Mathematical Modeling: Education and Its Method

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Abstract. Numerical modeling and simulation have revolutionized the industry over the past few decades, and these technics become invaluable to almost every fields. In this paper we mainly discuss the teaching of mathematical modeling, autonomous learning for students, and people who needs studying. Firstly, we introduce several fundamental models and how to integrate them easily. Secondly, we introduced how to effectively carry out group learning and case study method. Finally, three example is given to show the application of modeling processes and methods. Modeling examples verify our students' mastery of such ideas after the teaching and heuristic teaching practice.

Introduction

In recent years, more and more Chinese students and teachers have paid attention to a series of mathematical modeling competitions, which have attracted the attention of the community. For example, the American College Students' Mathematical Contest in modeling and the National Collegiate Mathematical Contest in modeling. The National Collegiate Mathematical Contest in modeling in China was founded in 1992. In 2017, nearly 110,000 college students from 34 provinces/municipalities (including Hong Kong, Macao and Taiwan) and 1418 colleges/campuses in Singapore and Australia, 36,375 teams (33,062 undergraduate and 3,313 specialist teams) participated in the competition. The participating universities, regions and countries are still growing.

![Figure 1. The numbers of participants and teams increased in 1996-2017.](http://www.mcm.edu.cn/)

Then how can we learn this useful and involves different types of customers in different industries? Some scholars believe that it is a specialized course in mathematics and should be corresponded through systematic teaching materials. However, faced with practical problems, sometimes for teachers, it is also very exhausting. Some scholars think that according to the problem background classification, we know that the problem-driven approach is a good method, but it takes a long time to supplement the background knowledge.
As a College Mathematics Teacher, I have a strong interest in studying and teaching on mathematical model, but also some suggestions and share with you. Firstly, we need to understand some common mathematical models and methods for solving different models, and then we need to practice how to use common models through a large number of examples. Secondly, we should encourage students of different disciplines to set up modeling groups to practice through online and offline communication and discussion, which can still invite interested teachers to participate.

Finally, the complex problems are interpreted and decomposed, and the common models and patterns are used to analyze the problems. The system, complete model solution and result analysis are given.

This article is divided into the following parts. The second chapter introduces the common mathematical models and the classification of solution problems in the process of mathematical modeling. The third chapter: modeling team members, group discussion and other suggestions, how to enhance students' autonomous learning ability and teamwork ability. In chapter 4, the modeling example verifies our students mastery in the teaching and heuristic teaching practice. The conclusion has been given in last chapter.

Basic Mathematical Models

As a general person, maybe you don't have enough background of Advanced Math, you may ask me what is mathematical model. Dubois in his book said [1]. A model of a system is an experienceable representation. It is an object (real or abstract) or a set of objects that we are willing to study. It is a very broad concept: for instance, a system may be a plane, an engine, or even a screw. Note that it may contain subsystems that are systems as well. So, in industry, teams work on systems on a daily basis (these systems will be products or parts of products marketed by the company), either to conceive, develop, or produce them.

In Undergraduate Mathematics and its applications, the Interdisciplinary Lively Application Project (ILAP)[2] is good for understanding the special application topic. The purpose is to solve a practical problem through interdisciplinary research teams Collectively.

The Classifications Basic Models

We can divide the mathematical model into different types by variables and the relationships can be described by operators, such as algebraic operators, functions, differential operators.

Linear and nonlinear Linearity is the simple relationship in mathematics. If the operators are all linear, we consider the model is linear, of course the complex relationship which cannot be precisely described, we often choose the main operators and linearity for initial assumption firstly. Such as statistical linear model, linear differential operators.

Discrete and continuous The time variable in the model is changed in a certain interval of the model called continuous time model, the above models described by differential equations are continuous time models. When dealing with lumped parameter model, the time variable can also be discretized. The obtained model is called discrete time model. The discrete time model is described by a difference equation.

Deterministic and probabilistic The relationship between variables in the stochastic model is given in the form of statistical value or probability distribution, while the relationship between variables in the deterministic model is determined.

How to Enhance the Autonomous Learning and Teamwork Ability

During the course, I often use the following methods to help them do better and faster.

Heuristic class mode, class through Weixin, QQ group, QQ space, discussion group and other network ways to share a lot of mathematical modeling of the United States, through competition to promote teamwork, regular face-to-face discussion, reasonable incentives and Practice are also used in coursing.
Heuristic Class Model

We use many examples in real society for breakthrough point, in class we do not tell them the answer or the model directly. It is important for students to think deeply and understand the background of the problems. Even teacher can be role as a novice, and gradually guide them to use modeling process and made to solve problems. Modeling course requires our teachers to spend a lot of energy preparing for a class.

Making Use of Internet

The Big difference distinguished with other courses is using the internet fluently. Because of the width and arbitrariness of knowledge in mathematical model. In Chinese people use phone very universal, construct the WeChat public address, QQ group, teachers often recommend knowledge for students and your friends. In summer vocation I used the qq group to talk with them, pointed out their shortcoming, and my advise conveniently. Moody classes, open classes can be also used easily for them. This is not the only way to find most of the knowledge they want to learn on the Internet.

The Modeling Example with Our Students

The projection of the workpiece and the reconstruction of the projected image by CT scanner are a pair of "dual" problems [3,4]. If the absorptivity of the workpiece is known, the projection image of the workpiece scanned by the CT machine can be calculated when the system parameters of the CT machine have been calibrated. On the other hand, the projection image can also be obtained to reverse the absorptivity distribution of the original workpiece.

