

# Mathematical Model on the Visualization of Axis Straightness Errors

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Lijie Chen and Mingyao Zhang

## ABSTRACT

The fundamental theory on the visualization was determined, mathematical model on visualization of axis straightness errors was established. Used the theory of computer graphics and three-dimensional wireframe models, the display of extracted derived feature and associated derived feature on measured axle were achieved; Adopted axonometric projection transformation, the basic geometric shapes was realized; Adopted real space hidden method, the problem of geometrical graph hidden was solved. On the basis of theoretical study, LabWindows/CVI used as a software development platform, the software was developed to realize visualization of axis straightness errors.

## INTRODUCTION

Axis straightness errors have a big performance impact on precision machines and instruments. Accurate values were obtained for axis straightness errors, and abstract sampling data were changed into visual error graph, so that the reasons for causing axis straightness errors of parts were analysed, in order to control errors well. We can also use the parts according to the errors of form and position, reducing errors influence on performance of parts. Therefore, visual mathematical model established for axis straightness errors has theoretical significance and practical significance [1-4].

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Lijie Chen, Mingyao Zhang. Northeastern University, Shenyang, Liaoning Province, P.R. China

## MATHEMATICAL MODEL

As shown in Fig.1, established space rectangular coordinate system  $OXYZ$ .  $Z$  is the rotary axis in measuring. Element tested was separated into  $m$  sampling sections which were vertical to  $Z$  axis and were equal in distance each other. Coordinate plane  $XOY$  was symmetrical with discrete sampling sections of the element tested[5-6]. There were  $n$  sampling points which were equal in angle each other on the profile measured in each sampling section. Least square circle center  $O_i$  of the profile measured was determined by  $n$  sampling points within each sampling section. Least square circle Centers of  $m$  sampling sections were connected and it was as actual axis measured.  $L$  was the least square axis of actual axis. Point  $O_1(a, b, 0)$  was intersection of the least square axis  $L$  and coordinate plane  $XOY$ . The direction vector for the least square axis  $L$  was  $\mathbf{S} = \{g, l, 1\}$ .

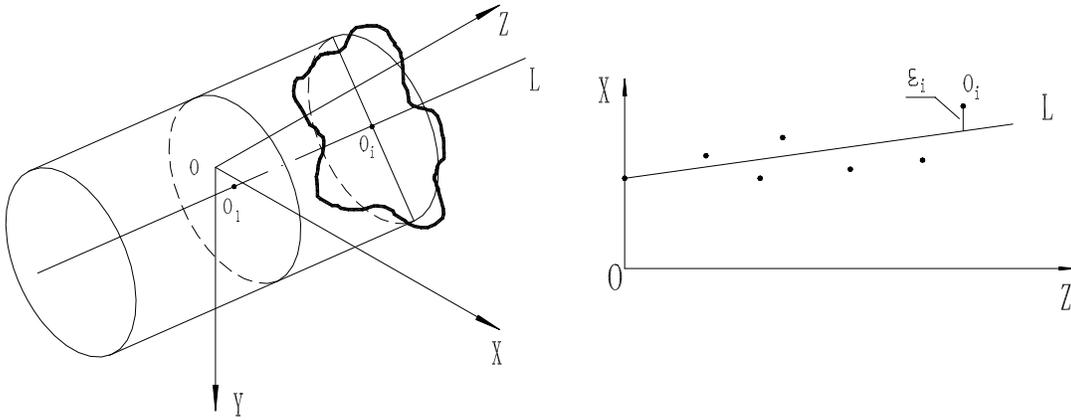


Figure 1.The principle diagram of producing axis straightness error graph.

Coordinates of the least square circle center  $O_i(a_i, b_i, z_i)(i = 1, 2, \dots, m)$  are

$$\begin{cases} a_i = \frac{2}{n} \sum_{j=1}^n \Delta r_{ij} \cos \theta_{ij} \\ b_i = \frac{2}{n} \sum_{j=1}^n \Delta r_{ij} \sin \theta_{ij} . \\ z_i = z_i \end{cases} \quad (1)$$

