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A Comparison of Dynamic Soil Models for Backcalculation of Engineering Soil Properties from Falling Weight Deflectometer Data

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ABSTRACT: The falling weight deflectometer (FWD) test is one of the most widely used nondestructive tests for backcalculation evaluation of pavement and soil properties. Despite the advantages in accounting for the time-dependent load and responses, the accuracy of evaluating elastic moduli based on dynamic backcalculation analyses is, however, significantly depended on the forward dynamic soil models employed in the backcalculation process. To serve this challenge, a comparison for advantages, limitations and the reliability of different dynamic soil models employed in the backcalculation of engineering soil properties is presented in this paper. Two types of dynamic soil forward models in the FWD backcalculation, namely, dynamic single-degree-of-freedom (SDOF) and dynamic half-space models are considered. The study in this paper provides a better understanding of dynamic backcalculation processes which is essential for the development of dynamic backcalculation program and its applications.

INTRODUCTION

Falling weight deflectometer (FWD) test is one of the most widely used methods for nondestructive evaluation of *in-situ* soil properties and the structural condition of in-service pavements (Shahin, 2005). The FWD backcalculation process for predicting the elastic properties of the subgrade soils/pavement basically consists of three main components, i.e., FWD field data (time-domain deflection basin), the forward soil model and the optimization process, as schematically presented in Fig. 1. FWD backcalculation procedure estimates pavement properties by matching measured (field deflection data) and calculated pavement surface deflections (from forward soil model) using optimization. The backcalculation elasticity modulus is determined by assuming a set of pavement layer moduli in the forward soil models that can produce deflections similar to those measured from the field test.

A great deal of efforts have been made in both theoretical soil mechanics for idealizing soil models and interpretation approaches for backcalculation based on FWD test (Pan *et al.*, 2008). Numerous backcalculation methods and commercial programs have been developed for the backcalculation problem of pavement moduli. Backcalculation methods can be broadly classified into static and dynamic approaches. Earlier studies are primarily focused on static backcalculation. Dynamic backcalculation are later developed in order to improve the reliability of the backcalculation interpretations (e.g. Uzan, 1994; Founquinos *et al.*, 1995; Liang and Zhu, 1998). Due to the dynamic nature of the FWD test, static backcalculation is known to be capable of producing erroneous estimates of the moduli associated with the neglect of dynamic effects (Mamlouk and Davies, 1984).

Despite the advantages in accounting for the time-dependent load and responses, the accuracy of evaluating elastic moduli based on dynamic backcalculation analyses is, however, significantly depended on the forward dynamic soil models employed in the backcalculation process. The dynamic interaction between a circular plate and a soil medium has been considered by a number of researchers in the literature (e.g., Krenk and Schmidt, 1981; Philippacopoulos, 1989; Zeng and Rajapakse, 1999; Senjuntichai and Sapsathiarn, 2003). The primary objective of this study is to investigate the influence of forward soil models for backcalculation of layer soil properties.

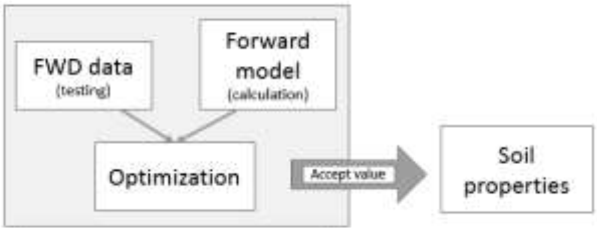


Figure 1. Falling weight deflectometer (FWD) backcalculation process.

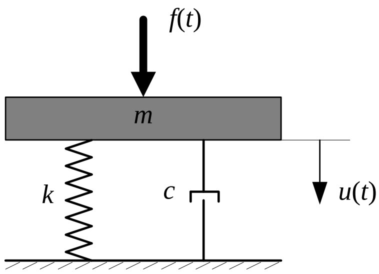


Figure 2. Schematic illustration of dynamic single degree of freedom (SDOF) model.

DYNAMIC SOIL MODELS

Two types of dynamic soil models are considered in the present study as forward models in the backcalculation process, namely, dynamic single degree of freedom (SDOF) and dynamic half-space models.

