

Coherent Combination Characteristics of Laser Array with Hexagonal Ring Distribution

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ABSTRACT

Coherent combination of laser beam is an important and challenging area of high power laser science, and many proof-of-principle experiments have been demonstrated. In this paper, we presented a mathematical model and derived the far field intensity distribution of a 2-D hexagonal ring distribution laser array using Fraunhofer diffraction theory. The effects of fill factor and phase errors on far field intensity patterns are numerically analyzed and simulated. The result shows that non-unity fill factor causes the side lobe, the fill factor is smaller, the central lobe angular width will be much narrower and the power in side lobes will be larger. To make sure the side lobe intensity less than $1/e^2$ central peak power, the fill factor should be larger than 64%. Strehl ratio is also referred as evaluation parameter of the effect of phase errors on far field intensity. To limit the degradation of the Strehl ratio caused by phase errors to 0.8, the rms phase errors must be limited to $\sim\pi/4$ or better.

INTRODUCTION

Coherent combination of multiple lasers is becoming a viable alternative for high power and high brightness laser source. In recent years, various coherent combination techniques have been brought forward and investigated to obtain ideal coherent combination beam. There are excellent reviews on these progress[1].Active

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phasing implementations mostly employ the master oscillator power amplifier (MOPA) architectures[2], which require additional phase locking electronics. Passive phasing implementations are generally explained by the self-organized mechanism, including interferometry resonator[3,4], the self-Fourier resonator[5], or fiber couplers [6]. As proof-of-principle experiments, most of these investigations have been demonstrated. The research of array distribution is mostly focused on 1-D linear or 2-D[7,8] square distribution in prior work. And the far field intensity distribution is mostly described by the propagation theory of the Gaussian beam [9] or Huygens-Fresnel integral[8]. In order to understand the effect of near field parameters on the far field intensity patterns and beam quality, we investigate the laser array of a 2-D hexagonal ring distribution laser array using Fraunhofer diffraction theory.

This paper presents a mathematical model and derives the far field intensity distribution of a 2-D hexagonal ring distribution laser array using Fraunhofer diffraction theory. The effects of fill factor and phase errors on far field intensity patterns are numerically analyzed and simulated. Finally, the requirements of fill factor and phase errors are presented to obtain the optimal coherent combination.

THEORY

The seven-element laser array is arranged in hexagonal ring, as shown in Fig. 1. The array consists of N elements. Each element is a single-mode polarization-maintaining laser, whose waist radius is ω_0 , and the separation distance between the adjacent elements is d . We suppose that m is the surrounding ring number and n is the element number in the mth surrounding ring. Then according to the hexagonal ring distribution, the position coordinate element (x_{mn}, y_{mn}) can be expressed as

$$x_{mn} = md \cdot \cos \left[(n-1) \frac{2\pi}{6m} \right] \quad y_{mn} = md \cdot \sin \left[(n-1) \frac{2\pi}{6m} \right] \quad (1)$$

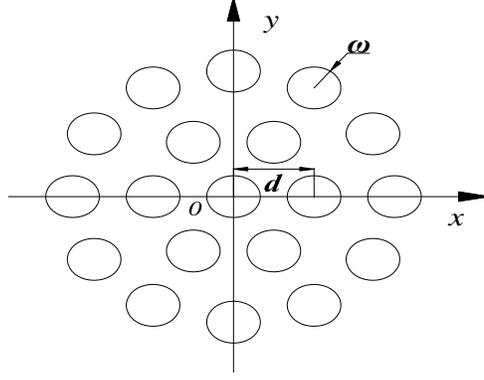


Figure 1. Schematic diagram of the laser array with hexagonal ring distribution.

To calculate the intensity propagating in the far field, we assumed that each element is identical with a Gaussian amplitude distribution and the wave fronts are collimated and co-aligned in the output plane. Then the field distribution in the output plane is given by

$$\begin{aligned}
 U(x, y) &= \sum_{m=0}^M \sum_{n=1}^{6m} A_{mn} \exp \left[-\frac{(x - x_{mn})^2 + (y - y_{mn})^2}{\omega_0^2} + i\phi_{mn} \right] \\
 &= \sum_{m=0}^M \sum_{n=1}^{6m} A_{mn} \exp \left(-\frac{x^2 + y^2}{\omega_0^2} + i\phi_{mn} \right) * \delta(x - x_{mn}, y - y_{mn})
 \end{aligned} \tag{2}$$

Where A_{mn} are axial amplitudes and ϕ_{mn} are the initial phase, and the point (x_{mn}, y_{mn}) are the central position of the element.

