

The Influence of Biased Signal Set on Stock Market—Based on the Shanghai Composite Index

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Abstract. Based on the two phenomena of over and underreaction, this paper introduces the characteristics of signal set into the model, which studies the price deviation of financial market caused by investors' signal set deviation. The text concludes that investors overreact to information of low weight and underreact to information of high weight, and uses cross-section analysis and time series analysis to verify the correctness of the results.

1. Introduction

Traditional economics researches in the field of classical theories believe that, investors are absolutely rational and markets tend to be efficient. However, with the continuous development of economic theories, more and more "market anomalies" have been discovered, among which the overreaction and underreaction of investors have become the most concerned direction.

It is generally believed in the academic circles that these two phenomena are due to the bias of investors' reaction to market information. In different situations, investors will show a specific reaction. This reaction would disturb the price of the stock in the financial market, and help to predict the future return rate of the stock. Therefore, our research has important value.

Investors usually have different forms of reaction when facing positive and negative market signals. Nam (2006) established an asymmetric model and used empirical data to prove that investors tend to underreact when facing positive returns and overreact when facing negative returns.^[1] Domestic scholars have also drawn similar conclusions. Cai Wenxin (2018) found that investors have different performances in the face of good news and bad news, and investors tend to react less to good news than bad news.^[2]

Modern behavioral finance tends to deal with market anomalies by assuming that investors are irrational. In such cases, investors tend to be influenced by many aspects, including their own sentiment and the external environment. Bagizzi (2000) uses representativeness bias and conservatism bias as the measurement standards. The more confident investors are, they will be overly optimistic and ignore the updated market information brought by such a situation.

In this paper, based on bounded rationality, the characteristics of the signal set, which investors will receive, is defined. The differences between signal set take the place of investors' irrationality of processing information, at the same time, by using the data from China's Shanghai composite for validation, the theory is justified of practical significance.

2. Model

In order to simplify the model, the processing process of the signal set is divided into two periods: $t_0 - t_1$, $t_1 - t_2$. In t_0 , investors can buy investment funds through the stock market and obtain the uncertain portfolio return R_m in t_2 , or buy national bonds to obtain the risk-free return R_f in t_2 . In t_1 , the investor gets a set of signals about the final return of the portfolio, and these signals will be reflected in the price during the period $t_1 - t_2$.^[3] Investors are risk neutral and have homogeneous beliefs. They are rational, and expect to maximize utility based on what they can expect. If the investors can obtain the complete information of the signal set, their beliefs be updated through the Bayesian updating; Otherwise, investors will update the expected price with their own beliefs.

2.1. Signal Set in Financial Markets

In most of the current studies, portfolio returns are assumed to be β distribution.

For random variables $X \sim \beta(n_1, n_2)$, in general, follow as:

$$E(x) = \frac{n_1}{n_1+n_2} \text{ and } \text{Var} = \frac{n_1*n_2}{(n_1+n_2)^2(n_1+n_2+1)} \quad (1)$$

In this specific model, the final price P_{M2} at time t_2 follows β distribution:

$$P_{M2} \sim 0.5 + \beta(n_M, n_M)$$

On the basis of the initial signal set, the first-order moment and second-order moment of P_{M2} are as:

$$E(P_{M2}) = 1 \quad \text{Var}_0(P_{M2}) = \frac{1}{4(2n_M+1)} \quad (2)$$

n_m determines the variance of the initial symmetric probability distribution for P_{M2} . Under this assumption, the beta distribution of P_{M2} is similar to the normal distribution of the stock market. The price of the portfolio in t_0 is:

$$P_{M0} = \frac{1}{R_{F1}R_{F2}} \quad (3)$$

Where, R_{F1} and R_{F2} are the total risk-free rate of return from t_0 to t_1 and from t_1 to t_2 with $R_{F1} = r_{f1} + 1$ and $R_{F2} = r_{f2} + 1$.