Dynamic single degree of freedom (SDOF) model

For the dynamic SDOF model, the loading plate-soil system is represented by a mass-spring-damper system as shown in Fig. 2, in which the dynamic governing equilibrium equation can be given by

$$m\ddot{u}(t) + c\dot{u}(t) + ku(t) = f(t) \qquad (1)$$

where $f(t)$ is the impulse load representing a time-dependent loading from the falling weight deflectometer device; $u(t)$, $\dot{u}(t)$ and $\ddot{u}(t)$ are the deflection, the velocity and the acceleration respectively; and the overdot denotes the derivative with respect to the time parameter t; and k, c and m are the elastic stiffness, the damping coefficient and the equivalent mass respectively.

The central finite difference integration approach is employed in the present study to determine the acceleration and the deflection from the known velocity. Details of the method for solving Eq. (1) by using the central finite difference integration approach can be found on Chopra (2007).

Dynamic half-space model

The schematic representation for the dynamic half-space model is presented in Fig. 3. The plate is subjected to axisymmetric time dependent loading and its response is governed by the classical thin-plate theory. The soil domain is represented by a poroelastic half-space. Vibration behavior of the poroelastic medium is governed by Biot's poroelastodynamic theory. The constitutive relations for a homogeneous poroelastic material can be expressed as (Biot, 1941):

$$\sigma_{rr} = 2\mu \frac{\partial u_r}{\partial r} + \lambda e - \alpha p \,; \quad \sigma_{zz} = 2\mu \frac{\partial u_z}{\partial z} + \lambda e - \alpha p \tag{2a}$$

$$\sigma_{zr} = \mu \left(\frac{\partial u_r}{\partial z} + \frac{\partial u_z}{\partial r} \right); \quad p = -\alpha M e + M \zeta \tag{2b}$$

where

$$e = \frac{\partial u_r}{\partial r} + \frac{\partial u_z}{\partial z} + \frac{u_r}{r} \,; \quad \zeta = -\left(\frac{\partial w_r}{\partial r} + \frac{\partial w_z}{\partial z} + \frac{w_r}{r} \right) \tag{2c}$$

In the above equations, σ_{rr}, σ_{zz} and σ_{zr} denote the total stress component of the bulk material; u_i and w_i are the average displacement of the solid matrix and the fluid displacement relative to the displacement of the solid matrix, in the $i-$ direction ($i = r, z$), respectively; p is the excess pore fluid pressure (suction is considered negative); ζ is the variation of fluid content per unit reference volume; e is the dilatation of the solid matrix; μ is the shear modulus and λ is a constant of the bulk material, respectively. In addition, α and M are Biot's parameters accounting for compressibility of the two-phased material (Biot, 1941).

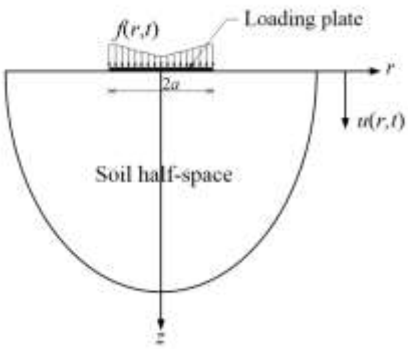


Figure 3. Schematic illustration of dynamic half-space model considered in the present study.

The equations of motion for a poroelastic medium undergoing axisymmetric deformations, in the absence of body forces (solid and fluid) and a fluid source, can be expressed according to Biot (1962) as

$$\mu \nabla^2 u_r + \left(\lambda + \alpha^2 M + \mu \right) \frac{\partial e}{\partial r} - \mu \frac{u_r}{r^2} - \alpha M \frac{\partial \zeta}{\partial r} = \rho \, \ddot{u}_r + \rho_f \ddot{w}_r \tag{3a}$$

$$\mu \nabla^2 u_z + \left(\lambda + \alpha^2 M + \mu \right) \frac{\partial e}{\partial z} - \alpha M \frac{\partial \zeta}{\partial z} = \rho \, \ddot{u}_z + \rho_f \ddot{w}_z \tag{3b}$$

$$\alpha M \frac{\partial e}{\partial r} - M \frac{\partial \zeta}{\partial r} = \rho_f \ddot{u}_r + m \ddot{w}_r + b \dot{w}_r \tag{3c}$$

$$\alpha M \frac{\partial e}{\partial z} - M \frac{\partial \zeta}{\partial z} = \rho_f \ddot{u}_z + m \ddot{w}_z + b \dot{w}_z \tag{3d}$$