Under Fraunhofer diffraction condition, the far field pattern is given by the Fourier transform of the electric field distribution at the output plane. The far field intensity is given by

$$I(\theta_x, \theta_y) \propto \left| \sum_{m=0}^M \sum_{n=1}^{6m} A_{mn} \exp \left\{ i \left[k(x_{mn}\theta_x + y_{mn}\theta_y) - \phi_{mn} \right] \right\} \right|^2 \times \exp \left[-\frac{k^2 \omega_0^2}{2} (\theta_x^2 + \theta_y^2) \right] \tag{3}$$

This equation shows that the far field intensity is represented by the product of the near field distribution function and single element far field distribution with Gaussian function. Therefore Eq. (3) is used to analyze the effect of these parameters on the far field intensity distribution.

SIMULATION AND DISCUSSION

The seven element hexagonal array is taken as example to simulate the numerical calculation of the far field distribution. The parameters used in simulation are $m = 2$, $\lambda = 1.064 \mu m$, $\omega = 0.5 mm$, $A_{ij} = 1$, d and ϕ_{ij} are varied, respectively.

Taken the seven element hexagonal array as an example, the far field intensity is

$$\begin{aligned}
 I(\theta_x, \theta_y) &\propto \left| \sum_{m=0}^M \sum_{n=1}^{6m} \exp\{ik(x_{mn}\theta_x + y_{mn}\theta_y)\} \right|^2 \times \exp\left[-\frac{k^2\omega_0^2}{2}(\theta_x^2 + \theta_y^2)\right] \\
 &\propto \left| 1 + 2\cos(kd\theta_x) + 4\cos\left\{kd\left[\cos\left(\frac{\pi}{3}\right)\theta_x + \sin\left(\frac{\pi}{3}\right)\theta_y\right]\right\} \right|^2 \times \exp\left[-\frac{k^2\omega_0^2}{2}(\theta_x^2 + \theta_y^2)\right]
 \end{aligned} \tag{4}$$

Eq. (4) indicates that the far field intensity pattern has peaks with angular separation. It means that the peaks separation just relates with the element distance in near field distribution. Moreover, the central lobe angular width, that is the zeros position, is definite by element number and element separation distance. If the separation distance remains constant and the element distribution is defined, just adding a ring, the peaks separation and the side lobe numbers are remain unchanged. If the separation distance or the element number increases, the central lobe angular width and the peaks separation would all reduce. That is, as reduce of the fill factor, the central lobe would contain the fewer power because the side lobes increase in numbers. It also shows that the far field intensity is not geometric symmetry in x-axis and y-axis, but circular symmetry, which corresponds with the hexagonal distribution construction.

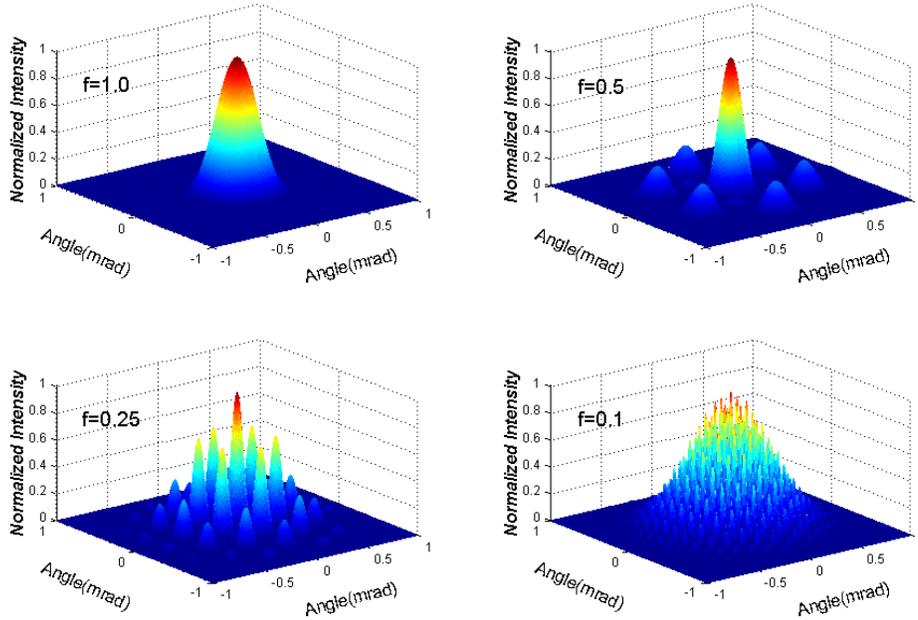


Figure 2. Normalized far-field profile of different fill factors.

Fig. 2 shows the far field intensity profile of different fill factors. We vary the fill factor by increase the element separation distance, which can also be realized by reduce by the spot size of element aperture. It can be seen visualized that, the fill factor affects the far field intensity patterns on central lobe angular width, power of central lobe and side lobes number. As the fill factor reduces, the central lobe angular width reduces, and side lobe number also increases, which causes the far field brightness weakened.