Matlab Parameter Calibration

Two functions, radon and iradon, are provided in the image processing toolbox of MATLAB. But MATLAB language pays attention to universality rather than particularity: its scanning center must be image center, the number of scanning units can not be preset. Therefore, it is still necessary to modify the result of function processing to restore the CT scanner in MATLAB and get the approximate numerical solution of the model.

After translating the absorptivity (center reset) of the template diagram in appendix 1, the projection image is scaled to 512x180 by using the radon function and the bilinear difference algorithm. The absorptivity image is shown in Figure (1-a). In order to verify the conjecture, the data from 90 to 95 angles are taken as explanatory variables, and the output data from MATLAB are used as regression variables to carry out unary linear regression, if the regression equation is used as following:

\[ P(\theta, t) = a + bP^m(\theta, t), \] (1)

thus six regression equations are obtained, and their regression coefficients, sum of squares of residuals and R^2 statistics are shown in Table 1:

<table>
<thead>
<tr>
<th></th>
<th>90</th>
<th>91</th>
<th>92</th>
<th>93</th>
<th>94</th>
<th>95</th>
<th>mean value</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>-0.430</td>
<td>-0.443</td>
<td>-0.449</td>
<td>-0.438</td>
<td>-0.441</td>
<td>-0.436</td>
<td>-0.440</td>
</tr>
<tr>
<td>b</td>
<td>1.438</td>
<td>1.438</td>
<td>1.438</td>
<td>1.438</td>
<td>1.438</td>
<td>1.438</td>
<td>1.438</td>
</tr>
<tr>
<td>MSE</td>
<td>2.499</td>
<td>2.244</td>
<td>2.076</td>
<td>2.027</td>
<td>2.087</td>
<td>2.215</td>
<td>2.191</td>
</tr>
<tr>
<td>R^2</td>
<td>0.998</td>
<td>0.998</td>
<td>0.999</td>
<td>0.999</td>
<td>0.999</td>
<td>0.999</td>
<td>0.999</td>
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</table>

Note that the first and second steps of the algorithm 2 both gain the data from the same absorber, and the filtering back-projection algorithm is linear to the value of the projected image. The two steps can also be written as follows:

\[ P_{22} = (k_1k_2)P_1, \] (2)
It is not difficult to verify the correctness of algorithms 1 and 2 by using the absorption data in appendix 1 and the projection data in appendix 2.

**Correction of Absorption Rate**

An algorithm 2 for projection data given in appendix 2 is given, and a density map is drawn for the obtained results (Fig. 2).

![Figure 2. Density graph.](image)

Observing the density map, we can find that there is still a small fluctuation between the calculated results and the true absorptivity (0 and 1), that is, there is truncation error in the numerical integration. Although some calculation errors are unavoidable, a reasonable correction model will improve the calculation results of absorption. In this paper, two different modified models are proposed and validated for the calculation results.

For direct clustering of absorbance data in question 2, the computational complexity and storage complexity are all up to $O(n^4)$ due to the need to compute the distance between any two points, where $n$ is the size of the image, so the calculation is time-consuming and laborious. The general computer cannot even implement the clustering model due to memory constraints. Therefore, a sample point can be taken every 1000 points in the template for clustering operation.

The hierarchical clustering method divides the data into five categories in the third layer. After obtaining the characteristics of the five types of data, the original data of absorption rate of 20 points and the data of recovery absorption rate of $I_m$ can be obtained by intra-class average (less than 0.01 thought absorption rate is 0) for each type of data (see Table 2):

<table>
<thead>
<tr>
<th></th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>Im</td>
<td>0.005</td>
<td>0.997</td>
<td>-0.001</td>
<td>1.190</td>
<td>1.049</td>
<td>1.464</td>
<td>1.320</td>
<td>0.018</td>
<td>0.025</td>
<td>0.028</td>
</tr>
<tr>
<td>Ir</td>
<td>0</td>
<td>1.001</td>
<td>0</td>
<td>1.184</td>
<td>1.001</td>
<td>1.474</td>
<td>1.296</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

The upper bound of the absolute error of the model is about 0.15 after observing Figure 2. Therefore, if the absorptivity difference in the 3x3 window is less than 0.3, it is considered that the absorptivity near this point is a kind of data and can be corrected by algorithm 2. After the noise removal operation of the projection map, the original data of absorptivity of ten points required by the title, Im of the original data, M of the 3x3 window where the ten points are located, and Ir of the absorptivity data after restoration are obtained as shown in Table 3:

<table>
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<tr>
<th></th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>Im</td>
<td>0.165</td>
<td>2.608</td>
<td>6.336</td>
<td>0.144</td>
<td>0.478</td>
<td>3.367</td>
<td>5.521</td>
<td>0.028</td>
<td>6.181</td>
<td>0.052</td>
<td></td>
</tr>
<tr>
<td>M</td>
<td>0.377</td>
<td>0.668</td>
<td>1.215</td>
<td>0.259</td>
<td>0.511</td>
<td>0.414</td>
<td>1.760</td>
<td>0.278</td>
<td>1.050</td>
<td>0.267</td>
<td></td>
</tr>
<tr>
<td>Ir</td>
<td>0.165</td>
<td>2.608</td>
<td>6.336</td>
<td>0.029</td>
<td>0.478</td>
<td>3.367</td>
<td>5.521</td>
<td>0.044</td>
<td>6.181</td>
<td>0.014</td>
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</tr>
</tbody>
</table>

**Conclusions**

In this paper we mainly discuss the teaching of mathematical modeling, autonomous learning for students, and people who needs studying. Firstly, we introduce several fundamental models and
how to integrate them easily. Secondly, we introduced how to effectively carry out group learning and case study method. Finally, three example is given to show the application of modeling processes and methods. Modeling examples verify our students' mastery of such ideas after the teaching and heuristic teaching practice.

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Reference