Do the Interest Rates Adjust Asymmetrically across Business Cycles?

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Keywords: Markov switching, Vector error correction, Term structure of interest rates, Business cycle, Asymmetric adjustments.

Abstract. Since the late nineties, literature devoted to the term structure of interest rates suggests that the dynamics of interest rate spread might be well described by nonlinear models. This paper examines this possibility for U.S. long-term and short-term interest rates between 1962M1 and 2015M11 by employing the Markov-switching VEC model, which allows for the asymmetric adjustment of interest rates to the long run equilibrium across regimes. Our results support the long-term interest rate adjusts asymmetrically towards the long run equilibrium in different regimes. Besides, our findings point to the regimes identified from the term structure of interest rate by MS-VEC model is closely related to the business cycle dating by the NBER.

Introduction

Since the late nineties, both theoretical and empirical analysis devoted to the term structure of interest rates have shift from a linear towards a nonlinear setup. Basically, the term structure models developed by, e.g., Naik and Lee (1997), Evans (1998), and Bansal and Zhou (2002) imply that it is necessary to consider the regime switching in the model setup. The underlying idea is that the aggregated economy is subject to discrete and persistent changes in the business cycle. The business cycle fluctuation combined with the corresponding monetary policy impact the term structure of interest rates, leading to the nonlinearity of the term structure.

In this paper we consider the possibility that a vector error corrected model with Markov switching in the adjustment parameter (MS-VECM hereafter) would provide a better empirical description of the term structure of interest rates. After its first application to the identification of expansion and recession regimes in economic activity, the Markov switching (MS, henceforth) model has quickly been applied to model other macroeconomic or financial variables, such as oil price, stock market return, exchange rate and interest rate etc. Since MS framework could be used to capture crucial statistical features of macroeconomic or financial variables such as skewness, volatility clustering or mean reverting, and it is suitable for modeling nonlinear dynamics with an understandable economic interpretation, hence it has been wildly employed in identifying business cycle or financial cycle. MS-VECM employed
in this paper, initially developed by Krolzig (1999), combines the Markov switching feature and the linear cointegration model. Basically, this paper investigates the relationship between the 3-Month Treasury Bill Rate (DTB3) and 10-Year Treasury Rate (DSG10) from January 1962 to November 2015 by employing the MS-VECM, which includes two regimes: low-and high-volatility regimes. It worth noting that in our analysis the two regimes are classified endogenously based on specific characteristic in each regime. First, our empirical findings point out the term structure is more likely to be in the high-volatility regime when the economy experiences a recession. Second, the long-term interest rate adjusts asymmetrically towards the long run equilibrium across regimes, whereas the short-term one does not.

The paper is organized as follows. Section 2 presents and discusses the methodology used in the empirical analysis. Section 3 describes the implementations of estimating the MS-VECM and discusses the empirical results. Section 4 concludes.

**Methodology**

Noting that the 3-Month Treasury Bill Rate (DTB3) and 10-Year Treasury Rate (DSG10) are potentially cointegrated, but their dynamic interactions may exhibit regime switching parameters, our analysis employs the following MS-VECM,

\[
\Delta X_t = \mu_{S_t} + \sum_{k=1}^{p-1} \Gamma_{k,S_t} \Delta X_{t-k} + \Pi_{S_t} X_{t-1} + \varepsilon_t, \quad t = 1, 2, \ldots, T
\]  

(1)

where \(p\) is the lag length of the MS-VAR model, \(\varepsilon_t \sim N(0, \Omega_{S_t})\), and \(\Omega_{S_t}\) is a \((2 \times 2)\) positive definite covariance matrix. \(S_t\) presents the unobserved state or regime variable, which is conditional on \(S_{t-1}\), independent of past \(X_s\), and assumed to follow a q-state Markov process. From the essence of economics, two-regime, that corresponds the recession-expansion cycle, is sufficient to describe the dynamic relationship between interest rates with different maturities, we thus set \(q=2\). Hence the transition probability matrix given by \(P = \begin{bmatrix} p_{ii} & p_{ij} \\ p_{ji} & p_{jj} \end{bmatrix}\), \(\sum_{j=1}^{2} p_{ij} = 1\) where \(p_{ij}\) represents the probability of transition from \(i\) state to \(j\) state. Besides, the MS-VECM also allows the variance matrix \(\Omega_{S_t}\) to dependent on the regime variable.