Equation of  $L$  is

$$\frac{x-a}{g} = \frac{y-b}{l} = z. \quad (2)$$

According to the geometry in Fig.1 and the principle of least square, we can obtain

$$\begin{cases} a = \frac{1}{m} \sum_{i=1}^m a_i, b = \frac{1}{m} \sum_{i=1}^m b_i \\ g = \frac{\sum_{i=1}^m a_i z_i}{\sum_{i=1}^m z_i^2}, l = \frac{\sum_{i=1}^m b_i z_i}{\sum_{i=1}^m z_i^2} \end{cases} \quad (3)$$

Distances of point  $O_i$  to Line  $L$  are

$$\begin{aligned} r_i &= \frac{|(\mathbf{O}_i - \mathbf{A}) \times \mathbf{S}|}{|\mathbf{S}|} \\ &\approx \sqrt{(a_i - a - gz_i)^2 + (b_i - b - lz_i)^2} \end{aligned} \quad (4)$$

A axis straightness error is

$$f = 2r_{\max}. \quad (5)$$

In order to clearly display the error graph,  $a_i, b_i$  will be enlarged or reduced  $M$  times (usually several times).  $R_a$  can be selected according to the size of the drawing area of a computer screen. In order to clearly reflect points of the actual axis,  $z_i$  will be enlarged or reduced  $N$  times (usually several times).

Axonometric coordinate of a space point is

$$\begin{cases} X = (x + z) \cos 30^\circ \\ Y = x \sin 30^\circ + y - z \sin 30^\circ \end{cases} \quad (6)$$

The coordinates of the points  $O_i$  in the coordinate system of the screen are

$$\begin{cases} X_i = (Ma_i + Nz_i) \cos 30^\circ \\ Y_i = Mb_i + (Ma_i - Nz_i) \sin 30^\circ \end{cases} \quad (7)$$

In screen coordinate system, axis equation of ideal cylindrical surface which contained the actual axis is

$$\begin{cases} X = (a + (g + 1)Nz) \cos 30^\circ \\ Y = b + a \sin 30^\circ + ((g - 1) \sin 30^\circ + l)Nz \end{cases} \quad (8)$$

Radius is

$$r = Mf / 2 . \quad (9)$$

## VISUALIZATION OF AXIS STRAIGHTNESS

As shown in Fig. 2, that is the resulting graph for axis straightness errors, which is generated by using the mathematical model and a set of sampling data in LabWindows/CVI[7].

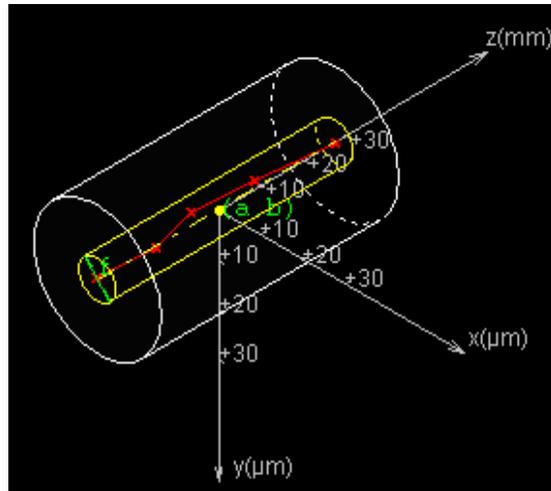


Figure 2. The error graph of axis straightness.

The magnification in the mathematical model is  $M=5$ ,  $N=7$ . Radium is  $R_a=15\text{mm}$ . The coordinate of the intersection point for ideal axis  $L$  and coordinate plane  $XOY$  is  $a=-0.55 \mu\text{m}$ ,  $b=0.39 \mu\text{m}$ . The direction vector for the least square axis  $L$  is  $g=-0.00$ ,  $l=0.00$ . the axis straightness error is  $f=1.73 \mu\text{m}$ .

## CONCLUSIONS

The basic theory has been proposed, which was applied to generate geometrical error graph. Selected three-dimensional wireframe models to realize visualization of geometrical error, it could clearly display extracted derived feature and associated derived feature on measured axle. Successfully has solved the hidden problems of dimensional error graph.

The mathematical model on visualization has been established. It is for axis straightness errors on least squares in rotary parts. The mathematical model meets the provision on axis straightness errors about geometrical tolerances in national standard. It has provided a theoretical basis for visualization on axis straightness errors.

The theory of error graph and the mathematical model have realized the visualization of axis straightness errors.

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