Mathematical Study on the Expectation Factors of the Deviation between
Internal Price and External Price of Non-International Currency

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Abstract. Based on the theoretical model of money in the utility function with micro-foundation, by means of the method of mathematical analysis, the expectation factors of the deviation between internal price and external price of non-international currency are analyzed, the clear interval of exchange rate expectation and price expectation which lead to the deviation between internal price and external price of non-international currency are calculated. This study finds that: without considering the stochastic impact, while the expected exchange rate is less than or equal to the current exchange rate, there will be no deviation between internal price and external price of non-international currency, regardless of the size of the expected price; while the expected exchange rate is greater than the current exchange rate, and the price expectation keeps in a certain range at the same time, there will be deviation between internal price and external price of non-international currency. On this basis, some new ideas about monetary policy and reform of exchange rate are put forward.

1 Introduction

Although the RMB has officially joined the SDR on October 1, 2016, there is still a long way to go for the full internationalization of the RMB. In the process of RMB internationalization, the divergence between the internal price and external price of RMB is an important factor. Specifically, the deviation between the internal and external prices of RMB may hinder the progress of RMB internationalization, aggravate the “Triffin Dilemma” of RMB internationalization, and shake the internationalization status of RMB (Tao and Liu, 2014[1]). According to Purchasing Power Parity (PPP), which was first explicitly put forward by Cassel (1918) [2], the exchange rate between local currency and foreign currency expressed by direct price method is generally close to or equal to the ratio of domestic general price level to foreign general price level. That is to say, the exchange rate between the local currency and foreign currency expressed by the direct pricing method should be in the same direction with the general price level of the country. When the general price level of the country rises, the exchange rate between the local currency and foreign currency expressed by the direct price method will also rise, and the local currency will depreciate; while when the general price level of the country falls, the exchange rate between the local currency and foreign currency expressed by the direct price method will also decline, and the local currency will appreciate accordingly. However, the reality is that the exchange rate between local currency and foreign currency expressed by the direct pricing method often changes in the opposite direction with the general price level of the country. Because the exchange rate between the local currency and the foreign currency reflects the external price of the currency, and the domestic general price level reflects the internal price of the currency, in this case, the reverse change between the exchange rate and the domestic general price level is called the deviation of the internal and external prices of a country’s currency. The monthly RMB exchange rate and the monthly consumer price index (CPI) of China from July 2005 to August 2019 are collected. The correlation coefficient between the

\(^{1}\)Data Source: Wind
change rates of them is -0.14. This means that the monthly RMB exchange rate and the monthly consumer price index have a reverse relationship, or there is a deviation between the internal and external prices of RMB. What causes the deviation between internal and external prices? What are the conditions for a domestic currency to deviate from its internal and external prices? This paper intends to use mathematical analysis to answer the above questions.

2 Literature Review

Reviewing the relevant literature, it can be found that the deviation between the internal and external prices of a country's currency has been widely concerned by scholars long ago. The most representative studies are Harrod (1933)[3], Balasa (1964)[4], Samuelson (1964)[5]. They found that the price index of rich and fast-growing countries is usually higher than that of poor and slow-growing countries, which makes it difficult for purchasing power parity theory to be true in reality. This discovery by Harrod, Balasa and Samuelson is also known as the Harrod-Balasa-Samuelson Effect.

Some studies pay more attention to overshooting of exchange rate. Under the same impact of monetary policy, the price may not change significantly in the short term, but the exchange rate may overshoot. Rudiger Dornbusch (1976)[6] developed a theory of exchange rate movements under perfect capital mobility, a slow adjustment of goods markets relative to asset markets, and consistent expectations. He found an initial overshooting of exchange rates is shown to derive from differential adjustment speed of markets. Sari, Hammoudeh and Soytas (2010)[7] examined the co-movements and information transmission among the spot prices of four precious metals (gold, silver, platinum, and palladium), oil price, and the US dollar/euro exchange rate. They found some evidence of market overreactions in the palladium and platinum cases as well as in the exchange rate market.

