  $\overline{S^N} > S_1^N - C_1^N; S_2^N - C_1^N < 0$ , it is easy to deduce that:  $\pi_{12}^N - \pi_{22}^N = S_2^N - C_1^N < 0$ ,  $\pi_{11}^N - \pi_{21}^N = S_1^N - C_1^N - \overline{S^N} < 0$ . Therefore,  $\pi_{12}^N - \pi_{22}^N$  might equal to  $\pi_{11}^N - \pi_{21}^N$ . We set it as  $k_1$ . Put it into Eq.1 and finally get the solution:

$$x(t) = \frac{ce^{k_1 t}}{1 + ce^{k_1 t}}. \quad (6)$$

In this case,  $k_1 < 0$ , the cooperation possibility of central bank(x) decrease with the increasing t.

Similarly, when  $\overline{S^N} < S_1^N - C_1^N; S_2^N - C_1^N > 0$ ,  $\pi_{12}^N - \pi_{22}^N$  might equal to  $\pi_{11}^N - \pi_{21}^N$ . We set it as  $k_1$  (now  $k_1 > 0$ ). Put it into Eq.1, we finally get the solution:

$$x(t) = \frac{ce^{k_1 t}}{1 + ce^{k_1 t}}. \quad (7)$$

Now that  $k_1 > 0$ , so cooperation possibility of central bank(x) will increase with the increasing t.

In addition, under this proposition,  $\pi_{21}^M - \pi_{22}^M$  will never equal to  $\pi_{11}^M - \pi_{12}^M$ .

*Proposition 3:* Central bank:  $\overline{S^N} > S_1^N - C_1^N; S_2^N - C_1^N > 0$  or  $\overline{S^N} < S_1^N - C_1^N; S_2^N - C_1^N < 0$ ; Regulation institutions:  $\overline{S^M} < S_1^M - C_1^M; S_2^M - C_1^M > 0$  or  $\overline{S^M} > S_1^M - C_1^M; S_2^M - C_1^M < 0$ .

When  $\overline{S^M} > S_1^M - C_1^M; S_2^M - C_1^M < 0$ ,  $\pi_{21}^M - \pi_{22}^M$  might equal to  $\pi_{11}^M - \pi_{12}^M$ . Assume it as  $k_2$ . In this case,  $k_2 < 0$ . Put it into Eq.5, we finally get the solution:

$$y(t) = \frac{ce^{k_2 t}}{1 + ce^{k_2 t}}. \quad (8)$$

$k_2 < 0$ , so the cooperation possibility of three institutions(y) will decrease as time goes on.

Similarly, when  $\overline{S^M} < S_1^M - C_1^M; S_2^M - C_1^M > 0$ ,  $\pi_{21}^M - \pi_{22}^M$  might equal to  $\pi_{11}^M - \pi_{12}^M$ . Set it as  $k_2$ , now the  $k_2 > 0$ . Put it into Eq.5, we finally get the solution:

$$y(t) = \frac{ce^{k_2 t}}{1 + ce^{k_2 t}}. \quad (9)$$

$k_2 > 0$ , so the cooperation possibility of three institutions(y) will increase as time goes on.

In this proposition,  $\pi_{12}^N - \pi_{22}^N$  will never equal to  $\pi_{11}^N - \pi_{21}^N$ .

*Proposition 4:* Central bank:  $\overline{S^N} < S_1^N - C_1^N; S_2^N - C_1^N > 0$  or  $\overline{S^N} > S_1^N - C_1^N; S_2^N - C_1^N < 0$ ; Regulation institutions:  $\overline{S^M} < S_1^M - C_1^M; S_2^M - C_1^M > 0$  or  $\overline{S^M} > S_1^M - C_1^M; S_2^M - C_1^M < 0$ .

With the deduction above, we can conclude that: when  $\overline{S^N} < S_1^N - C_1^N; S_2^N - C_1^N > 0$ , there is a positive correlation between x and t. when  $\overline{S^N} > S_1^N - C_1^N; S_2^N - C_1^N < 0$ , there is a negative correlation between x and t. Similarly, when  $\overline{S^M} < S_1^M - C_1^M; S_2^M - C_1^M > 0$ , there is a positive correlation between y and t. When  $\overline{S^M} > S_1^M - C_1^M; S_2^M - C_1^M < 0$ , there is a negative correlation between the y and t.

## Summary

As we can conclude from discussion above, in proposition 1, we can get the fifth equilibrium solution through Zhang's model.

In proposition 2, the fifth equilibrium solution does not exist as for central bank. But we find that when  $k_1 > 0$ , the cooperation possibility of central bank will increase as time goes on. When  $k_1 < 0$ , the cooperation possibility of central bank decrease as time goes on

In proposition 3, the fifth equilibrium solution does not exist as for three regulatory institutions. However, when  $k_2 > 0$ , the cooperation possibility of three regulatory institutions increase as time goes on. When  $k_2 < 0$ , the cooperation possibility of three regulatory institutions decrease as time goes on.

In proposition 4, the fifth equilibrium solution does not exist for both sides. The relations between  $t$  and the two sides' cooperation decisions depend on  $k_1$  and  $k_2$ .

Taking all propositions together, it is easy to find it is optimal to both parties when  $k_1 > 0$ ,  $k_2 > 0$ . Because the profit that uncooperative party gains due to free ride is smaller than that when both of the parties cooperate. Besides, both of parties get positive profit when there is no cooperation.

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