In the above equations, ρ and ρ_f are the mass densities of the bulk material and the pore fluid respectively; and $m = \rho_f / \beta$ (β = porosity), is a density-like parameter. In addition, b is a parameter accounting for the internal friction due to the relative motion between the solid matrix and the pore fluid. The parameter b is defined as the ratio between the fluid viscosity and the intrinsic permeability of the porous medium. In addition $\nabla^2 = \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{\partial^2}{\partial z^2}$. Introducing the Fourier-Hankel integral transform of a temporal field $g(r,t)$ via

$$G(\xi,\omega) = \int_{-\infty}^{\infty}\int_{0}^{\infty} g(r,t) r J_n(\xi r) e^{-i\omega t} dr dt \quad (4a)$$

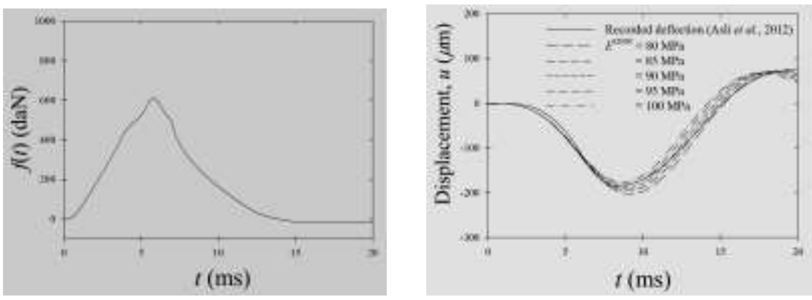
and the inverse Fourier-Hankel integral transform can be expressed as

$$g(r,t) = \int_{-\infty}^{\infty}\int G(\xi,\omega)\xi J_n(\xi r) e^{i\omega t} d\xi d\omega \quad (4b)$$

The governing partial differential equations, Eqs. (2) and (3), can be solved by applying the Fourier-Hankel integral transform, Eq. (4a), together with appropriate boundary conditions along the free surface (Zeng and Rajapakse, 1999). The loading elastic plate can be included in the analysis system by considering the energy of the plate-soil interaction system. Details of plate-soil interaction analysis system are given elsewhere (Senjuntichai and Sapsathiarn, 2003 and 2006).

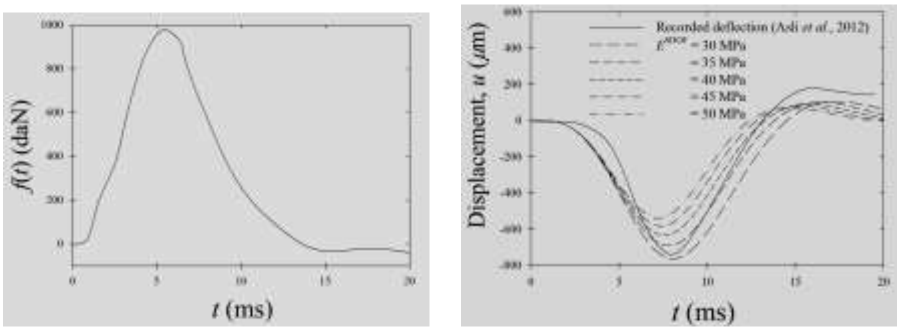
RESULTS AND DISCUSSION

A computer code has been developed in the present study for evaluating the backcalculation elastic modulus from the applied load pulse and the recorded deflection based on the formulation of dynamic soil models presented in the preceding section. The accuracy of the dynamic soil models' solution are first verified through the comparison with existing solutions reported in the literature. In order to expose the advantages, limitations and reliability of different forward models to the FWD backcalculation, a selected set of numerical solutions is presented in this section. Two types of dynamic soil models—namely, dynamic single degree of freedom (SDOF) and dynamic half-space models—are considered as forward models in the FWD backcalculation process. The applied load pulse and FWD displacement profiles for clayey soil and concrete crushed aggregates (CCA) reported in the literature are served as the field data in the FWD backcalculation process (Asli *et al.*, 2012).



(a) (b)

Figure 4. (a) The applied load pulse *f*(*t*) by FWD test on CCA soil and (b) displacement profiles from the recorded field data (*Asli et al.*, 2012) and the dynamic SDOF model for CCA.



(a) (b)

Figure 5. (a) The applied load pulse *f*(*t*) by FWD test on clayey soil and (b) displacement profiles from the recorded field data (*Asli et al.*, 2012) and the dynamic SDOF model for clayey soil.

