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Dynamic Upper Approximation Fuzzy Information Rough Communication Model

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Abstract. According to practical situations that any potentially useful information cannot be lost and the communicated concept is a dynamic fuzzy set, using the two-direction S-rough fuzzy set with dynamic characteristics, a dynamic fuzzy rough communication model is constructed. The concept of dynamic fuzzy information communication quantity is defined by using the cardinality of a fuzzy set, and the condition of information fidelity in the process of information communication is obtained. Dynamic fuzzy rough communication is a new application of two-direction S-rough fuzzy set theory.

Introduction

Using Z.Pawlak rough sets theory[1], Mousavi put forward information rough communication[2] of a classical concept between multiple agents for the first time. Ref.[3] discussed the properties of this kind of information rough communication. But in ref.[1,3], the communicated concept is a static classical set on the universe U. However, in real life, we often meet the communicated concept is a fuzzy set, that is, the communicated concept $\tilde{X}$ is a fuzzy set on the universe U. Information rough communication of a fuzzy concept is called fuzzy information rough communication[4,5]. Ref. [4,5] discussed this kind of fuzzy information rough communication. In [4,5], the disscussed fuzzy set is ststic. However in reality, the communicated fuzzy concept $\tilde{X}$ is often changed dynamically. For example, for $x \in U$, $\tilde{X}(x) = a$, where a is the membership degree of x to the fuzzy set $\tilde{X}$, however, with the development and change of the system, the membership degree of x to the fuzzy set $\tilde{X}$ is changed into b from a, where $a \neq b$, and $a, b \in [0,1]$. Such fuzzy rough communication that the communicated fuzzy concept $\tilde{X}$ is dynamic is called dynamic fuzzy rough communication. Therefore, the dynamic fuzzy rough communication is more in line with the actual situation.

In practical information communication, we often meet the following situation: in order to avoid losses, we cannot lose any potentially useful information in the process of dynamic fuzzy information communication. Such dynamic fuzzy information communication has not been discussed in the previous references. Therefore, it is natural for us to ask the following question: can we construct a dynamic fuzzy rough communication model in this case? How to construct? If the model can be contructed, what are the characteristics of the model?

We know that rough fuzzy sets[6,7] proposed by Dubois and Prade as a theoretical tool to discuss these problems are not appropriate, because the rough fuzzy sets are static, they are no longer suitable for the dynamic fuzzy problems. Thus, dynamic rough fuzzy sets, namely two-direction S-rough fuzzy sets were given in ref.[8,9]. In the paper we use two-direction S-rough fuzzy sets to discuss the above questions, and construct a new dynamic fuzzy rough communication model.

To accept this paper easily, first we will introduce the concepts of two-direction S- rough fuzzy sets.

Two-direction S-rough Fuzzy Sets

Definition 2.1 Let $\mathcal{F}(U)$ be a fuzzy power set on the universe U, $U=\{u_1,u_2,\cdots,u_l\}$ be a finite universe. $\tilde{X}\in\mathcal{F}(U)$ and $\tilde{X}=\{a_1/u_1,a_2/u_2,\quad\cdots,a_l/u_l\}$, where $a_i\in[0,1]$, $i=1,2,\cdots,l$. Call $F=\{f_1,f_2,\cdots,f_m\}$ and $\overline{F}=\{\overline{f_1},\overline{f_2},\cdots,\overline{f_n}\}$ the fuzzy elementary transfer families on U, if $f_s\in F$ and $\overline{f}_t\in\overline{F}$ satisfy: $\exists u_i\in U$, $\tilde{X}(u_i)=a_i\Rightarrow f_s(\tilde{X}(u_i))=b_i$, where $b_i>a_i$ and $b_i\in[0,1]$. And $\exists u_j\in U$, $\tilde{X}(u_j)=a_j\Rightarrow\overline{f}_t(\tilde{X}(u_j))=c_j$, where $c_j<a_j$ and $c_j\in[0,1]$. Call $f_i\in F$ and $\overline{f}_j\in\overline{F}$ ($i=1,2,\cdots,m$, $j=1,2,\cdots,n$) fuzzy elementary transfer [10, 11, 12, 13].

