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With Given Tangent Polygon the Quartic Generalized Ball Closed Curves

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ABSTRACT

An algorithm for constructing the close quartic generalized Ball curves which are tangent to the given polygon is described. The control points of the close quartic generalized Ball curves to be constructed are computed simply by the vertices of the given polygon. The constructed curves are shape-preserving to the polygon. The local modification to the close quartic generalized Ball curves can be completed by simply adjusting the corresponding control parameters. One example is also given.

INTRODUCTION

B spline curve and Bézier curve in CAD, CAGD and the approximation is very useful. A given tangent polygon spline curve research, there have been many research results [1-9]. This paper discusses the quartic generalized Ball closed curves with given tangent polygon, the curve is G2 continuous, it has shape preserving property to the tangent polygon, the control points of each segment of the generalized Ball curve can be calculated directly from the vertices of the given tangent polygons, the local changes of the curve are more convenient. The quartic generalized Ball curve with shape parameters can control the shape of the curve, the curve design is more flexible to meet the design requirements.

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QUARTIC GENERALIZED BALL CLOSED CURVE

For a given closed polygon $< V_0, V_1, \cdots, V_n >$, where $V_0 = V_n$. This paper is assumed to be constructed piecewise quartic generalized Ball curves tangent point in the article i edge $V_i V_{i+1}$ of the tangent polygon for:

$$Q_i = (V_i + V_{i+1})/2, \quad (i = 0,1\cdots,n-1) \tag{1}$$

The purpose of this paper is in the 2 adjacent tangent point between Q_i, Q_{i+1} to structure quartic generalized Ball curve $r_i(t)\ (i = 0,1,\cdots,n-1)$, the corresponding

$$\begin{cases} P_i = Q_i , \\ P_{i+1} = Q_i + \alpha_i (V_{i+1} - V_i) , \\ P_{i+2} = V_{i+1} , \\ P_{i+3} = Q_{i+1} - \alpha_{i+1} (V_{i+2} - V_{i+1}) , \\ P_{i+4} = Q_{i+1} . \end{cases} \tag{2}$$

where $Q_0 = Q_n$, α_i is adjusting parameters of control vertices, it satisfies $0 \le \alpha_i \le 1/2\ \ (i = 0,1,\cdots,n-1), \alpha_0 = \alpha_n$.

The corresponding quartic generalized Ball Curve segments[10] :

$$r_i(t) = \sum_{j=0}^{4} B_{j,4} P_{i+j} \quad (0 \le t \le 1). \tag{3}$$

where

$$\begin{cases} B_{0,4} = (1-\lambda t)(1-t)^3 \\ B_{1,4} = (3+\lambda)(1-t)^3 t \\ B_{2,4} = 6(1-t)^2 t^2 \\ B_{3,4} = (3+\lambda)(1-t)t^3 \\ B_{4,4} = (1-\lambda+\lambda t)t^3 \end{cases}, \lambda \text{ is a shape parameter}, 0 \le \lambda \le 1.$$

According to the formula (3), endpoint properties of quartic generalized Ball curve are

$$\begin{cases} r_i(0) = P_i \ , \ r_i(1) = P_{i+4} \ , \\ r_i'(0) = (3+\lambda)\cdot(P_{i+1} - P_i) \ , \\ r_i'(1) = (3+\lambda)\cdot(P_{i+4} - P_{i+3}) \ , \\ r_i''(0) = 6(1+\lambda)P_i - 6(3+\lambda)P_{i+1} + 12P_{i+2} \ , \\ r_i''(1) = 6(1+\lambda)P_{i+4} - 6(3+\lambda)P_{i+3} + 12P_{i+2i} \ . \end{cases} \tag{4}$$

The formula (1) and the formula (2) into the formula (4), the results are as follows:

$$\begin{cases} r_i(0) = Q_i \ , \ r_i(1) = Q_{i+1} \ , \\ r_i'(0) = (3+\lambda)\alpha_i(V_{i+1} - V_i) \ , \\ r_i'(1) = (3+\lambda)\alpha_{i+1}(V_{i+2} - V_{i+1}) \ , \\ r_i''(0) = 6(1 - 3\alpha_i - \lambda\alpha_i)(V_{i+1} - V_i) \ , \\ r_i''(1) = -6(1 - 3\alpha_{i+1} - \lambda\alpha_{i+1})(V_{i+2} - V_{i+1}) \ . \end{cases} \tag{5}$$

In the arbitrary two section of the splicing process of quartic generalized Ball curve, shape parameter λ is the same value. By the formula (5), constructed the segments quartic generalized Ball curve is G 1 continuous.