Some studies are more concerned about price stickiness. Under the same monetary policy shocks, the exchange rate may be significantly impacted in the short term, but the price may not change significantly due to the stickiness characteristics. Obstfeld and Rogoff (1995)[8] developed a two-country model that marries a full account of global macroeconomic dynamics to a supply framework based on monopolistic competition and sticky nominal prices. The model offers simple and intuitive predictions about exchange rates and current accounts. Harald Hau (2000)[9] introduced factor (labor) markets into the intertemporal monetary model of Obstfeld and Rogoff and combines this richer market structure with a new utility-independent representation of nontradeables. He found that factor price rigidities imply similar properties for the international transmission mechanism as domestic producer price rigidities. Nontradeables give raise to interesting new effects under asymmetric monetary shocks: They create short-run PPP deviations, increase exchange rate volatility relative to price level volatility and reduce (positive) consumption and (negative) output comovements.

In addition, many studies also focus on the transmission effect between exchange rate and price. The research of Cheung and Lai (2000)[10] is representative. Their research shows that the exchange rate of a country's currency will spontaneously recover to PPP after being subjected to exogenous shocks, but this recovery process is long-term and non-monotinous, which often leads to deviations between the internal and external prices of the currency. Another possibility is that the transfer effect from exchange rate to price is gradually weakening (Marazzi and Sheets, 2007)[11], which results in the change of exchange rate not being fully transmitted to domestic prices in time. Forbes, Hjortsoe and Nenova (2018) [12] built a standard open-economy model shows that the relationship between exchange rates and prices depends on the shocks which cause the exchange rate to move. Then they built on this to develop a structural Vector Autoregression (SVAR) framework for a small open economy and apply it to the UK. They found that prices respond differently to exchange rate movements based on what caused the movements.

By reviewing the existing studies, the author finds that there is no special study to analyze the deviation between the internal and external prices of a currency from the perspective of
expectations, nor from the micro-basis to analyze the deviation between the internal and external prices of a currency. Based on this, this paper intends to establish a theoretical model with microcosmic basis based on monetary utility function and use mathematical analysis method to analyze the expectation factors of the deviation between the internal and external prices of non-international currencies, and clearly calculate the specific range of expectation factors which lead to deviation between the internal and external prices of non-international currencies.

3 Theoretical Model Construction

Sidrauski (1967) [13] introduced currency variables into the inter-period utility function for the first time, thus establishing the Money in the Utility Function or the MIU model. The money model in utility function assumes that the utility of representative residents arises not only from the consumption of goods, but also from the holding of money. Since the money model in utility function was put forward, it has attracted extensive attention and related research has been produced (Brock, 1974[14]; Lucas, 1982 [15]; Bordo and Choudhri, 1982 [16]; Obstfeld and Rogoff, 1996 [17], etc.). Next, based on the money model in the utility function, this paper establishes a theoretical model to analyze the conditions of the deviation between the internal and external prices of non-international currencies.

Suppose there are two countries in the world: home country and foreign country. The currency of home country is non international currency and the currency of foreign country is international currency. The representative resident of home country get utility through consumption and holding real compound currency. Assuming the homogeneity of the residents, in this case, we can grasp the overall behavior of the residents by analyzing the behavior of the representative resident. Assuming that the representative resident have the ability of rational expectation, that is to say, there will be no systematic deviation between their expectation of the future and the real situation of the future. It is assumed that domestic and foreign currencies can be freely converted, that is to say, the representative residents of the country can voluntarily choose to hold local or international currencies, which is unregulated. Based on the above assumptions, the inter-temporal utility function of the representative resident in the t period can be set as follows:

\[ U_t = E_t \{ \sum_{s=1}^{\infty} \beta^{t-s} [u(C_s) + v(M_s)] \} \]  

(1)