Definition 2.2[8, 9] if

$$\tilde{X}'=\tilde{X}-\{[1-\overline{f}(\tilde{X}(u))]/u\mid u\in U,\overline{f}(\tilde{X}(u))=c<\tilde{X}(u)\},\tag{1}$$

$$\tilde{X}^*=\tilde{X}'\bigcup\{f(\tilde{X}(u))/u\mid u\in U,f(\tilde{X}(u))=b>\tilde{X}(u)\}.\tag{2}$$

Call $\tilde{X}'$ the fuzzy loss set of $\tilde{X}$. Call $\tilde{X}^*$ two-direction S-fuzzy set of $\tilde{X}$, where $f\in F$ and $\overline{f}\in\overline{F}$.

Obviously, the two-direction S- fuzzy set $\tilde{X}^*$ is a dynamic fuzzy set.

Definition 2.3[8, 9] Let $K=(U,R)$ be a knowledge base (or a Pawlak approximation space). The fuzzy lower and upper approximation of two-direction S-fuzzy set $\tilde{X}^*$ in (U,R), denoted by $(R,\mathcal{F})_o(\tilde{X}^*)$ and $(R,\mathcal{F})^o(\tilde{X}^*)$, are defined as fuzzy sets on U, and their membership functions are given as follows:

$$(R,\mathcal{F})_o(\tilde{X}^*)\ (x)=\inf\{\tilde{X}^*(y)\mid y\in[x]_R\},\ x\in U,\tag{3}$$

$$(R,\mathcal{F})^o(\tilde{X}^*)\ (x)=\sup\{\tilde{X}^*(y)\mid y\in[x]_R\},\ x\in U.\tag{4}$$

Where $\mathcal{F}=F\bigcup\overline{F}$, $F\neq\phi$ and $\overline{F}\neq\phi$.

Definition 2.4[8, 9] The ordered pair $((R,\mathcal{F})_o(\tilde{X}^*),(R,\mathcal{F})^o(\tilde{X}^*))$ is called two-direction S-rough fuzzy sets of $\tilde{X}^*$ in (U,R).

Definition 2.5[8, 9] Call $(R,\mathcal{F})_o(\tilde{X}^*)$ the positive region of $\tilde{X}^*$ in (U,R), denoted by $pos_{(R,\mathcal{F})}(\tilde{X}^*)$, that is, $pos_{(R,\mathcal{F})}(\tilde{X}^*)=(R,\mathcal{F})_o(\tilde{X}^*)$. Call $\sim(R,\mathcal{F})^o(\tilde{X}^*)$ the negative region of $\tilde{X}^*$ in (U,R), denoted by $neg_{(R,\mathcal{F})}(\tilde{X}^*)$, that is, $neg_{(R,\mathcal{F})}(\tilde{X}^*)=\sim(R,\mathcal{F})^o(\tilde{X}^*)$.

Definition 2.6 The ordered fuzzy sets pair ($pos_{(R,\mathcal{F})}(\tilde{X}^*)$, $neg_{(R,\mathcal{F})}(\tilde{X}^*)$) is called an agent's two-direction S-dynamic fuzzy cognition of $\tilde{X}^*$ with the knowledge R.

Definition 2.7 Let ($\tilde{A}_1,\tilde{A}_2$) and ($\tilde{B}_1,\tilde{B}_2$) be two fuzzy sets pair. If $\tilde{A}_1\subseteq\tilde{B}_1$, $\tilde{A}_2\subseteq\tilde{B}_2$, call $(\tilde{A}_1,\tilde{A}_2)\subseteq(\tilde{B}_1,\tilde{B}_2)$. If $\tilde{A}_1=\tilde{B}_1$, $\tilde{A}_2=\tilde{B}_2$, call $(\tilde{A}_1,\tilde{A}_2)=(\tilde{B}_1,\tilde{B}_2)$.

Dynamic Upper Approximation Fuzzy Information Rough Communication Model

Let (U,R_1), (U,R_2), $\cdots$, (U,R_n) be n knowledge bases. Let A_1, A_2, $\cdots$, A_n be n agents from $(U,R_1),(U,R_2),\cdots,(U,R_n)$, that is, they have the knowledge $R_1,R_2,\cdots,R_n$, respectively. Suppose the dynamic fuzzy concept $\tilde{X}^*$ is communicated between the n agents, the fuzzy sets pair

($pos_{(R,\mathrm{F})}(\tilde{X}^*)$, $neg_{(R,\mathrm{F})}(\tilde{X}^*)$) is the agent A_1's two-direction S-dynamic fuzzy cognition of $\tilde{X}^*$ with his knowledge R_1. In order to write conveniently, let $D\tilde{B}_1^+ = pos_{(R_1,\mathrm{F})}(\tilde{X}^*)$ and $D\tilde{B}_1^- = neg_{(R_1,\mathrm{F})}(\tilde{X}^*)$.