During periods t_0 to t_1 , investors receive signal set that has two components: positive and negative. A positive signal indicates that the price of investors' portfolio will rise, bringing positive returns. A negative signal indicates that the price of investors' portfolio will fall, resulting in a loss. In the signal set received by investors, it is composed of a_m positive signal and b_m negative signal. When the price of P_{M2} is 1.5, all signals in the signal set are positive signals. When the price of P_{M2} is 0.5, all the signals are negative signals, and the reduction process is linear. Griffin and Tversky (1992) suggest that one new signal set mainly consist of strength and weight. They define strength as $S_M = a_m / (a_m + b_m)$ and weight as the total number of received signals, $w_m = a_m + b_m$.^[4]

When people update price information, they usually use the standard Bayesian updating formula:

$$\Pr(P_{M2}|S_M) = \frac{\Pr(S_M|P_{M2})\Pr(P_{M2})}{\Pr(S_M)} \quad (4)$$

From the above hypothesis, it can be deduced that on the basis of the signal set with S_m strength and w_m weight, the price of the market portfolio follows the following β distribution:

$$P_{m2} \sim 0.5 + \beta(n_m + w_m S_m, n_m + w_m(1 - S_m)) \quad (5)$$

At time t_1 , the expectation of P_{M2} is as:

$$E(P_{M2}) = 1 + \frac{S_M - 0.5}{1 + 2n_m/w_m} \quad (6)$$

In the above formula, we can get an intuitive feeling that when the intensity of the signal set S_m is greater than 0.5, the price of the portfolio in t_2 will rise, bringing positive returns to investors. When the weight of signal set obtained by investors is larger (w_m is bigger), the initial uncertainty of portfolio price is higher (n_m is smaller), and the expected portfolio price will increase.

2.2. Influence of Cognitive Bias on Earnings

In the process of the model, P_{M0} will not be affected by the signal of information concentration, because the period $t_0 - t_1$ is the process of signal collection by investors, and it will not involve information processing. But in t_1 investors' information collecting is often biased. They are often unable to collect all the information on the market, so in the process of processing information will use perceived weight level \widetilde{w}_m . Market price P_{M1} and the market rate of return do not reflect the actual weight.

$$P_{M1} = \left(1 + \frac{S_M^{-0.5}}{1+2n_m/\bar{w}_m}\right) R_{F2}^{-1} \quad (7)$$

$$r_{M1} = \left(1 + \frac{S_M^{-0.5}}{1+\frac{2n_m}{\bar{w}_m}}\right) R_{F1} - 1 \quad (8)$$

In the case perceived weight level total is different from the actual total as $w_m \neq \bar{w}_m$. The price updating process will not follow the standard Bayesian update formula, and the price of P_{M1} will be different from its standard price. According to the results of Griffin and Tversky (1992), there is a certain relationship between the perceived weight and the actual weight. Investors respond to different signal weights with some investors overreacting and some investors underreacting. That indicates that investors use relatively average weight to estimate in their actual judgment. Such an assumption would not lead to a popularity of overreaction and underreaction.

In the process of building the model, the paper assumes that overreaction and underreaction occur at the same frequency in the same time, and takes the average as he average weight. We regard the perceived weight as the combination of the actual weight and the average weight:

$$\bar{w}_m = \alpha \bar{w}_m + (1 + \alpha) w_m \quad (9)$$

Where α is the parameter, $\alpha \in (0,1)$. Under this assumption, the excess returns of t_1 to t_2 portfolios can be deduced as:

$$E(r_{M2} - r_{F2}) = (r_{M1} - r_{F1})(\bar{w}_M - \bar{w}_M) \frac{\alpha}{1-\alpha} \frac{2R_{F2}n_M}{\bar{w}_M(w_M+2n_M)(r_{M1}+1)} \quad (10)$$

In the above formula, when the last two terms of excess return are positive and the perceived weight level is not equal to the average weight level, the excess return from t_1 to t_2 can be predicted with the excess return from t_0 to t_1 . It also indicates that when the perceived weight level is different from the average weight level, the subsequent earnings will continuously increase or reverse. For low information weight ($w_m < \bar{w}_m$), $E(r_{M2} - r_{F2})$ and $(r_{M1} - r_{F1})$ have opposite sign. Subsequent returns will be reversed; For high information weight ($w_m > \bar{w}_m$), $E(r_{M2} - r_{F2})$ has identical sign as $(r_{M1} - r_{F1})$. Subsequent gains will persist.

