A Stochastic Model of Empty Container Transport on Sea

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ABSTRACT

A chance-constrained programming model for empty container transport on sea is built. The objective function is to minimize the cost of empty container transport including shipping, renting and shortage cost. In the model, shipping cost is determined by the number of ship used for empty container transport. The constraints to the model include satisfying the need of empty containers, limit to the ability of empty containers provided and the capacity of transport. Lingo9.0 is used to solve the model and simulation is done under varied parameters to better transport strategy. The results show that the model can provide an effective program of empty container repositioning for a shipping company and it is a good way to raise shipping efficiency.\(^1\)

KEYWORDS

Empty container; line; chance-constrained programming; simulation.

INTRODUCTION

Container shipping transport on sea plays key role in international trade, especially with China’s entry into WTO and the development of world trade. But at the same time, number of empty container has increased rapidly with the development of container shipping, it produces only cost.

Many scholars did much study in the subject. ShiX [1] used integer programming to research empty container shipping on sea, and he did simulation to

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**MATHEMATICAL MODEL**

**Parameters**

\[ T: \text{Set of planning period} \quad T = \{1, 2, \cdots, T\} \]

That is to say, \( T \) is composed of all time periods. \( L: \text{Set of lines.} \)

\[ O_l: \text{The origination of line} \quad l \quad (l \in L) \]

The set of ports supplying empty containers during \( t \) \((t \in T)\). \( S^t_i: \text{The number of empty containers provided from port} \quad i \quad \text{during} \quad t \)

\( D^t_j: \text{The set of ports requiring empty containers during} \quad t \quad (t \in T) \). \( D^t_2: \text{The number of empty containers required from port} \quad j \quad \text{during} \quad t \)

\( Sli: \text{The set of ships which can be used in empty container repositioning of line} \quad l \quad \text{during} \quad t \).

\( \tau^t_{ij}: \text{The shipping time from port} \quad i \quad \text{to port} \quad j \quad \text{by line} \quad l \).

\( C: \text{Cost of a ship used for empty containers repositioning.} \)

\( C^{R(t)}_{ij}: \text{Cost caused by an empty container rented from port} \quad j \quad \text{during} \quad t \).

\( C^{S(t)}_{ij}: \text{Cost caused by shortage of an empty container from port} \quad j \quad \text{during} \quad t \).

**Variables**

\( y^{sl} \): \( y^{sl} = 1 \) means that ship \( s \) is used in empty container repositioning by line \( l \) during \( t \). Otherwise \( y^{sl} = 0 \).

\( x^{sli}_{ij}(t_1, t_2) \): The number of empty containers shipped from port \( i \) to port \( j \) in ship \( s \) by line \( l \). The empty containers are started from port \( i \) during \( t_1 \) and arrive at port \( j \) no later than port \( j \). Ship \( s \) sets off from \( O_l \) during \( t \) by line \( l \) and arrive at \( i \), \( j \) during \( t_1 \) and \( t_2 \) respectively. \((i \in S_t, j \in D_t, s \in Sli, l \in L, t_1, t_2, t \in T)\)

\( x^{R(t)}_{ij} \): The number of empty containers rented from port \( j \) during \( t \). \((j \in D_t, t \in T)\)

\( x^{S(t)}_{ij} \): The number of shortage of empty containers in port \( j \) during \( t \). \((j \in D_t, t \in T)\)
Objective Function

\[ Min \sum_{n \in T} \sum_{j \in D_j} x_j^S \]

(1)

\[ Min(\sum_{n \in T} \sum_{l \in L} \sum_{s \in S_{lt}} \sum_{t \in T} \sum_{j \in D_j} C_{ij}^R x_j^R + \sum_{n \in T} \sum_{j \in D_j} C_{ij}^S x_j^S) \]

(2)

Turn multi objective function into a linear programming.

\[ Max \lambda_1 (1) + \lambda_2 (2) \]

(3)