When $\lambda = 1$, and because

$$K_i(1) = \frac{|\, r_i'(1) \times r_i''(1) \,|}{\left| r_i'(1) \right|^3} = 0 \ , \quad K_{i+1}(0) = \frac{|\, r_{i+1}'(0) \times r_{i+1}''(0) \,|}{\left| r_{i+1}'(0) \right|^3} = 0$$

we know that, when $\lambda = 1$, they have the same curvature at the connection point, therefore, the entire four generalized Ball curve is G 2 continuous.

Attention to the range of adjusting the parameter α_i is $0 \le \alpha_i \le 1/2 \quad (i = 0, 1, \cdots, n-1)$, therefore, $P_i, P_{i+1}, P_{i+2}, \ P_{i+3}, P_{i+4}$ form a convex quadrilateral, and the convexity of it and both sides in form $< V_i, V_{i+1}, V_{i+2} >$ are the same. By the convexity of quartic generalized Ball curves, we know $r_i(t)$ is convex, and the convexity of it and both sides in form $< V_i, V_{i+1}, V_{i+2} >$ are the same.

Let V_{i+1} be the turning point of the tangent polygon, namely the vector $\boldsymbol{V_{i-1}V_i} \times \boldsymbol{V_i V_{i+1}}$ and the vector $\boldsymbol{V_i V_{i+1}} \times \boldsymbol{V_{i+1} V_{i+2}}$ in the opposite direction, then the convex of the section i-1 curve and the section i curve is the opposite, and the curve form an inflection point at point Q_i.

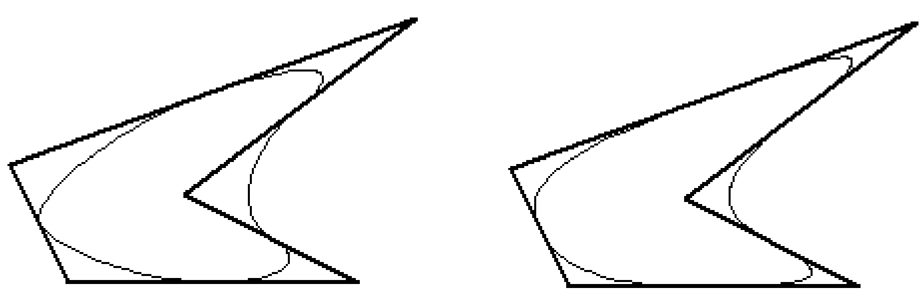
Therefore, inflection point number of the constructed Piecewise quartic generalized Ball curve is equal to the number of the turning point for tangent polygon, namely the curve for its given tangent polygon is shape preserving.

By Piecewise quartic generalized Ball curve control points of the structure, it is not difficult to judge the local modification of subsection four of generalized Ball curve is also very convenient.

For example, when modifying the i segment quartic generalized Ball curve of $r_i(t)$, we can be achieved by adjusting the parameters of α_i and α_{i+1} $(0 \le \alpha_i \le 1/2,\ 0 \le \alpha_{i+1} \le 1/2)$, and each adjusting parameter α_i only affects the curve segment the $r_i(t)$ and its neighboring curve segment $r_{i+1}(t)$.

THE APPLICATION EXAMPLES AND DISCUSSION

Using the proposed method in this paper were constructed on the quartic generalized Ball curve, taking respectively, $(\alpha_i, \lambda) = (0.2, 0)$, $(0.4, 0.5)$, we get quartic generalized Ball curves as shown in figure 1. The shape of the curves is determined by the parameters α_i and λ. Inner control point of each section curve can be chosen arbitrarily in a certain range. When the α_i increases, the inner control point will be near to the vertices of the given tangent polygon, composite curve generated correspondingly will be closer to the vertices of the given tangent polygons, and vice versa curve will move away from the vertex. Because the quartic generalized Ball closed curve has 2 free parameters, which has good adjustability and has a good approximate effect on tangent polygon.



(a) $(\alpha_i, \lambda) = (0.2, 0)$ (b) $(\alpha_i, \lambda) = (0.4, 0.5)$

Figure 1. The 2 extended quartic generalized Ball closed curve.

CONCLUSION

This paper deals with the quartic generalized Ball curves with given tangent polygon, it is an extension of references in [10] curve. In references [10], another quartic generalized Ball closed curve with tangent polygon will be discussed in another paper writing.

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