In Formula (1), \( U_t \) denotes the intertemporal total utility of the representative resident of the home country in the period \( t \). \( U_t \) is the conditional expectation of the discount value of the utility of the representative resident of the home country in the future from the beginning of the period \( t \). \( E_t \) represents conditional expectation of the representative resident of the home country based on all available information sets in period \( t \). \( \beta \) is the discount coefficient of the future utility of the representative resident in home country. The future utility of each period of the representative resident since the \( t \) period comes from two aspects: \( u(C_s) \) and \( v(M_s) \). \( u(C_s) \) indicates the level of utility brought by consumption in the period \( s \) to the representative resident of the home country. \( C_s \) is the amount of consumption of the representative resident in the period \( s \). Marginal utility \( u(C_s) \) is always positive but has a decreasing feature. \( v(M_s) \) denotes the utility level of the real compound currency holdings in the period \( s \) to the representative resident of the home country, while \( M_s \) denotes the actual compound currency holdings in the period \( s \) by the representative resident of the home country. Specific real compound currency can be defined by a CES function, as shown in Formula (2):

\[ M_s = \left( \frac{M_{s-1}}{P_{s-1}} \right)^{\frac{\theta}{\theta-1}} + \left( \frac{M_{foreign_s}}{P_{foreign_s}} \right)^{\frac{\theta}{\theta-1}} \]  

(2)

In formula (2), \( M_s \) denotes the nominal amount of the currency of the home country held by
the representative resident in the $s$ period, $P_s$ denotes the price level of the home country in the $s$ period, and $M_s / P_s$ denotes the actual amount of the currency of the home country held by the representative resident in the $s$ period. $M_{\text{foreign},s}$ denotes the nominal number of foreign currency (world currency) held by the representative resident in the period $s$. $e_s$ is the exchange rate in the period $s$ denoted by the direct pricing method, and $M_{\text{foreign},s} e_s / P_s$ is the actual number of foreign currency (world currency) held by the representative resident in the period $s$. $\theta$ denotes the substitutional elasticity of the two real currency balances held by the representative resident of the home country, and $\theta > 1$. Similarly, the marginal utility $v'(\tilde{M}_s)$ of the real compound currency balance held by the representative resident of the home country is always positive but has a decreasing feature.

Assuming that the domestic economy is an endowment economy, that is to say, the level of domestic output is determined by exogenous factors. In this case, the stage constraints of the optimal decision-making of the representative resident can be expressed by the following equation:

$$B_s + \frac{M_s}{P_s} + \frac{M_{\text{foreign},s} e_s}{P_s} = B_{s-1}(1 + r_{\text{foreign},s-1}) + \frac{M_{\text{foreign},s-1} e_{s-1}}{P_s} + Y_s - C_s - T_s,$$

In formula (3), $B_{s-1}$ represents the actual balance of foreign bonds held by the representative residents at the end of the $s-1$ period, $r_{\text{foreign},s-1}$ represents the actual interest rate of foreign bonds in the $s-1$ period, and $B_{s-1}(1 + r_{\text{foreign},s-1})$ represents the actual balance of foreign bonds held by the representative residents at the beginning of the $s$ period. $Y_s$ represents the actual output level of the representative resident in the $s$ period, and $T_s$ represents the total actual tax paid by the representative resident in the $s$ period.

From formulas (1) to (3), we can get three first-order conditions for maximizing the utility of the representative resident of the home country, as follows:

$$u(C_s) = (1 + r_{\text{foreign},s}) \beta E_t[u(C_{s+1})]$$

$$\frac{1}{P_t} u(C_s) = v(M_t)(\tilde{M}_t)^{\theta} \left( \frac{M_s}{P_s} \right)^{1-\theta} \frac{1}{P_t} + \beta E_t[u(C_{s+1})] - \frac{1}{E_t(P_{s+1})}$$

$$\frac{e_s}{P_s} u(C_s) = v(M_t)(\tilde{M}_t)^{\theta} \left( \frac{M_{\text{foreign},s} e_s}{P_s} \right)^{1-\theta} \frac{e_s}{P_s} + \beta E_t[u(C_{s+1})] - \frac{1}{E_t(P_{s+1})}$$

Formulas (4) to (6) are the three first-order conditions for maximizing the utility of the representative resident of the home country. Further, by combining formulas (4) to (6), we can obtain that:

$$\frac{(1 + r_{\text{foreign},s}) E_t(P_{s+1}) - P_t}{(1 + r_{\text{foreign},s}) E_t(P_{s+1}) - p_t E_t(e_{s+1})} = \left( \frac{M_s}{M_{\text{foreign},s}} \right)^{\frac{1-\theta}{\theta}}$$