The \(\Pi_{S_t}\) matrix, which equals to \(\alpha_{S_t} \beta_{S_t}^{\prime}\), contains the long-run relationships between short-term and long-term interest rates \(\beta_{S_t}^{\prime}\) and the adjustment parameter when the interest rate goes beyond the long-run relationship \(\alpha_{S_t}\) in the MS-VEC model. Since this paper is aim to exam whether the adjustment of interests rates is asymmetry or not in two different regimes. This matrix \(\Pi_{S_t}\) in this paper is assumed as \(\Pi_{S_t} = \alpha_{S_t} \beta_{S_t}^{\prime}\), where \(\beta\) is the state-independent long-run cointegration matrix and \(\alpha_{S_t}\) are the state-dependent short-term adjustment matrix.

To estimate the MS-VECM model, the Bayesian MCMC estimation based on the Gibbs sampling is employed. The MCMC takes the regimes as distinct sets of parameter, the data augmented approach is applied to deal with the estimation issue for the latent state variables.
Data and Empirical Findings

Data

Our data set comprises monthly observations spanning from 1962:01 to 2015:11 for 3-Month Treasury Bill Rate (DTB3) and 10-Year Treasury Rate (DSG10), coming from the Fed St Louis Database (The 10-Year Treasury Rate (DSG10) data begins from January 1962, which corresponds to the earliest date in the Fed St Louis Database). The two series are plotted in Figure 1.

![3-month and 10-year interest rate](image)

Figure 1. 3-Month treasury bill rate and 10-year treasury rate.

Asymmetric Adjustment across Regimes

We estimate both linear VEC and MS-VEC models over the full sample period 1962:01-2015:11 with 647 observations. Table 1 reports estimation results and model selection criteria for the MS-VEC model given in equations (1) and (3). The lag order is selected as 2 according to the minimum AIC value in a VAR for both linear VEC and MS-VEC models. The MS-VEC model is estimated by Bayesian Monte Carlo Markov Chain (MCMC) method with Gibbs sampling.

First, we use the likelihood ratio statistic to test the nonlinearity, in which the linear VECM as the null hypothesis and MS-VECM as the alternative one. Since unidentified parameters exist under the null, the LR test is not standard. The \( \chi^2 \) p-values (in square brackets) with degrees of freedom equal to the number of restrictions as well as the number of restrictions plus the numbers of parameters unidentified under the null are reported, and the p-value of the Davies’ test is presented in squared brackets as well. We find that the linearity is rejected. Henceforth, we focus on discussing the estimates of the nonlinear model, MS-VECM.

One of the aims of this paper is to see the asymmetrical adjustments toward the long-run equilibrium across regimes. According to the estimates of the short run correction parameters, \( \alpha \), in Table 1, we could notice that the adjustment of long-term interest rate exhibit significantly asymmetric across regimes. More specifically, the speed of long-term interest rate adjustment to long-run equilibrium is quicker under high-volatility regime than that under low-volatility one, \( |\alpha_{S=2}| > |\alpha_{S=1}| \). By contrast, we find no evidence supporting the asymmetricity for short-term interest rate.
Table 1. Estimation results for the MS-VEC model.

<table>
<thead>
<tr>
<th>Model selection criteria</th>
<th>MS(2)-VEC</th>
<th>Linear VEC(2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Log likelihood</td>
<td>-82.83726</td>
<td>-499.34215</td>
</tr>
<tr>
<td>AIC criterion</td>
<td>-1695.26638</td>
<td>-1308.76149</td>
</tr>
<tr>
<td>HQ criterion</td>
<td>-1713.65929</td>
<td>-1315.82501</td>
</tr>
<tr>
<td>BIC criterion</td>
<td>-1630.81987</td>
<td>-1279.00749</td>
</tr>
<tr>
<td>LR linearity test</td>
<td>Statistic</td>
<td>p-value</td>
</tr>
<tr>
<td></td>
<td>833.00977</td>
<td>(\chi^2(9) = [0.0000]^{***})</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(\chi^2(11) = [0.0000]^{***})</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(Davies = [0.0000]^{***})</td>
</tr>
<tr>
<td>Short run correction parameters</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(\alpha_{s=1} = \begin{pmatrix} -0.02^{**} \ 0.003 \end{pmatrix}^{(0.03)} )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(\alpha_{s=2} = \begin{pmatrix} -0.06^{**} \ 0.057 \end{pmatrix}^{(0.027)} )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Transition probability matrix</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(P = \begin{bmatrix} 0.955 &amp; 0.1088 \ 0.045 &amp; 0.8912 \end{bmatrix} )</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Regime properties</th>
<th>Probability</th>
<th>Observations</th>
<th>Duration (months)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Low-volatility regime (Regime 1)</td>
<td>0.71</td>
<td>460</td>
<td>22.22</td>
</tr>
<tr>
<td>High-volatility regime (Regime 2)</td>
<td>0.29</td>
<td>187</td>
<td>9.19</td>
</tr>
</tbody>
</table>

