On the Basic Course of Inquiry-based Mathematics Learning

Yan-Wu DU

School of Mathematics and Statistics, Linyi University, Shandong, P. R. China
dyw6668@163.com

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Abstract. Inquiry-based mathematics learning takes problem solving as its main form. This paper aims at the self-constructing course of mathematics knowledge in inquiry-based learning from problems representation, conjecture, evidence collection, forming explanation and survey, which has important theoretical and practical significance to the further understanding of the nature of the inquiry-based mathematics learning.

Introduction

Inquiry-based learning, with problem solving as its main form, is the course of the connection between new information and prior knowledge by using logical ideas and imaginable ideas. It is also the process of the independent mathematical knowledge construction. It is not the course of trial errors repetition, not the routine “do” without thinking. During this course, students, by positive thinking, make the new questions and the existing cognitive structure work together from the representation of information. On the one hand, we reasonably generate new problems and try to solve them. On the other hand, the existing cognitive structure will also be adjusted and changed.

Representing Problems, Forming Conflicts

In the inquiry-based mathematics learning, the learners find and realize the problems, only showing that there is a problem in his mind. In order to continue the exploration, problems should be presented further. i.e., learners should understand the problems, be clear of the structure, confirm the variations related to problems, and the relations among them based on the background information. They can identify the key information and understand problems correctly and accurately through representing them. The subjects decide what to extract knowledge or what strategies they adopt to solve the problem based on the mental representation formed by problems. If the problems are not suitable for the presentation, then they are difficult to solve or cannot be solved.

Therefore, research on problem presentation of mathematical problem has a vital role in solving the problems; its quality directly affects the degree of difficulty to solve the problem. Due to the nature of different characterization of the problems, students usually have two results about the problem presentation: one is that the problem of the initial state and the goal state is clear through the representing, thus, the pathway to achieve the targets is familiar. That is, the subjects have already accumulated the corresponding schema in mind to solve the problems. The solutions to the problems can be extracted directly from their schema. So the role of problem presentation only lies in confirming the corresponding conditions and promoting the migration of schema in a similar situation; the other is that the subjects have made the problem of initial state and target state clear, but it can't use the existent schema for
problem solving. That is, by characterization of the main body, we can found the gap between
the existent knowledge, ideas and new problems, and the existent knowledge can’t solve the
new problems. The problems exist in Inquiry-based mathematics learning is ill-structured, and
thus the results of problem representation should belong to the latter. In this case, the students
will produce a kind of cognitive imbalance, enter into cognitive conflict state, produce the
need to solve the problem, which in turn caused to eliminate the exploratory behavior of
conflict, so the inquiry-based learning can go on.

Thinking Carefully, Proposing Conjecture

Under the action of the exploring need and mental set produced by conflict stage, students
will have an effort to eliminate the behavior of the inner conflict; this is the second phase of
inquiry-based learning: thinking carefully and presenting a conjecture. Mathematical
conjecture is based on certain mathematical knowledge and mathematical facts, to make a true
judgment on the unknown variables and their relationships; it is the embodiment of scientific
hypothesis in mathematics. Any formation of the scientific theory has a process of being from
potential to obvious; need the development from hypothesis (guess) to theory. As the latent
form of mathematics, mathematical conjecture is often the forerunner of the theory, once it
was proven; it will rise to the mathematical theory. Therefore, mathematical conjecture plays
an important role in the development of mathematics theory and innovation; it is also an
important element of inquiry-based mathematics learning.

Forming and putting forward the mathematical conjecture needs a complex process of
thinking activity. It needs to take positive thinking on the basis of existing mathematical
knowledge and materials in the embryonic stage of conjecture, not only some logical thinking
methods such as analysis, synthesis, induction, analogy, but some imaginable thinking
methods as association and imagination should be used. Through multi-level association,
imagination and colorful activities such as induction, analogy, to form a variety of speculative
assumption, and then using the method of analysis, comparison, comprehensive those way to
screen, amend or supplement, to confirm the mathematical conjecture. Therefore, the process
of formation of mathematical conjecture is a process of consciously applying theory thinking
method; mathematical guess is the result of mathematical theory thinking activity. It is clear
that, mathematical conjecture is not general conjecture in the inquiry-based mathematics
learning, nor blind speculation and subjective assume, it is based on the mathematics
empirical fact that students have, based on mathematical knowledge, to make the speculation
and judgment for the unknown nature of mathematical object, quantity, and the mutual
relationship.