Assuming that the Uncovered Interest Parity (UIP) holds, i.e.

$$1 + r_t = \frac{E_t(e_{s+1})}{e_t} (1 + r_{\text{foreign},s})$$

In formula (8), $r_t$ denotes the real interest rate of the home country in the $t$ period. By combining formulas (7) and (8), we can obtain that:

$$\frac{(1 + r_t) e_t E_t(P_{s+1}) - P_t E_t(e_{s+1})}{(1 + r_t) e_t E_t(P_{s+1}) - P_t E_t(e_{s+1})} = \left( \frac{M_s}{M_{\text{foreign},s}} \right)^{\frac{1-\theta}{\theta}}$$

By calculating the derivatives of formula (9) to $P_t$ on both sides, and considering $e_t$ as an implicit function of $P_t$, we can obtain that:
Formula (10) shows that the partial derivative of spot exchange rate $e_i$ of non-international currency issuing country with respect to spot price level $P_i$ is a function of expected exchange rate $E_i(e_{i+1})$, currency substitution elasticity $\theta$, interest rate level $r_i$ of non-international currency issuing country and expected general price level $E_i(P_{i+1})$. According to the previous settings, the pricing method of spot exchange rate $e_i$ of non-international currency issuing country is direct pricing method. That is to say, the increase of spot exchange rate $e_i$ means that the non-international currency depreciates relative to international currency, while the decrease of spot exchange rate $e_i$ means that the non-international currency appreciates relative to international currency. In this case, if the partial derivative of spot exchange rate $e_i$ of non-international currency with respect to spot price level $P_i$ expressed in formula (10) is greater than or equal to zero, that is to say, spot exchange rate $e_i$ and spot price level $P_i$ will not change in reverse, then the internal and external prices of non-international currency meet the purchasing power parity law, or there is no deviation between the internal and external prices of non-international currency. Conversely, if the partial derivative of spot exchange rate $e_i$ of non-international currency with respect to spot price level $P_i$ expressed in formula (10) is less than zero, that is to say, spot exchange rate $e_i$ and spot price level $P_i$ will change inversely, then the internal and external prices of non-international currency will no longer meet the purchasing power parity law, or there is a deviation between internal and external prices of non-international currency.

4 Mathematical Analysis

Through the theoretical model of the second part, the partial derivative of spot exchange rate $e_i$ of non-international currency with respect to spot price level $P_i$ is obtained, as shown in formula (10). Next, the author will further analyze the expectation conditions for the deviation between internal and external prices of non-international currency. It should be noted that the impact of random shocks will not be taken into account in the next analysis. Based on formula (10), we can see that the condition for the deviation between the internal and external prices of non-international currency is that the partial derivative of spot exchange rate $e_i$ of non-international currency with respect to spot price level $P_i$ is less than zero. Then, three situations will be discussed.

4.1 The Situation of $E_t(e_{t+1}) = e_t$

When $E_t(e_{t+1}) = e_t$, the expected exchange rate is equal to the spot exchange rate, and the local currency is neither expected to appreciate nor depreciate. By substituting this item into formula (10), the molecular part on the right side of formula (10) is equal to 0, and then:

$$\frac{\partial e_t}{\partial P_t} = \frac{(e_t - E_t(e_{t+1} + 1)\theta}{P_i(1-2\theta) + \frac{P_i(1+\theta)(P_i^2E_i(e_{t+1})(1-\theta))}{E_i(e_{t+1})(1+\theta)E_i(P_{t+1})(e_t)^2}} = 0$$

(11)

Formula (11) shows that the partial derivative at this situation is 0, and the spot exchange rate will not change with the current price, so there will be no deviation between the internal and external prices of the currency.

