A Novel Fuzzy Methodology for Quality Function Deployment Based on TOPSIS

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Abstract. Quality function deployment (QFD) is widely accepted as a product-designing tool. QFD could turn customer quality requirements into the engineering measures of a product. Ranking the importance of engineering measures according to the input data is a critical procedure in this fuzzy methodology. When the relative weights of quality requirements of customer and the relationship measures between quality requirements and engineering measures are evaluated by Interval-valued intuitionistic fuzzy sets (IVIFS), calculating the importance of each engineering measure depends on the operations between IVIFSs. With the target of prioritizing engineering measures in this fuzzy methodology, this paper presents a novel method by integrating the new aggregation operators based on the Łukasiewicz triangular norm, which is monotone with respect to the total order of IVIFS, and the TOPSIS-based nonlinear-programming methodology. In addition, an application example of diesel engine design is provided to test the practicality and extensibility of the novel fuzzy methodology.

Introduction

The development of new products play an extremely significant role in improving the competitiveness of enterprises. However, products design is commonly seen as a badly complex and burdensome mission. Quality function deployment (QFD) is a widespread customer driven product development approach initiated in the 1960s [1]. As QFD is a well-targeted work coordination method, QFD has been widely used in industry. In the process of using this method, all the key points related to customers’ needs are systematically taken into account.

The most common QFD development pattern is Four-stage mode defined by American Supplier Institute. The four stages of interconnection are as follows: in stage 1, translate quality requirements (QRs) into engineering measures (EMs) or product characteristics, and certify the evaluation values of the respective engineering measures (EMs) or product characteristics by experts according to marketing competing assessment and technique competing assessment; in stage 2, translate important engineering measures into component characteristics; in stage 3, translate critical component characteristics into process parameters; in stage 4, translate key process parameters into detailed production control standards. According to the definition of the four-stage model, each stage have inputs (WHATs) and output (HOWs), and the inputs of each stage come from the outputs of pre-stage. The core
procedure of each stage is the constructing of house of quality (HOQ), which is the basic tools as well as essence of quality function deployment systems. The HOQ of each stage can be expressed by a two dimensional matrix of inputs and output [2]. For instance, the quality requirements of customer (WHATs) distinguished for the product could be definitely translated into corresponding engineering measures (HOWs) through establishing HOQ of stage 1. The expression and translating approach of the else three QFD stages are no different in essence with the first one [3]. Thus, a majority of QFD researches didn't cover all four stages, but concentrated on the first stage to prioritize EMs from QRs [4,5,6], which is also the focus of our current research in this paper.

Properly defining the input and output of a HOQ is often subject to the human subjective preference and heterogeneity of the customer and the company’s technical staff. A number of studies have been accomplished with the purpose of dealing with the uncertainties of definition and evaluation. In earlier researches, the aim is finding a solution to prioritize EMs evaluated by fuzzy sets. Later, interval-valued fuzzy set (IVFS) extended from fuzzy sets used in Fuzzy QFD, where the evaluation values of an element were represented by an interval instead of exact numerical value. In recent researches, interval-valued intutionistic fuzzy set (IVIFS) was developed as further generalization of Atanassov’s intuitionistic fuzzy set (AIFS). Then, A few aggregation operators for IVIFS introduced by Xu (2007) which were used in Multi-objective decision-making problem [7]. With the development of fuzzy system theory, the potential demand for fuzzy system theory is increasing, especially in the operation and management, there are many complex problems to be solved using effective tools.

Based on this analysis, in order to minimize the effects of human subjective perception and customer heterogeneity, a novel method for prioritizing EMs in fuzzy QFD is proposed by integrating the TOPSIS-based nonlinear-programming methodology and a new IVIFS aggregation operators based on the Łukasiewicz triangular Norm.

We have organized the remainder of this paper as follows: Section 2 introduces the basic concepts of IVIFS as well as TOPSIS method. Section 3 proposes a fuzzy method for Quality Function Deployment. Section 4 presents an example to demonstrate the proposed fuzzy QFD method. In the last section, conclusions and suggestions for future research are drawn.

Preliminaries

Interval-valued intuitionistic fuzzy set

At first, we explain some basic Definition related to AIFSs and IVIFSs. The AIFS [8], proposed by Atanassov (1986), was an extension of the fuzzy set.

**Definition 1.** Let $X$ be a nonempty set; a AIFS $A$ drawn from $X$ is defined by an object of the following form:

$$A = \{(x, \mu_A(x), \nu_A(x)) | x \in X\},$$  \hspace{1cm} (1)

Where $\mu_A(x)$ and $\nu_A(x)$ are the membership function and non-membership function of $x$ in $A$ respectively, which subject to $0 \leq \mu_A(x) + \nu_A(x) \leq 1$, $\forall x \in X$.