In the whole process of proposing the mathematical conjecture, students should always pay
attention to analyze and understand the mathematical problems; firmly grasp the purpose and
the main characteristics of the problem, to focus on the problem and its answer standard.
Students can put forward problem solving conjecture, can also made guess through
cooperation. Students discuss the possibility of problem solving through cooperation, and put
forward all kinds of guess, discuss the guess is reasonable, this not only can fully arouse the
enthusiasm of learning, also can make up for the defect of personal knowledge and the lack
of considering, make the guess more reasonable. The process of putting forward conjecture,
for students, is not only the process of integration of knowledge; it is also the process that
beyond the existing knowledge, update the understanding. It is visible that inquiry-based
mathematics study is given priority to students’ exploration, even through, the role of the
teacher also should not be neglected, teachers should pay attention to teach students some thinking methods, especially the often-used methods in mathematical inquiry, such as analogy, induction and deduction. Except that there also have opposite thinking method, the transformation thinking method, divides thinking method and so on, teacher should consciously foster and train students' mathematical thinking quality.

The correct degree differs from each other since there are in great varieties in students’ own ideas, thus, many kinds of conjectures existing. The internal consistency of conjecture and premise needs further testing, which makes the construction process into third link: collecting evidence and analyzing reasons.

**Collecting Evidence, Analyzing Reasons**

Further investigation is an important way to confirm whether the mathematical conjecture is right. Collecting evidence is a process for the students to find a way to gain the resources to confirm their own assumptions. Collecting evidence methods include experiment, observation, questionnaire, interview method, classroom quiz method, etc. For example, in the exploration of the volume of a "ball", the students can make use of the half spherical of a radius R, the barrel of radius R and high R, and a cone and some fine sand that the teacher prepared, students observe the relationship between the size of their volume under the guidance of teachers ((V\text{cylinder}>V\text{hemisphere}>V\text{con}, 3/3 \pi R^3>V\text{hemisphere}>1/3 \pi R^3), so what’s the size of V\text{hemisphere}? Students may guess V_{\text{hemisphere}}=2/3 \pi R^3. Then use experiments to test hypotheses: (1) put the cone into the barrel; (2) the hemisphere containers filled with fine sand, pour into the bucket, then students will find the barrel is filled up, this is because V\text{cylinder}-V\text{cone}=V\text{hemisphere}, then V_{\text{hemisphere}}= 2/3 \pi R^3, so V_{\text{sphere}}=4/3 \pi R^3. Then students can use logic to prove further validate assumptions, that is to say, the students can prove it with Zu Heng principles under the guidance of the teacher, here, the "watch - guess - experiment", is just the enrichment of mathematicians thought activity, in the experiment, the students also real like a small mathematicians that involved in the problem solving process, through careful observation, bold guess, experimental verification, the theory has been proved that it is scientific conclusion, students in the osmosis, accept the idea of a mathematician, and cultivate the rigorous doing scholarly research attitude and courageous enough to probe into the scientific spirit, so that students in mathematics experiments personally involved in the process of constructing mathematics, learned the real math.

After students’ proposing the assumption of solving problem, the assumption will be the guide for exploring, and guide the students to verify each factor of the problem, to see whether these factors were associated with the proposed solution, and thus decided whether the solution should be insisted or other solutions should be put forward. Students need to collect the related information according to the assumption. There are a lot of methods to collect data, they can consult books, query network and borrow books through the library, and also can consult experts or negotiate with partners.

In the process of collecting data, students are not simply needed to find out the information of solving problems, meanwhile, they also need to refine and organize these information, so that can facilitate the extraction and migration of the information in the process of solving problem. The refinement and organization of information is to integrate the obtained
information with existing knowledge content in appropriate clues, and form the networked cognitive structure. Refined and organization methods include: to find out link or similarities between new information and prior knowledge content (the concept of upper concepts and subordinate concept), to understand the meaning of new information (use their own words to explain the information, give examples that appropriate for the situation), to collect data in critical way, to make the appropriate classification of data collected, and extract the key points, etc. Students outline their learning materials, draw diagram and concept map, those are the specific methods of the refinement and organization of the learning content. In teaching, students generally collect and process the data in small groups as one unit. Data collection can be done in the classroom, can also be carried out at before class, after class or playtime.