4.2 The Situation of $E_t(e_{t+1}) > e_t$

When $E_t(e_{t+1}) > e_t$, the expected exchange rate is greater than the spot exchange rate, and the local currency is expected to depreciate. By substituting this item into formula (10), the molecular part
part on the right side of formula (10) is less than 0. At this situation, the deviation between the internal and external prices of the local currency requires that the denominator part on the right side of Formula (10) is greater than 0, thus ensuring \( \frac{\partial E_t}{\partial t} < 0 \). It is:

\[
P_t(1−2\theta) + \frac{P_t(1+\theta)}{E_t(e_{t+1})} - \frac{(P_t)^2E_t(e_{t+1})(1−\theta)}{(1+\tau_t)E_t(P_{t+1})(e_t)^2} > 0
\]

(12)

In order to find the conditions for the establishment of Formula (12), the first and third terms of Formula (12) are first extracted, as shown on the left side of the inequality sign in the following formula. On this basis, the conditions for the establishment of the following formula are calculated:

\[
P_t(1−2\theta) + \frac{(P_t)^2E_t(e_{t+1})(1−\theta)}{(1+\tau_t)E_t(P_{t+1})(e_t)^2} > P_t(1−2\theta) - \frac{(P_t)^2E_t(e_{t+1})(1−\theta)}{E_t(e_{t+1})} \]

(13)

By deforming Formula (13), the following results can be obtained:

\[
P_t(1−2\theta)(1+\tau_t)E_t(P_{t+1})e_t + (P_t)^2E_t(e_{t+1})(\theta−1) > (P_t)^2E_t(P_{t+1})E_t(e_{t+1})(\theta−1)
\]

(14)

By simplifying Formula (14), we can obtain that:

\[
(\frac{e_t}{E_t(e_{t+1})})^2 > (\frac{e_t}{E_t(e_{t+1})})^2
\]

(15)

Because of \( E_t(e_{t+1}) > e_t \) and \( \theta > 1 \), a new inequality can be obtained for the left part of Formula (15):

\[
(1−2\theta)(1+\tau_t)E_t(P_{t+1})e_t + (P_t)^2E_t(e_{t+1})(\theta−1) > (1−2\theta)(1+\tau_t)E_t(P_{t+1})E_t(e_{t+1}) + P_tE_t(e_{t+1})(\theta−1)
\]

(16)

In this case, based on Formula (15) and Formula (16), the establishment of Formula (15) can be ensured by the establishment of the following formula.

\[
(1−2\theta)(1+\tau_t)E_t(P_{t+1})E_t(e_{t+1}) + P_tE_t(e_{t+1})(\theta−1) > (1−2\theta)(1+\tau_t)E_t(P_{t+1})E_t(e_{t+1}) + P_tE_t(e_{t+1})(\theta−1)
\]

(17)

Further look for the conditions for the establishment of Formula (17). By simplifying Formula (17), we can obtain that:

\[
(\frac{e_t}{E_t(e_{t+1})})^2 > \frac{(1−2\theta)(1+\tau_t)E_t(P_{t+1})E_t(e_{t+1})}{E_t(e_{t+1})^2}
\]

(18)

Since there is \( E_t(e_{t+1}) > e_t \), the further conditions for the establishment of Formula (18) are as follows:

\[
(1−2\theta)(1+\tau_t)E_t(P_{t+1}) + P_t(\theta−1) > 0
\]

(19)

By simplifying Formula (19), Formula (20) can be obtained:

\[
E_t(P_{t+1}) < \frac{P_t(\theta−1)}{(2\theta−1)(1+\tau_t)}
\]

(20)

Formula (20) is an inequality condition about the expected price. Formula (20) is the condition for the establishment of Formula (13). Furthermore, under the condition that Formula (13) holds, combining inequality function \( E_t(e_{t+1}) > e_t \), Formula (12) can be rewritten as follows:

\[
\frac{e_t}{E_t(e_{t+1})} + \frac{P_t(1+\theta)}{E_t(e_{t+1})} - \frac{(P_t)^2E_t(e_{t+1})(1−\theta)}{(1+\tau_t)E_t(P_{t+1})(e_t)^2} \]

(21)

Therefore, if Formula (21) is established, the deviation of internal and external prices of non-national currency will occur. Next, the author continues to look for the conditions for the establishment of Formula (21). First, simplify Formula (21) to obtain:

\[
P_t(2−\theta) + \frac{(P_t)^2(\theta−1)}{1+\tau_t}E_t(P_{t+1}) > 0
\]