In practical information communication, we often meet the following situation: in order to avoid losses, we cannot lose any potentially useful information.. For example, in the risk investment system, we would like consider any risk factors affecting the success of the investment. According to this situation, in this paper, a new dynamic fuzzy rough communication model is established by using the upper approximation operator in the two-direction S- rough fuzzy sets.

Definition 3.1 let (U,R_1), (U,R_2), $\cdots,(U,R_n)$ be n knowledge bases, R_1, R_2, $\cdots,R_n$ be the knowledge of the agents A_1, $A_2,\cdots,A_n$, respectively. Two-direction S-fuzzy set $\tilde{X}^* \in \mathrm{F}(U)$, Call

$$(D\tilde{B}_1^+, D\tilde{B}_1^-) = (pos_{(R_1,\mathrm{F})}(\tilde{X}^*), neg_{(R_1,\mathrm{F})}(\tilde{X}^*)), \tag{5}$$

$$(D\tilde{B}_i^+, D\tilde{B}_i^-) = \overline{apr}_{(R_i,\mathrm{F})}(D\tilde{B}_{i-1}^+, D\tilde{B}_{i-1}^-) = ((R_i,\mathrm{F})^o(D\tilde{B}_{i-1}^+), (R_i,\mathrm{F})^o(D\tilde{B}_{i-1}^-)),\ i = 2,3,\cdots,n \tag{6}$$

dynamic upper approximation fuzzy information rough communication of the two-direction S-fuzzy set $\tilde{X}^*$ along the information flow path $\tilde{X}^* \to A_1 \to A_2 \to \cdots \to A_n$, briefly called dynamic upper approximation fuzzy rough communication.

Similarly, we can define dynamic upper approximation fuzzy rough communication of $\tilde{X}^*$ along other information flow paths.

Definition 3.2 Let

$$|(D\tilde{B}_i^+, D\tilde{B}_i^-)| = |\overline{apr}_{(R_i,\mathrm{F})}(D\tilde{B}_{i-1}^+, D\tilde{B}_{i-1}^-)| = |(R_i,\mathrm{F})^o(D\tilde{B}_{i-1}^+)| + |(R_i,\mathrm{F})^o(D\tilde{B}_{i-1}^-)|. \tag{7}$$

Call $|(D\tilde{B}_i^+, D\tilde{B}_i^-)|$ dynamic upper approximation fuzzy rough communication information quantity of $\tilde{X}^*$ from the agent A_{i-1} to A_i, where $|(R_i,\mathrm{F})^o(D\tilde{B}_{i-1}^+)| = \sum_{x\in U}(R_i,\mathrm{F})^o(D\tilde{B}_{i-1}^+)(x)$ is the cardinality of the fuzzy set $(R_i,\mathrm{F})^o(D\tilde{B}_{i-1}^+)$, and $(R_i,\mathrm{F})^o(D\tilde{B}_{i-1}^+)(x)$ is the membership degree of $x \in U$ to the fuzzy set $(R_i,\mathrm{F})^o(D\tilde{B}_{i-1}^+)$.

From Definition 2.3, we can get the meaning of the operator $\overline{apr}_{(R_i,\mathrm{F})}(D\tilde{B}_{i-1}^+, D\tilde{B}_{i-1}^-)$: In order to avoid losing any potentially useful information, A_i accepts information which certainly and possibly included in $(D\tilde{B}_{i-1}^+, D\tilde{B}_{i-1}^-)$ according to his knowledge R_i.

