Application of a Power Function to Allocating Seats in the European Parliament Before and After Brexit

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Abstract. The paper puts forward a proposal how to apply a power function to allocate the seats in the European Parliament. First, a sequence of populations representing the entitlements of member states is modified by a power function, and then the allocation is performed using the classical, divisor method of apportionment. The results are compared with those obtained by a well-known allocation method called Cambridge Compromise. Our construction and conclusions regard a specific problem of establishing the composition of the European Parliament, but they can be easily generalized to any case of degressively proportional allocation of goods and obligations.

Introduction

The principle of degressively proportional distribution of goods and benefits was legally endorsed by the Treaty of Lisbon. It is a compromise between the classical principle of proportionality originating from Aristotle and the willingness to respect a common interest of all participants of distribution, who are approached as one social community. It may happen that a common interest of all participants recommends a deviation from the classical rule for the sake of accomplishing common benefits. In such cases some adjustments of claims of individual agents are necessary, so as to achieve the goal that was mutually determined. In case of distribution of goods, larger agents, i.e. those with greater claims, abandon a part of their entitlements for the good of smaller agents.

Such a case occurred when deciding on the parliamentary representation of member states in the European Union. Significant differences in sizes of populations in member states made it impossible to allocate the mandates proportionally to the number of population. Such allocation is always possible in theory, but its application would peripheralize the roles of the smallest member states. A common good, i.e. the solidarity of community’s members having consideration for unbiased representation, was the cause that more populous states abandoned parts of their claims for the good of less populous states, so that the minimum representation is six mandates, and the maximum – ninety-six mandates. More precisely, the rule expressed in Article 9 of the Treaty stipulates: “The European Parliament shall be composed of representatives of the Union’s citizens. They shall not exceed seven hundred and fifty in number, plus the President. Representation of citizens shall be degressively proportional, with a minimum threshold of six members per Member State. No Member State shall be allocated more than ninety-six seats” [1].

Degressive Proportionality as a Rule of Allocating Mandates in the European Parliament

The application of the degressively proportional allocation generates many problems in practice because its formulation is very broad. In spite of an effort to define the term of degressive proportionality in a more precise way by the resolution. Proposal to amend the Treaty provisions concerning the composition of the European Parliament” [2], there are still many possible interpretations. The mentioned resolution lays down merely, that “the larger the population of a country, the greater its entitlement to a large number of seats”, and “the larger the population of a
country, the more inhabitants are represented by each of its Members of the European Parliament”. Therefore, we can formally put the idea given by the Treaty in the mathematical form as the four conditions:

1. for every \(1 \leq i < n\), \(s_i \leq s_{i+1}\),

2. for every \(1 \leq i < n\), \(\frac{p_i}{s_i} \leq \frac{p_{i+1}}{s_{i+1}}\),

3. \(s_1 = m = 6, s_n = M = 96\) and

4. \(s_1 + s_2 + \cdots + s_n = H = 751\),

where the sequence of numbers \(P = (p_1, p_2, \ldots, p_n)\) is the sequence of populations in \(n\) member states, and the sequence \(S = (s_1, s_2, \ldots, s_n)\) determines the numbers of mandates allocated to them. The conditions (1) and (2) define the idea of degressive proportionality, while (3) and (4) provide the so-called boundary conditions of an allocation. In this case, subject to boundary conditions for \(m, M, H\) there still exist many sequences \(S\), which satisfy them, and a problem arises how to select a concrete allocation. In consequence, an area of both political as well as scientific discourse [3, 4, 5, 6, 7, 8, 9, 10] is opened.

Adjustment of Claims and Weakly Degressive Proportionality

The idea of adjusting the claims of agents consists in transforming the sequence of claims \(P\) into another sequence \(P'\), with the result that a proportional allocation performed regarding the sequence \(P'\) is degressively proportional regarding the sequence \(P\) [11]. This approach differs from those reported earlier in the literature, as the burden of solving the problem is shifted from finding the sequence \(S\) to finding the sequence \(P'\), because given this sequence, it suffices to apply conventional methods of proportional distribution in order to find out a concrete allocation. This idea is validated by the above-mentioned concept of abandoning a part of one’s entitlement for the sake of attaining the common goal that consists in pledging a fair representation to every member of the community.

This is in agreement with the provisions of the Lisbon treaty and its further legal amendments. One of the most significant proposals of allocating seats in the European Parliament, the so-called Cambridge Compromise, abandons the postulate of satisfying the condition \(\frac{p_1}{s_1} \leq \frac{p_2}{s_2} \leq \cdots \leq \frac{p_n}{s_n}\) by the sequence \(s_1, s_2, \ldots, s_n\) and only requires its satisfaction before rounding to integers. The degressive proportionality in this sense is called “unrounded degressive proportionality” in the literature [12]. It is also legally supported by the report [13], that puts forward a proposal of the European Parliament resolution, stipulating that “the ratio between the population and the number of seats of each Member State before rounding to whole numbers shall vary in relation to their respective populations in such a way that each Member of the European Parliament from a more populous Member State represents more citizens than each Member from a less populous Member State and, conversely, that the larger the population of a Member State, the greater its entitlement to a large number of seats”. As a consequence, if a sequence \(P'\) is degressively proportional regarding the sequence \(P\), then obviously any other sequence that is proportional with respect to \(P'\) is also degressively proportional with respect to \(P\), thus satisfying the condition of weakly degressive proportionality.