(22)

Continuing to deform Formula (22), it can be obtained that:

\[
P_t(2−\theta)(1+\tau_t)E_t(P_{t+1}) + (P_t)^2(\theta−1) − [(1+\tau_t)E_t(P_{t+1})]^2 > 0
\]

(23)

After factorization of formula (23), we can get that:

\[
[−(1+\tau_t)E_t(P_{t+1}) + P_t](1+\tau_t)E_t(P_{t+1}) + P_t(\theta−1) > 0
\]

(24)

From Formula (24), the following two groups of inequalities are obtained:
\[
\begin{align*}
(1 + r_t)E_t(P_{t+1}) + P_t(\theta - 1) & > 0 \\
(1 + r_t)E_t(P_{t+1}) + P_t(\theta - 1) & < 0 \\
(1 + r_t)E_t(P_{t+1}) + P_t(\theta - 1) & < 0 \\
\end{align*}
\]

(25)

(26)

Observing Formula (26), the inequality conditions of Formula (26) are impossible because of interest rate \( r_t > 0 \), expected price \( E_t(P_{t+1}) > 0 \), current price \( P_t > 0 \) and \( \theta > 1 \). Therefore, the author further investigates the inequality condition of Formula (25). After calculation, the author obtains another inequality condition about the expected price:

\[
\frac{P_t(1-\theta)}{(1+r_t)} < E_t(P_{t+1}) < \frac{P_t}{(1+r_t)}
\]

(27)

Because of \( \theta > 1 \), we can further combine the two inequality conditions (20) and (27) about the expected price, and obtain the following inequality conditions about the expected price:

\[
\frac{P_t(1-\theta)}{(1+r_t)} < E_t(P_{t+1}) < \frac{P_t(\theta-1)}{(2\theta-1)(1+r_t)}
\]

(28)

Formula (28) is the expectation price condition for the deviation between internal and external prices of non-international currency when the expectation exchange rate meets \( E_t(e_{t+1}) > e_t \).

### 4.3 The Situation of \( E_t(e_{t+1}) < e_t \)

In the previous analysis, the author inspected the two cases of \( E_t(e_{t+1}) = e_t \) and \( E_t(e_{t+1}) > e_t \) respectively. Next, the author will continue to inspect the situation of \( E_t(e_{t+1}) < e_t \). When \( E_t(e_{t+1}) < e_t \), the expected exchange rate is less than the spot exchange rate, and the local currency is expected to appreciate. By substituting this item into formula (10), the molecular part on the right side of formula (10) is greater than 0. At this point, the deviation between the internal and external prices of the local currency requires that the denominator on the right side of Formula (10) is less than 0, thus ensuring \( \frac{\partial E_t}{\partial P_t} < 0 \). That is to say, the following condition is required:

\[
\frac{P_t(1-2\theta)}{e_t} + \frac{(P_t)^2E_t(e_{t+1})(\theta-1)}{(1+r_t)E_t(P_{t+1})(e_t)^2} - \frac{(1+r_t)E_t(P_{t+1})e_t}{[E_t(e_{t+1})]^2} < 0
\]

(29)

In order to find the conditions for the establishment of Formula (29), the first and third terms of Formula (29) are first extracted, as shown on the left side of the inequality sign in the following formula. On this basis, the conditions for the establishment of the following formula are calculated:

\[
\frac{P_t(1-2\theta)}{(1+r_t)E_t(P_{t+1})(e_t)^2} < \frac{P_t(1-2\theta)}{(1+r_t)E_t(P_{t+1})(e_t)^2} + \frac{(P_t)^2E_t(e_{t+1})(\theta-1)}{(1+r_t)E_t(P_{t+1})(e_t)^2}
\]

(30)

By deforming Formula (30), the following results can be obtained:

\[
\frac{P_t(1-2\theta)(1+r_t)E_t(P_{t+1})e_t}{(1+r_t)E_t(P_{t+1})(e_t)^2} < \frac{(1+r_t)E_t(P_{t+1})e_t}{(1+r_t)E_t(P_{t+1})(e_t)^2} + \frac{(P_t)^2E_t(e_{t+1})(\theta-1)}{(1+r_t)E_t(P_{t+1})(e_t)^2}
\]