Theorem 3.1 Let (U,R_1), (U,R_2), $\cdots,(U,R_n)$ be n knowledge bases, R_1, R_2, $\cdots,R_n$ be the knowledge of the agents A_1, $A_2,\cdots,A_n$, respectively. Two-direction S-fuzzy set $\tilde{X}^* \in \mathrm{F}(U)$, and the information flow path is $\tilde{X}^* \to A_1 \to A_2 \to \cdots \to A_n$. If $U/ind(R_1) \supset U/ind(R_2) \supset \cdots \supset U/ind(R_n)$, then the communicated information remains unchanged in the process of information communication , that is,

(1) $$(D\tilde{B}_1^+, D\tilde{B}_1^-) = (D\tilde{B}_2^+, D\tilde{B}_2^-) = \ldots = (D\tilde{B}_n^+, D\tilde{B}_n^-), \tag{8}$$

(2) $$\left|(D\tilde{B}_1^+, D\tilde{B}_1^-)\right| = \left|(D\tilde{B}_2^+, D\tilde{B}_2^-)\right| = \cdots = \left|(D\tilde{B}_n^+, D\tilde{B}_n^-)\right|. \tag{9}$$

Proof: (1) Because $U/ind(R_1) \supset U/ind(R_2) \supset \cdots \supset U/ind(R_n)$, then $[x]_{R_i} \subset [x]_{R_{i-1}}$. If $y \in [x]_{R_i}$, then $y \in [x]_{R_{i-1}}$, and $[x]_{R_{i-1}} = [y]_{R_{i-1}}, i = 2,3,\cdots,n$. Therefore, for any $x \in U$, we have

$$(D\widetilde{B}_2^+)(x)=(R_2,\mathsf{F})^o(D\widetilde{B}_1^+)\ (x)=\sup\{D\widetilde{B}_1^+(y)\mid y\in[x]_{R_2}\}=\sup\{(R_1,\mathsf{F})_o(\widetilde{X}^*)(y)\mid y\in[x]_{R_2}\}$$

$$=\sup\{\inf\{\widetilde{X}^*(z)\mid z\in[y]_{R_1}\}\mid y\in[x]_{R_2}\}=\sup\{\inf\{\widetilde{X}^*(z)\mid z\in[x]_{R_1}\}\mid y\in[x]_{R_2}\}. \quad (10)$$

Since $[x]_{R_2}\subset[x]_{R_1}$, then

$$(D\widetilde{B}_2^+)(x)=\inf\{\widetilde{X}^*(z)\mid z\in[x]_{R_1}\}=D\widetilde{B}_1^+(x). \quad (11)$$

Therefore $D\widetilde{B}_2^+=D\widetilde{B}_1^+$. Moreover,

$$(D\widetilde{B}_2^-)(x)=(R_2,\mathsf{F})^o(D\widetilde{B}_1^-)\ (x)=\sup\{D\widetilde{B}_1^-(y)\mid y\in[x]_{R_2}\}=\sup\{\sim(R_1,\mathsf{F})^o(\widetilde{X}^*)(y)\mid y\in[x]_{R_2}\}$$

$$=\sup\{1-(R_1,\mathsf{F})^o(\widetilde{X}^*)(y)\mid y\in[x]_{R_2}\}=\sup\{1-\sup\{\widetilde{X}^*(z)\mid z\in[y]_{R_1}\}\mid y\in[x]_{R_2}\}$$

$$=1-\inf\{\sup\{\widetilde{X}^*(z)\mid z\in[x]_{R_1}\}\mid y\in[x]_{R_2}\}=1-\sup\{\widetilde{X}^*(z)\mid z\in[x]_{R_1}\}$$

$$=1-(R_1,\mathsf{F})^o(\widetilde{X}^*)(x)=\sim(R_1,\mathsf{F})^o(\widetilde{X}^*)(x)=D\widetilde{B}_1^-(x). \quad (12)$$

Therefore $D\widetilde{B}_2^-=D\widetilde{B}_1^-$. From Definition 2.7, we can get $(D\widetilde{B}_1^+,D\widetilde{B}_1^-)=(D\widetilde{B}_2^+,D\widetilde{B}_2^-)$.