2.3. Verification Principle of Information Weight

In order to explore the impact of perceived weight on portfolio prices, it is necessary to calculate the actual weight. The actual weight is reflected in investors' variance expectations. In the model, investors form their own cognition after acquiring the signal set. The variance is as follows:

$$\text{Var}_1(P_{M2}) = \frac{(\bar{w}_M S_M + n_M)(\bar{w}_M(1-S_M) + n_M)}{(\bar{w}_M + 2n_M)^2(\bar{w}_M + 2n_M + 1)} \quad (11)$$

This formula reflects the relationship between the perceived weight level and the variance of the portfolio. We can observe that if the signal set contains more signals and has a higher weight, the uncertainty of future investment portfolio can be reduced and the price fluctuation can be reduced, which is consistent with the reality. The expression of the perceived weight can be obtained through Formula (2) and (11):

$$\bar{w}_M = R_{F1}^2 R_{F2}^2 \left(\frac{\left(0.5 + \frac{r_{M1} - r_{F1}}{R_{F1}}\right) \left(0.5 - \frac{r_{M1} - r_{F1}}{R_{F1}}\right)}{(1+r_{M1})^2 \text{Var}_1(r_{M2})} - \frac{1}{4\text{Var}_0(r_{M0,2})} \right) \quad (12)$$

Since all the parameters on the right side of the equation can be obtained, it is possible to calculate the perceived weight and use it for empirical analysis.

3. Empirical Research

3.1. Data

Before estimating the perceived weight, it is necessary to estimate the forward-looking variance. Since option is one of the important choices of investors and the variance of future return rate is an

important determinant of option price, this paper uses the variance of the observable option price in the option to calculate the investor's expected variance.

In terms of data selection, the paper selects the SSE Composite Index as the market portfolio. Meanwhile, in order to calculate the perceived weight and measure the uncertainty of market returns as much as possible, paper collects the data of the Shanghai Composite Index 50 ETF since February 9, 2015 to December 31, 2020. The risk-free rate r_f is replaced by the one-year Treasury rate

In terms of time range, the model applies the time of information collection $t_0 - t_1$ to one day. Based on past experience, paper assumes that when investors do not get the complete signal set, they will update their price belief according to the biased information weight, which leads to a market mispricing. After t_1 , investors gradually get all information, market gradually correct the price, and the entire process is not expected to exceed 30 days. During the period $t_1 - t_2$, more and more repeated information is collected by investors, making it difficult to reliably measure subsequent expected returns. The model pays attention to the returns of the next day and the following week after t_1 to test the predictability of returns.

Table 1. Descriptive Analysis.

	r	$\widetilde{w}_M - \overline{w}_M$	$r_m - r_f$	r_f	MOM	REV	MV
mean	0.0036	-0.0053	-0.0285	0.0287	-0.0004	0.0003	11.1462
max	0.0576	25.3202	0.0389	0.0425	0.2576	0.1657	12.9800
min	-0.0849	-2.4758	-0.1152	0.0144	-0.1465	-0.1714	10.0293

Table 1 shows the descriptive analysis about the variable table, from 2015 to 2020. r denotes the market portfolio return, $\widetilde{w}_M - \overline{w}_M$ denotes total information error, the r_f denotes risk-free interest rate, the $r_m - r_f$ denotes market portfolio excess yields, MOM,REV and MV denote the momentum effect, reversal effect, and the portfolio's market capitalization.

3.2. Empirical Analysis

Previous studies have shown that when a single stock has a price shock and releases a profit announcement, it tends to produce overreaction and underreaction. In this model, the perceived weight is low often when price shake, and the perceived weight is large when profit announcement occurs. In addition, paper uses the perceived information weight to forecast the future portfolio returns.

3.2.1. Cross-section Analysis

Since there is no option based on individual stocks in the Chinese market, it is impossible to discuss individual stocks, so the research is limited to the broad market. This paper divides the data into two cases: increase and decrease, and discuss the phenomena of underreaction and overreaction in these cases.

In empirical research, there are 656 revenue-increasing dates and 561 revenue-decreasing dates. In Table 2, the return rate on 561 return decreasing dates is arranged according to the size, and its quantile is divided into five levels to track the return in the following day and week.

Table 2. Revenue decrease.

	t+1	t+2	t+3	t+4	t+5	week
Low	-0.028	-0.013	-0.029	-0.043	-0.037	-0.150
2	-0.049	-0.026	-0.030	-0.053	-0.029	-0.187
3	-0.046	-0.040	-0.034	-0.023	-0.012	-0.155
4	-0.034	-0.015	-0.022	-0.031	-0.010	-0.112
high	-0.035	-0.011	-0.010	-0.029	-0.018	-0.104

The table shows that in the case of a reduction of income, the price impact of the worst situation than the price impact the lightest weekly revenue by 4.6%.