(31)

By simplifying Formula (31), we can obtain that:

\[
\frac{(1-2\theta)(1+r_t)E_t(P_{t+1})e_t}{(e_t)^2} < \frac{(1-2\theta)(1+r_t)E_t(P_{t+1})e_t}{(e_t)^2} + \frac{(P_t)^2E_t(e_{t+1})(\theta-1)}{[E_t(e_{t+1})]^2}
\]

(32)

Because of \( E_t(e_{t+1}) < e_t \) and \( \theta > 1 \), a new inequality can be obtained for the left part of Formula (32):

\[
\frac{(1-2\theta)(1+r_t)E_t(P_{t+1})e_t}{(e_t)^2} + \frac{(P_t)^2E_t(e_{t+1})(\theta-1)}{[E_t(e_{t+1})]^2} < 0
\]

(33)

In this case, based on Formula (32) and Formula (33), the establishment of Formula (32) can be ensured by the establishment of the following formula:

\[
\frac{(1-2\theta)(1+r_t)E_t(P_{t+1})e_t}{(e_t)^2} + \frac{(P_t)^2E_t(e_{t+1})(\theta-1)}{[E_t(e_{t+1})]^2} < 0
\]

(34)

Further look for the conditions for the establishment of Formula (34). By simplifying Formula (34), it can be concluded that:

\[
\frac{(1-2\theta)(1+r_t)E_t(P_{t+1})e_t}{(e_t)^2} + \frac{(P_t)^2E_t(e_{t+1})(\theta-1)}{[E_t(e_{t+1})]^2} < \frac{(1-2\theta)(1+r_t)E_t(P_{t+1})e_t}{(e_t)^2} + \frac{(P_t)^2E_t(e_{t+1})(\theta-1)}{[E_t(e_{t+1})]^2}
\]

(35)

Since there is \( E_t(e_{t+1}) < e_t \), the condition for the establishment of Formula (35) is as follow:

\[
(1 - 2\theta)(1 + r_t)E_t(P_{t+1}) + P_t(\theta - 1) > 0
\]

(36)

By simplifying Formula (36), Formula (37) can be obtained:
\[ E_t(P_{t+1}) < \frac{P_t(\theta-1)}{(2\theta-1)(1+r_t)} \]  

Formula (37) is an inequality condition about the expectation price. Formula (37) is the condition for the establishment of Formula (30). Further, under the condition that formula (30) holds, combining inequality \( E_t(e_{t+1}) > e_t \), formula (29) can be rewritten as follow:

\[
\begin{align*}
\frac{P_t(1-\theta)}{e_t} + \frac{P_t(1+\theta)}{E_t(e_{t+1})} + \frac{(P_t)^2 E_t(e_{t+1})(\theta-1)}{(1+r_t)E_t(P_{t+1})e_t} - \frac{(1+r_t)E_t(P_{t+1})}(){E_t(e_{t+1})}^2 < 0
\end{align*}
\]

Therefore, if Formula (38) is established, the deviation of internal and external prices of non-international currency will occur. Next, the author continues to look for the conditions for the establishment of Formula (30). After simplifying Formula (38), we can get that:

\[
\begin{align*}
\frac{P_t(2-\theta)}{E_t(e_{t+1})} + \frac{(P_t)^2(\theta-1)}{(1+r_t)E_t(P_{t+1})} - \frac{(1+r_t)E_t(P_{t+1})}{E_t(e_{t+1})} < 0
\end{align*}
\]

By simplifying Formula (39), we can obtain that:

\[
\begin{align*}
P_t(2-\theta) + \frac{(P_t)^2(\theta-1)}{(1+r_t)E_t(P_{t+1})} - (1+r_t)E_t(P_{t+1}) < 0
\end{align*}
\]

Continuing to deform Formula (40), we can obtain that:

\[
\begin{align*}
P_t(2-\theta)E_t(P_{t+1})(1+r_t) + (P_t)^2(\theta-1) - [(1+r_t)E_t(P_{t+1})]^2 < 0
\end{align*}
\]