Application of Power Function for Adjustment of Claims

Applying the idea of claims adjustment does not eliminate the problem of non-uniqueness in the task of degressively proportional allocation. The non-uniqueness is merely transferred to the task of finding the sequence \(P'\) that can be determined in many ways. One approach is an attempt to apply methods used in case of the construction of allocation function [14]. Allocation functions
considered in the literature are transformations of the interval \([p_1, p_n]\) to the interval \([m, M]\), i.e. they describe a desired distribution. In case of adjustment of claims, one can seek such function \(f: [p_1, p_n] \rightarrow [p'_1, p'_n]\) that the sequence \(P' = f(P) = (f(p_1), f(p_2), \ldots, f(p_n))\) is degressively proportional with respect to the sequence \(P\), and additionally, \(p'_1 = f(p_1) = \frac{M}{H}p_n = f(p_n) = \frac{M}{H} p'_n\) and \(f(p_1) + f(p_2) + \cdots + f(p_n) = p_1 + p_2 + \cdots + p_n = V\). The conditions \(p'_1 = f(p_1) = \frac{M}{H} p_n\) and \(p'_n = f(p_n) = \frac{M}{H}\) obviously represent an answer to the question about the numbers of population of the smallest and of the largest member state allowing them to be allocated \(m\) and \(M\) mandates, respectively, as required by the boundary conditions in the proportional distribution. Therefore, they can be considered equivalent to boundary conditions.

Due to the basic principle of proportionality the most natural function to solve this problem is definitely a linear function \(f(x) = ax + b\). However, it cannot be used because it only depends on two parameters, while the three independent conditions must be satisfied: \(f(p_1) = \frac{M}{H}p_n\), \(f(p_n) = \frac{M}{H}\) and \(f(p_1) + f(p_2) + \cdots + f(p_n) = p_1 + p_2 + \cdots + p_n = V\). As a consequence, most frequently this results in a contradiction. A natural generalization of a linear function is a power function \(f(x) = ax^c + b\). In this case we deal with three parameters and the three above-mentioned conditions typically yield a unique solution.

Case study

A power function \(f(x) = ax^c + b\) with parameters \(a, b,\) and \(c\) which are solutions of equations \(f(p_1) = \frac{M}{H}p_n\), \(f(p_n) = \frac{M}{H}\) and \(f(p_1) + f(p_2) + \cdots + f(p_n) = V\), where \(m = 6, M = 96\), \(H = 751\), and \(V\) is equal to the total population of all member states in the European Union, was applied to determine the allocation of mandates in the European Parliament, taking into account the sequence of populations based on the Council Decision 2016/2353 [15]. The task was executed twofold, i.e. firstly, assuming that the United Kingdom is a member state of the European Union, and secondly, assuming that the Brexit was accomplished [16]. The estimated functions are of the form: \(f(x) = 2.8999x^{0.9238} + 3599875.925\) and \(f(x) = 40.8940x^{0.7737} + 2617837.116\), respectively. Then, adjusted sequences of populations \(P'\) were determined, and based on them, the allocations were found by means of a classical divisor method, that is known in case of proportional allocations [17]. The rounding rule applied was to the nearest integer. In each case the obtained results (PO) were compared with allocations determined by means of the Cambridge Compromise (CC). The results are presented in Table 1.

**Table 1. Allocation of mandates in the European Parliament by means of a power function and the Cambridge Compromise (Source: own works).**

<table>
<thead>
<tr>
<th>Lp.</th>
<th>Country</th>
<th>P</th>
<th>P’</th>
<th>PO</th>
<th>CC</th>
<th>P’</th>
<th>PO</th>
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<td>6</td>
<td>3789503, 29</td>
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<tr>
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Conclusion

In each of the two analyzed cases the sequence $P'$ is degressively proportional with respect to the sequence $P$. This means that the determined sequences of allocations satisfy the conditions required by the treaty of Lisbon in the sense of unrounded degressive proportionality. Obviously, these sequences are uniquely determined. As compared with the Cambridge Compromise, they favour medium-populous countries at the expense of most populous countries.

This bias is an evident consequence of the properties of a power function. The more the exponent $c$ deviates from unity, i.e. the more a power function deviates from a linear function, the more noticeable differences in allocation at a disadvantage of the most populous countries. In cases under study, i.e. with the UK, and without the UK, the respective numbers are roughly 0.9238 and 0.7737, and that is why in the second case the largest countries suffer the loss of more mandates as compared with the Cambridge Compromise.

The obtained results allow for a statement that application of a power function better reflects the idea of sharing entitlements of larger agents with smaller ones. Given the current structure of populations in the member states of the European Union compared with the Cambridge Compromise, the number of countries losing their mandates is decisively smaller than the number of countries gaining mandates. This is an advantageous fact from the viewpoint of a group’s solidarity that leads to a more balanced distribution of seats in the European Parliament. Thus, if a consensus is reached regarding an abandon of proportionality, then the proposal to apply a power function can turn out to be interesting for practical reasons.

Both in a case with the UK, and without the UK, as compared to the Cambridge Compromise, the same countries are losers, but in the first case the number of lost seats is 14, while in the other one it is 42. This suggests a conclusion that after Brexit the composition of the European Parliament can be reduced with no harm to the relative, internal system of forces. Nevertheless, political endorsement is still necessary to indicate the extent of reduction.

References