Table 3. only observe the low weight.

	t+1	t+2	t+3	t+4	t+5	week
Low	-0.029	-0.033	-0.024	-0.029	-0.036	-0.151
2	-0.049	-0.026	-0.030	-0.053	-0.029	-0.187
3	-0.046	-0.040	-0.034	-0.023	-0.012	-0.155
4	-0.022	-0.031	-0.018	-0.029	-0.015	-0.115
high	-0.283	2.525	-0.871	-1.059	-0.402	-0.089

Table 3 shows the returns of only observing low weight under the same circumstances. The results show that if the sample is limited to low weight, the yield difference will expand to 6.2%.

For sustained earnings, investors often react inadequately and fail to consider the high reliability of earnings information. The paper ranks 656 revenue-increasing dates and selects their quartiles. The revenue over the next day and week is tracked, and shown in Table 4.

Table 4. Revenue increase.

	t+1	t+2	t+3	t+4	t+5	week
Low	-0.402	-1.404	0.673	-0.027	0.854	-0.306
2	0.715	-6.147	1.226	-3.422	-4.272	-11.901
3	-0.027	0.854	-0.077	1.164	0.332	2.247
4	-0.081	-1.382	-0.780	1.642	-0.340	-0.942
high	4.542	2.388	-1.156	-3.027	0.459	3.206

Due to the positive news of rising prices, this chart shows that in the event of a loss in returns, the difference in weekly returns between the most positive and the most negative cases is 3%.

Table 5. only observe the situation of high weight.

	t+1	t+2	t+3	t+4	t+5	week
Low	-0.402	-1.404	0.673	-0.027	0.854	-0.306
2	1.226	-4.272	-8.491	-7.631	-1.271	-20.439
3	0.854	1.164	0.332	1.150	-0.012	3.489
4	-0.780	1.642	1.422	0.513	-0.138	2.658
high	4.542	2.388	-3.027	0.459	3.510	7.871

Table 5 shows the returns of observing the high weight in the process of subsequent selection under the same circumstances. It found that limiting the sample to high weight widened the yield spread to 7% with an increase of 4%.

3.2.2. Time Series Analysis

After the cross-section analysis, in order to prove the predictability of the model for short-term returns, the paper uses the return of the portfolio on next day as the explained variable, the difference of weight and excess return as the explanatory variables, and momentum effect, reversal effect and the logarithm of index trading volume as the control variable.

Table 6. Regression results.

	1	2	3	4	
C	0.00012 (0.00044)	0.00009 (0.00043)	0.01403*** (0.00078)	0.01406*** (0.00079)	0.04082*** (0.00779)
$(r_{M1} - r_{F1})(\widetilde{w}_M - \overline{w}_M)$	0.03722*** (0.00417)	0.06659*** (0.00586)	0.03996*** (0.00508)	0.03985*** (0.00510)	0.03923*** (0.00507)
$(\widetilde{w}_M - \overline{w}_M)$		0.00184*** (0.00026)	0.00087*** (0.00023)	0.00086*** (0.00023)	0.00090*** (0.00023)
$(r_{M1} - r_{F1})$			0.47675*** (0.02370)	0.47732*** (0.02375)	0.49024*** (0.02390)
MOM				0.00164 (0.00986)	0.00256 (0.0098)
rev				-0.00307 (0.00617)	0.00348*** (0.00642)
MV					-0.00237*** (0.00069)

**** means P value is less than 0.01, *** means P value is less than 0.05; ** indicates a p-value less than 0.1

The regression model shows that the interaction between the difference of weight and excess return has obvious positive effect on portfolio return. Meanwhile, the difference of weight and excess return also have a significant impact on the portfolio. With gradually adding control variables, the results show that the cross section of portfolio return increases, in which excess return contributes the most, while momentum effect and reversal effect have no significant influence.

4. Conclusion

This paper uses the difference of the signal set to replace the irrationality of investors and introduces the difference into financial market. It is concluded that for high weight, information is underestimated, which means the response is insufficient. If the weight is low, people will overreact by assuming that the signal set is more reliable than it really is. Then the paper verifies the predictability of the returns.

The results indicate that the biased treatment of information weight not only affects investors' beliefs, but also has a great impact on market prices.

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