Here we prove $(D\widetilde{B}_2^+,D\widetilde{B}_2^-)=(D\widetilde{B}_3^+,D\widetilde{B}_3^-)=\cdots=(D\widetilde{B}_n^+,D\widetilde{B}_n^-)$

If $i\ge 3$, then for any $x\in U$, we have

$$(D\widetilde{B}_i^+)(x)=(R_i,\mathsf{F})^o(D\widetilde{B}_{i-1}^+)\ (x)=\sup\{D\widetilde{B}_{i-1}^+(y)\mid y\in[x]_{R_i}\}=\sup\{(R_{i-1},\mathsf{F})^o(D\widetilde{B}_{i-2}^+)(y)\mid y\in[x]_{R_i}\}$$

$$=\sup\{\sup\{D\widetilde{B}_{i-2}^+(z)\mid z\in[y]_{R_{i-1}}\}\mid y\in[x]_{R_i}\}=\sup\{\sup\{D\widetilde{B}_{i-2}^+(z)\mid z\in[x]_{R_{i-1}}\}\mid y\in[x]_{R_i}\}. \quad (13)$$

Since $[x]_{R_i}\subset[x]_{R_{i-1}}$, then

$$(D\widetilde{B}_i^+)(x)=\sup\{D\widetilde{B}_{i-2}^+(z)\mid z\in[x]_{R_{i-1}}\}=(R_{i-1},\mathsf{F})^o(D\widetilde{B}_{i-2}^+)\ (x)=(D\widetilde{B}_{i-1}^+)(x),\ i\ge 3. \quad (14)$$

Therefore, $D\widetilde{B}_2^+=D\widetilde{B}_3^+=\cdots=D\widetilde{B}_n^+$.

Similarly we can prove $D\widetilde{B}_2^-=D\widetilde{B}_3^-=\cdots=D\widetilde{B}_n^-$. Then

$$(D\widetilde{B}_2^+,D\widetilde{B}_2^-)=(D\widetilde{B}_3^+,D\widetilde{B}_3^-)=\ldots=(D\widetilde{B}_n^+,D\widetilde{B}_n^-). \quad (15)$$

In conclusion, we have

$$(D\widetilde{B}_1^+,D\widetilde{B}_1^-)=(D\widetilde{B}_2^+,D\widetilde{B}_2^-)=\ldots=(D\widetilde{B}_n^+,D\widetilde{B}_n^-). \quad (16)$$

(2) From (1) and Definition 3.2, we can obtain $\left|(D\widetilde{B}_1^+,D\widetilde{B}_1^-)\right|=\left|(D\widetilde{B}_2^+,D\widetilde{B}_2^-)\right|=\cdots=\left|(D\widetilde{B}_n^+,D\widetilde{B}_n^-)\right|$ easily.

Theorem 3.2 Let (U,R_1), (U,R_2), $\cdots,(U,R_n)$ be n knowledge bases, R_1, R_2, $\cdots,R_n$ be the knowledge of the agents A_1, $A_2,\cdots,A_n$, respectively. Two-direction S-fuzzy set $\widetilde{X}^*\in\mathsf{F}(U)$, and the information flow path is $\widetilde{X}^*\to A_1\to A_2\to\cdots\to A_n$. If they have the same knowledge, that is, $U/ind(R_1)=U/ind(R_2)=\cdots=U/ind(R_n)$, then the communicated information remains unchanged in the process of information communication , that is,

(1) $$(D\widetilde{B}_1^+,D\widetilde{B}_1^-)=(D\widetilde{B}_2^+,D\widetilde{B}_2^-)=\ldots=(D\widetilde{B}_n^+,D\widetilde{B}_n^-), \quad (17)$$

(2) $$\left|(D\widetilde{B}_1^+,D\widetilde{B}_1^-)\right|=\left|(D\widetilde{B}_2^+,D\widetilde{B}_2^-)\right|=\ldots=\left|(D\widetilde{B}_n^+,D\widetilde{B}_n^-)\right|. \quad (18)$$

The proof is similar to Theorem 3.1, omitted.

Theorem 3.1 and Theorem 3.2 show that the knowledge of agents satisfies the conditions in which the information will not be lost and increased, and the information will not be distorted. In this case the information communication is reliable

Summary

Because people's knowledge of many things in reality is incomplete and imprecise, and it's difficult to know them exactly, this results in the roughness of information communication . According to the two actual situations of information communication: on the one hand, we cannot lose any potentially useful information, on the other hand, the concept of being communicated in reality is a fuzzy set and dynamic, in the paper, a new dynamic fuzzy rough communication model is constructed by using the two-direction S-rough fuzzy sets with dynamic characteristics. The concept of dynamic fuzzy information communication quantity is defined, and the condition of information fidelity in the process of information communication is obtained. Dynamic fuzzy rough communication is a new application of S- rough fuzzy sets. Considering the limitations of the length, as for the properties and application of the model, these will be our next task.

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