Constraints

\[ \Pr\{ \sum_{n \in T} \sum_{l \in L} \sum_{s \in S_{lt}} \sum_{t \in T} \sum_{l \in L} n \in S_{lt} \sum_{t \in T} \sum_{j \in D_j} x_{ij}^{sl(t,t_2)} + \sum_{n \in T} \sum_{j \in D_j} x_j^S \leq D_j^{t_2} \} = \alpha \]

(4)

(4) shows that the requirement of empty containers during every time period from every supplying ports are satisfied.

\[ \sum_{t \in T} \sum_{l \in L} \sum_{s \in S_{lt}} \sum_{j \in D_j} x_{ij}^{sl(t,t_2)} \leq S_i^{t_2} + W_i^{t_2} \]

(5)

(5) shows the limit to number of empty containers supplied from supplying port.

\[ S_i^{t_2+1} = S_i^{t_2} - \sum_{n \in T} \sum_{l \in L} \sum_{s \in S_{lt}} \sum_{j \in D_j} \sum_{t \in T} x_{ij}^{sl(t,t_2)} + W_i^{t_2+1} \]

(6)

(6) shows the equation of supplying empty containers transferred.

\[ \sum_{t \in T} \sum_{l \in L} \sum_{s \in S_{lt}} \sum_{j \in D_j} \sum_{t \in T} x_{ij}^{sl(t,t_2)} \leq y_{slt}^{slt} * P_{slt} \]

(7)

(7) shows the limit to capacity of a ship used for empty container repositioning.

\[ l \in L, t \in T, \sum_{n \in S_{lt}} y_{slt} \leq Slt \]

(8)
(8) shows the limit to the number of ship used for empty containers repositioning. $X^{slt}_{ij}$, $X^{Rt}_{j}$, $X^{St}_{j}$ are non-negative integers and $y^{slt}$ is 0-1 negative integers.

**NUMERICAL EXAMPLES**

The net of lines is described in Fig. 1.

Figure 1. Net of lines.

Line $l_1: 1-3-5-7$, $l_2: 2-4-5-7$, $l_3: 2-4-5-6-8$, $l_4: 2-4-5-6-7$ and $l_5: 1-3-5-7-6-8$. $P^{slt} = 4 (slt \in Slt), T = \{t_1, \cdots, t_5\}, \text{Every time period has 3 days,}$

That is to say, $t_1 = \{1,2,3\}, t_2 = \{4,5,6\}, t_3 = \{7,8,9\}, t_4 = \{10,11,12\}, t_5 = \{13,14,15\}$. $\forall Slt = 2$, $(l = 1, \cdots, 5, t = t_1, \cdots, t_5)$. $P^{lat}_{l_1} = 235TEU$, $P^{lat}_{l_2} = 228TEU$, $P^{lat}_{l_3} = 219TEU$, $P^{lat}_{l_4} = 209TEU$, $P^{lat}_{l_5} = 206TEU$, $P^{lat}_{l_6} = 197TEU$, $P^{alt}_{l_1} = 218TEU$, $P^{alt}_{l_2} = 203TEU$, $P^{alt}_{l_3} = 209TEU$, $P^{alt}_{l_4} = 211TEU$. The renting cost and shortage cost of ports that need empty containers are shown below.

$C_4^{R(t_i)} = 7$, $C_6^{R(t_i)} = 7$, $C_7^{R(t_i)} = 11$, $C_8^{R(t_i)} = 9$, $C_4^{S(t_i)} = 13$, $C_6^{S(t_i)} = 15$, $C_7^{S(t_i)} = 14$, $S_1^l = 18$.

The solving result is shown below. $y^{lat}_{l_1} = 0$, $y^{alt}_{l_2} = 1$, $y^{lat}_{l_3} = 1$, $y^{alt}_{l_4} = 0$, $y^{alt}_{l_5} = 1$, $x^{2l_{(t_2)}}_{24} = 215$, $x^{3l_{(t_2)}}_{24} = 245$, $x^{2l_{(t_3)}}_{26} = 255$, $x^{3l_{(t_3)}}_{26} = 267$, $x^{4l_{(t_4)}}_{57} = 254$, $x^{5l_{(t_4)}}_{58} = 258$.

**REFERENCES**