As a note, the effect of fill factor on the far field intensity distribution analyzed by Fraunhofer diffraction intensity is different with the Gauss beam propagation theory in Ref.[9]. The latter supposed the Gauss beam with some divergence angle, and just calculated the intensity distribution of some far distance from the near field (20 meters). In fact, the beams just overlap in some plane, is not the true far filed intensity distribution. While the Fraunhofer diffraction intensity expressed by a Fourier transform of the near field distribution is the true far field in infinite distance. Therefore the side lobe number in Ref. [9] does not enlarge with reduction of the fill factor. As a fact, smaller fill factor may bring more side lobes, while reduce power in the central lobe.

Next, consider the phase control requirement for coherent combination. In coherent combination system, each element is independently locked to a reference arm instead of being locked to nearest neighbors. For laser array system, environmental vibration, changes in temperature or the pumping current, all may

introduce the phase noise. And as the output power is increased, the phase noise is worsened. Therefore, phase error ϕ_{ij} is a random variable of statistical independence, no need to consider the coupling effect among elements.

The Strehl ratio, which characterizes the on-axis far-field intensity of a beam propagated from a near-field hard aperture, is defined as the ratio of the on-axis far field intensity to that of an ideal, equal-power top-hat beam filling the same hard aperture. In a similar manner, Strehl ratio of far field intensity in coherent combination system with rms phase error of σ_ϕ is defined as the ratio of the far field on-axis intensity of actual coherent combination to that of ideal coherent combination. Then the coherent combination system with rms phase error of σ_ϕ is given by

$$S = \exp(-\sigma_\phi^2) + \frac{1}{N} [1 - \exp(-\sigma_\phi^2)] \quad (5)$$

A contour plot of the Strehl ratio is shown in Fig. 3. For a certain phase error σ_ϕ , the Strehl ratio is degraded as the increase of element number. For a certain element number, the Strehl ratio is degraded as the enlargement of the rms phase errors. When $\sigma_\phi = 0$, the Strehl ratio reaches 1, which is the case the perfect coherent combination. While $\sigma_\phi \rightarrow \infty$, the Strehl ratio tends to zero, suggesting that this is the case of incoherent combination. To limit the degradation of the Strehl ratio caused by phase errors to 0.8, the rms phase errors must be limited to $\sim \pi/4$ or better.

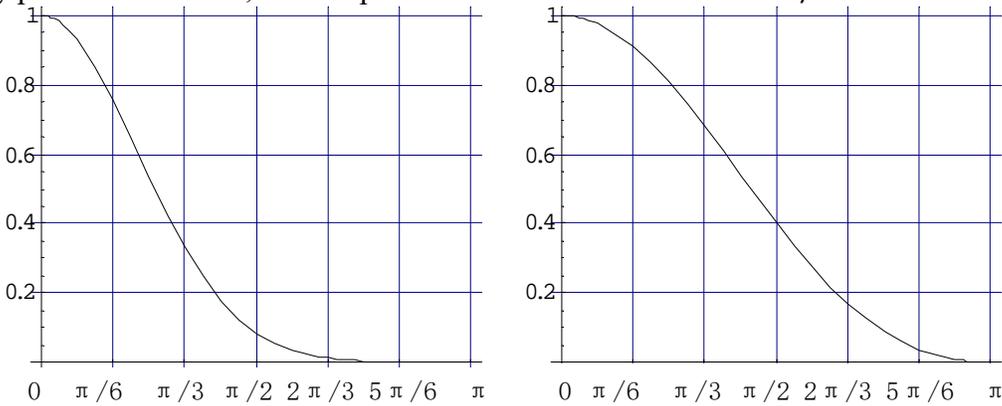


Figure 3. Contour plots of Strehl ratios with rms phase errors σ_ϕ .

CONCLUSION

In conclusion, the far field intensity of a 7-element array of hexagonal ring distribution is presented by Fraunhofer diffraction theory. The effects of fill factor and phase errors on far field intensity pattern are theoretically analyzed and simulated. The result shows that non-unity fill factor causes the side lobe, the fill factor is smaller, the central lobe angular width will be much narrower and the power in side lobes will be larger. To make sure the side lobe intensity less than $1/e^2$ central peak power, the fill factor should be larger than 64%. Although the phase errors make no effect on the central and side lobe distribution, but phase errors may attenuate the intensity to disperse the power in central lobe into side lobes until the interference fringe is fade, which is incoherent combination. Strehl ratio is referred as evaluation parameter of the effect of phase errors on far field intensity. To limit the degradation of the Strehl ratio caused by phase errors to 0.8, the rms phase errors must be limited to $\sim \pi/4$ or better.

According to different application requirement, the parameter in coherent combination system may be focused on differently. For example, in beam direction coherent controlling system, the phase difference may be controlled to some direction. However, the requirements on fill factor and phase errors are essential in active phasing coherent combination system to obtain ideal coherent combination.

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