Figs. 4 and 5 present a comparison between field data recorded during the FWD tests (*Asli et al*., 2012) and those calculated using the dynamic SDOF model for CCA and clayey soil, respectively, for different values of equivalent elastic modulus E^{SDOF}. The relation between the SDOF stiffness (k) and E^{SDOF} is based on the Boussinesq's theory (1885), i.e., $E^{SDOF} = (1-\nu^2)k/\beta a$, where ν is Poisson's ratio, a is the radius of the loading plate, and k is the elastic stiffness of the plate-half-space system. In addition, β is the shape factor depending on the stress distribution under the loading plate, i.e., $\beta = \pi/2$, 2 and $3\pi/2$ for uniform, inverse parabolic and parabolic stresses distribution, respectively. The maximum deflections obtained from dynamic SDOF and half-space models are presented in Tables 1 and 2 for CCA and clayey soil, respectively. The corresponding maximum deflections under the loading plate obtained from the FWD test are also given in Table 1 for CCA and Table 2 for clayey soil. Different optimization techniques can be employed in the backcalculation procedure to predict the elastic modulus. The peak value method has been routinely used for the estimation of the soil elastic modulus by considering only the maximum values of the recorded (field) and calculated (model) deflections. Based on the applied load pulse and recorded displacement data, the backcalculation elastic moduli obtained from dynamic SDOF and half-space models using peak value method are 93.8 and 97.9 MPa for CCA respectively, and 31.1 and 42.7 MPa for clayey soil respectively.

Table 1. Maximum soil deflections from forward models and FWD deflection data for CCA.

Elastic modulus (MPa)	**Dynamic SDOF model (μm)**	**Dynamic half-space model (μm)**	**Recorded field deflection (*Asli et al.*, 2012) (μm)**
80.0	202.4	222.2	184.1
85.0	195.4	209.7	
90.0	189.2	198.6	
95.0	182.2	188.6	
100.0	176.3	179.6	

Table 2. Maximum soil deflections from forward models and FWD deflection data clayey soil.

<table>
<tr><th>Elastic modulus (MPa)</th><th>Dynamic SDOF model (μm)</th><th>Dynamic half-space model (μm)</th><th>Recorded field deflection (Asli et al., 2012) (μm)</th></tr>
<tr><td>30.0</td><td>762.7</td><td>998.0</td><td rowspan="5">736.0</td></tr>
<tr><td>35.0</td><td>690.0</td><td>860.1</td></tr>
<tr><td>40.0</td><td>634.0</td><td>755.9</td></tr>
<tr><td>45.0</td><td>583.2</td><td>674.3</td></tr>
<tr><td>50.0</td><td>537.9</td><td>608.1</td></tr>
</table>

The differences between the predicted elastic moduli obtained from the two backcalculation models could be observed from the numerical results. The value of predicted backcalculation elastic moduli based on the dynamic SDOF model are lower than those based on dynamic half-space model for both CCA and clayey soils implying that higher equivalent stiffness of the system is required in the backcalculation process based on the half-space model when compared to those based on the SDOF model. This can probably be explained by the fact that the deflection obtained from the dynamic SDOF model is considered from the lumped mass-dashpot-spring system, whereas the deflection from the half-space model is calculated locally based on the continuum framework. Consequently, the half-space model requires stiffer soil in order to generate identical maximum deflection when compared to the SDOF model. The application of predicted elastic modulus from the available commercial backcalculation software requires careful interpretation and in-depth understanding from the user.

CONCLUSIONS

In this paper, a computer code has been developed for evaluating the backcalculation elastic modulus from the applied load pulse and the recorded deflection based on two types of forward models, namely, dynamic single-degree-of-freedom (SDOF) and dynamic half-space models. It can be seen from the numerical examples that the accuracy of evaluating elastic moduli based on dynamic backcalculation analyses is significantly depended on the forward dynamic soil models employed in the backcalculation process. The value of predicted backcalculation elastic moduli based on the dynamic SDOF model is generally lower than those based on dynamic half-space model. The discrepancy is due to the fact that deflection from the dynamic half-space model is calculated locally based on the continuum theory and therefore requires higher stiffness than those of the dynamic SDOF model in order to generate identical peak deflection. The study in this paper and further investigation are useful for the development of dynamic backcalculation program and its applications.

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