**Confirming Assumptions, Forming Explanations**

The exploration needs abundant evidence, and also corresponding reasonable explanations to the evidence. There is only “the existence of matter” through the evidence collection in observations and surveys of mathematics, that is, their value cannot be realized without verifications or connecting conjectures, variations and problems. And the evidence explanation is a series of thinking activities including analyzing, comparing, abstracting the evidence gained from observations and surveys of mathematics. It aims to demonstrate contents of evidence and the internal relations between evidences and assumption. Thus, new understanding can be formed beyond prior knowledge and observation results by connecting the evidence gained from mathematics experiments and prior knowledge. Learners deduct relations among different variations according to theory logics based on the information and the fact. In other words, it is a kind of transformation from “facts” to “theory”. If the result deduction conforms to the assumption, the thinking is the answer to the problem. Then the assumption is reasonable, or the assumption is false. At the same time, learners need to present a new assumption on the basis of results and restart activities.

For example, after students in groups dealing with data collections, they should make sure practical consequences conform to assumptions and make reasonable explanation. Besides, many factors should be considered, for example, whether the data are reliable and objective, whether the results are universal and applied to other situations.

However, during inquiry-based mathematics learning, the subjects cannot drive rules from many original data of mathematics experiments. Consequently, the data change is necessary. The specific strategies are as follows: (1) drawing the data into the bitmap; (2) Using the tabulation in representing data; (3) Showing relations among variations in mathematical expressions. To ensure complex relations among variations, sometimes the subjects have to make multiple changes for data, such as, representing data in diagram, performing functional relations among data according to the diagram. Usually, data explanation is based on assumption. On the premise of data received from prior knowledge and transformation, data explanation constructs contents and internal connections. In fact, it is a process that perceptual knowledge sublimates into rational knowledge. Furthermore, some abnormal condition may occur, for example, there are unusual data. This means, not all data follow some kind of rule. Even the trends that some data give do not match the hypothesis. In this circumstance, firstly, students should think reasonably and compare different kinds of data. Secondly, they should try to analyze various characterizations of data and the reason of phenomena. Finally, what kind of data should be kept is decided. When the trend which the evidences showed deviates from the assumption, learners have to make decisions about their existing. That is, learners
must judge if the experimental data are accurate, meanwhile, they have to do second thoughts about the assumption's correctness. Learners should give up the conjecture and put forward new one if the analysis indicates the conjecture is false, and then collect evidence anew, explain renew, that means reenter the constructive stage.

Giving up does not mean failure; it is just a choice after full account and systematic contrast among different conjectures, evidences and existing knowledge. Thus it can be seen that learners should make extensive use of their ability of thinking in the process of conjecture and forming explanation.

Rethinking Survey, Subliming New Information

Inquiry-based learning does not stop at this point after learners had participated actively in constructing course and got the new meaning of new problems by means of the final explanation. Only when the new meaning connects with the already known information and builds a new follow-up learning, the Inquiry-based learning is effective. Integration is the process of creating links between the meaning of new problems and existing construction and forming a cognitive structure that can be used for follow-up study. Generally, it is divided into two stages:

In the first stage, learners should expound their study to provide opportunities to others for raising questions, examining evidences, showing clearly the point that is against the evidences or giving different explanations to the same phenomena. Communication is essential to expand understanding to the study and trigger new problems and prompt the study to next circulation. Learners do self-examination to reasoning and thinking activity in the process from finding new problems to gaining its meaning. The self-examination includes the affirmation of new problems' condition and target, the related knowledge and experience and forming explanation on this basis. The implied principle and key points are extracted and purified based on the communication and reflection in the process of inquiry to make all kinds of meanings crystallized. One is the intensively understanding to the problem itself, another is the understanding to the nature and exploration of science and how to acquire the ability of scientific exploration. The latter means exploration is not just the combination of some facts and their forming process; it also emphasizes the scientific spirit and exploring ability.

In the second stage, according to the logical route for the meaning ratiocination in the problem's inferential process, learners are linked various principles and key points that purified in the first stage, and built relationship with conjecture and experimental data, and already known related backgrounds and experiences to form a new united and integrated cognitive structure, and finally achieve to the overall integrated goal. The new cognitive structure indicates the forming of new knowledge in the current Inquiry-based learning, but it also represents the already known cognitive structure as a footstone of next circulation.

Surveying makes the Inquiry-based learning amount to its final sublimation. So far, from a cognitive process point of view, a whole explorative circulation has been finished, and at last, the new cognitive structure as a basis takes part in next inquiry-basic process.

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References