Note: 1. The MCMC estimates are based on 20000 burn-in and 50000 posterior draws. 2. Regime properties include ergodic probability of a regime, observations falling in a regime based on regime probabilities, and average duration of a regime. 3. ***, ** and * represent significance at 1%, 5% and 10% levels, respectively.

**Term Structure Regime vs. the Business Cycle**

The long-run average probabilities of a low and high-volatility regime are 0.71 and 0.29, respectively. That is, for our 647 observations, we expect the low-volatility regime to occur on 460 occasions, and 187 for high-volatility one, implying an average 22.22-month duration in the low-volatility regime and 9.19 in the high-volatility one. The actual outcomes over our sample is 468 occasions in the low-volatility regime, and 179 in high-volatility regime, which are quite close to the expectation of the MS-VECM.

In order to see the links between the term structure regime and the recession-expansion cycle, Figure 2 plots the estimates of the smoothed probabilities of high-volatility regime...
(regime 2) with shaded (grey) bars that correspond to NBER business cycle recession. Visually, Figure 2 reveals a strong correspondence between the high-volatility regime and the business cycle recession.

![Smoothed probability estimates of high volatility and recession](image)

Figure 2. Smoothed probability estimates of high-volatility regime.

Besides, as can be seen in Table 2, we contact a comparison of the term structure regime from MS-ECM and the NBER business cycle for the entire sample period of 647 observations. The high- (low-) volatility regime occurs when its smoothed probability is larger than or equal to (less than) 0.5. Recessions and expansions depend on the NBER business cycle dates. The ratio indicates that when the economy is experiencing a recession, the probability of the term structure of interest rates staying in the high-volatility regime is 67%, twice as large as that in the low-volatility one, 33%. By contrast, the possibility of term structure in the low-volatility regime conditional on business cycle expansion, 79%, is triple larger than that in the high-volatility regime, 21%. The results showed by Table 5 imply a significant relationship between recession and high-volatility regime, or between expansion and low-volatility regime.

<table>
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<tr>
<th></th>
<th>High Volatility</th>
<th>Low Volatility</th>
<th>Count</th>
</tr>
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<tbody>
<tr>
<td>Recession</td>
<td>59 (67%)</td>
<td>29 (33%)</td>
<td>88</td>
</tr>
<tr>
<td>Expansion</td>
<td>120 (21%)</td>
<td>439 (79%)</td>
<td>559</td>
</tr>
</tbody>
</table>

**Table 2. High- and low-volatility VS recession and expansion outcomes.**

**Conclusion**

This paper examines the relationship between the 3-Month Treasury Bill Rate (DTB3) and 10-Year Treasury Rate (DSG10) from January 1962 to November 2015. We find that the 3-Month Treasury Bill Rate (DTB3) and 10-Year Treasury Rate (DSG10) exhibit non-stationary characteristic. Moreover, these two series are proved to be cointegrated, leading to the methodology of the MS-VECM. We find that the high-volatility regime more frequently exists prior to the 1990s, and the low-volatility one occurs more often after 1990s. Secondly, with allowing for the distinct short-run adjustment parameters in the MS-VECM

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1. The smoothed probabilities equal the means of the 30000 posterior draws for each time period based on the FFBS algorithm.
setup, we find that the evidence of asymmetric adjustments of interest rates to the long run equilibrium in difference regimes. More specifically, the long-term interest rate adjusts more quickly towards the long run equilibrium in the high-volatility regimes than in the low-volatility one. Thirdly, using the NBER business cycle dates, we examine the relationship between the recessions and the smoothed probability of a high-volatility regime identified from MS-VECM. We find it more likely that the economy experiences a recession when high-volatility regime occurs, while the economy is more possible stays in an expansion period when low-volatility regime takes place. This finding suggests that the regimes in the model are related to the NBER business cycle indicator, implying that the term structure regimes confirm and complement the real business cycles.

Acknowledgement

The views expressed in this paper reflect those of the authors. The usual disclaimer applies. This research has been conducted as part of the NSFC Project No. 71403295.

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