After factorization of formula (41), we can get that:

\[
\begin{align*}
[-(1+r_t)E_t(P_{t+1}) + P_t][1 + r_t]E_t(P_{t+1}) + P_t(\theta - 1) < 0
\end{align*}
\]

From formula (42), the following two groups of inequality conditions are obtained:

\[
\begin{align*}
\begin{cases}
(1 + r_t)E_t(P_{t+1}) + P_t > 0 \\
(1 + r_t)E_t(P_{t+1}) + P_t(\theta - 1) < 0 \\
(1 + r_t)E_t(P_{t+1}) + P_t < 0 \\
(1 + r_t)E_t(P_{t+1}) + P_t(\theta - 1) > 0
\end{cases}
\end{align*}
\]

Observing formula (43), the inequality conditions of Formula (43) are impossible because of interest rate \( r_t > 0 \), expectation price \( E_t(P_{t+1}) > 0 \), current price \( P_t > 0 \) and \( \theta > 1 \). Therefore, the author further investigates the inequality conditions of Formula (44). After calculation, the author obtains another group of inequality conditions about the expectation price:

\[
\begin{align*}
\begin{cases}
E_t(P_{t+1}) > \frac{P_t}{(1+r_t)} \\
E_t(P_{t+1}) > \frac{-P_t(\theta-1)}{(1+r_t)}
\end{cases}
\end{align*}
\]

In formula (45), because \( \theta > 1 \), so \( \frac{P_t}{(1+r_t)} > \frac{-P_t(\theta-1)}{(1+r_t)} \), then we can get:

\[
E_t(P_{t+1}) > \frac{P_t}{(1+r_t)} \]

Combining the inequality conditions of formula (37) and formula (46), we can obtain that:

\[
\frac{P_t}{(1+r_t)} < E_t(P_{t+1}) < \frac{P_t(\theta-1)}{(2\theta-1)(1+r_t)}
\]

But because \( \theta > 1 \), formula (47) does not hold. This means that when \( E_t(e_{t+1}) < e_t \), no matter what the specific value of expectation price \( E_t(P_{t+1}) \) is, there will be no deviation between the internal and external prices of non-international currency.

To sum up, when the expectation exchange rate is less than or equal to the current exchange rate without considering random shocks, no matter the size of the expectation price, it will not lead to the deviation between the internal and external prices of non-international currency. The expected conditions for the deviation between the internal and external prices of non-international currency can be expressed by the following inequalities:

\[
\begin{align*}
\begin{cases}
P_t(1-\theta) < E_t(e_{t+1}) > e_t \\
(P_t(1-\theta)(1+r_t)) < E_t(P_{t+1}) < \frac{P_t(\theta-1)}{(2\theta-1)(1+r_t)}
\end{cases}
\end{align*}
\]
5 Conclusions

Based on the theoretical model of money in the utility function with micro-foundation, by means of the method of mathematical analysis, the expectation factors of the deviation between internal price and external price of non-international currency are analyzed, the clear interval of exchange rate expectation and price expectation which lead to the deviation between internal price and external price of non-international currency are calculated. Based on the previous analysis, the author mainly draws the following two main conclusions:

First, when the expectation exchange rate is less than or equal to the current exchange rate without considering random shocks, no matter the size of the expectation price, it will not lead to the deviation between the internal and external prices of non-international currency.

Second, when the expectation exchange rate is greater than the current exchange rate, and the expectation price is in a specific range of \[ P_t(1-\theta) \frac{(1+\tau_t)}{(1+\tau_t)} < E_t(\frac{P_{t+1}}{P_t}) < P_t(\frac{\theta-1}{(2\theta-1)(1+\tau_t)}) \] at the same time, it will lead to the deviation between the internal and external prices of non-international currency.

The policy implication of this paper is that the governments of non-international currency issuing countries can adopt positive non-traditional monetary policies to guide the public's expectations of exchange rates and prices, and then correct the negative effects of excessive deviation between internal and external prices of local currency. Specifically, the governments of non-international currency issuing countries should guide the public to avoid devaluation expectations of their currencies, so as to avoid deviation between internal and external prices of their currencies.

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