However, due to the complexity and diversity of practical product-design problems, it is complicated for product-designers to accurately determine the exact value of membership degree and non-membership degree, so it is convenient to give the approximate range. Consequently, Atanassov and Gargov (1989) proposed IVIFS, as an extension of AIFS.
Definition 2. Let $X$ be a nonempty set. An interval-valued intuitionistic fuzzy set (IVIFS) $A$ in $X$ is denoted in the follow form:

$$A = \left\{(x, \mu_A(x), v_A(x)) \mid x \in X \right\},$$

(2)

where $\mu_A(x) = [\mu_A^L(x), \mu_A^U(x)] \subseteq [0,1]$ and $v_A(x) = [v_A^L(x), v_A^U(x)] \subseteq [0,1]$ satisfy $0 \leq \mu_A^L(x) + v_A^U(x) \leq 1$ for all $x \in X$. $\mu_A(x)$ and $v_A(x)$ are named the membership degree and the non-membership degree of the element $x \in U$ drawn from $A$, respectively.

Xu (2007) called each $(\mu_A(x), v_A(x))$ pair in $A$ an interval valued intuitionistic fuzzy number (IVIFN), and, for convenience, each IVIFN can be simply denoted by $\alpha = (\mu_\alpha, v_\alpha)$, where $\mu_\alpha = [\mu_\alpha^L, \mu_\alpha^U] \subseteq [0,1], v_\alpha = [v_\alpha^L, v_\alpha^U] \subseteq [0,1]$, and $0 \leq \mu_\alpha^L + v_\alpha^U \leq 1$.

The TOPSIS method

The Technique for Order of Preference by Similarity to Ideal Solution (TOPSIS) is a Multi-objective decision-making approach, which was originated by Hwang and Yoon in 1981. This method is widely used in various fields, such as process design, energy, and environment, health care and so on. It has improved the scientificity, accuracy and maneuverability of decision analysis obviously. There are two important notion in this method: positive ideal solution (PIS) and negative ideal solution (NIS). The basic principle is that prioritize objects according to distances of the object to the PIS and the NIS. If the object is closest to the PIS and the farthest away from the NIS, it is the best. Otherwise it is not optimal. The value of each PIS is the optimal value of each evaluation criteria. And the value of each NIS is the worst value of each evaluation criteria.

The Proposed Fuzzy Method For Qfd

The novel method consist of four steps. At first, we need to acquire quality requirements of customer by the new IVIFN aggregation operators. Although it is not the main coverage of this research, obtaining available and correct data is important in this fuzzy QFD procedure. Then, evaluate the relationship measures between QRs and EMs, which are subjectively assessed by technique experts. Next, calculate the fuzzy importance of engineering measures that is determined by the TOPSIS-based nonlinear-programming methodology. Finally, EMs can be prioritized according to the IVF set obtained from the third step.

Acquisition of quality requirements of customer

The main difference between fuzzy QFD and traditional QFD is that the evaluation values are expressed and represented in fuzzy sets instead of exact numbers. As previously described, values of QRs or EMs evaluated by IVIFSs are more accurate and more convenient to be evaluated by customers or experts.

The relative weight of each QR is one of critical data to fuzzy QFD. According to the definition of HoQ, The relationships between QRs and EMs compose the body of HoQ expressed in the relationship matrix. The target of this matrix is that obtain the important associations between each QR and its corresponding EMs rather than simply recognize whether there is a relationship.

Supposing that in a product development, there are $m$ QRs denoted by QR$j$ , $j = 1, 2, ..., m$, and $n$ EMs denoted by EM$i$ , $i = 1, 2, ..., n$. Let $s$ be the amount of customers investigated in the
A questionnaire about the product, and their respective assessment on the jth QR is denoted by $QR_j = \left( [r^{L}_{jk}, r^{U}_{jk}], \left[ \rho^{L}_{jk}, \rho^{U}_{jk} \right] \right)$, k=1, 2,...,s.

There are several different averaging operators defined for IVIFSs to obtain the importance of quality requirements of customer. These operators are not monotone with respect to the total order of IVIFS, which is undesirable. Wang et al. (2012) researched the averaging operators that can be indicated by using additive generators of the product triangular norm [9]. Moreover, a novel aggregation operators based on the Łukasiewicz triangular norm are proposed, which are monotone with respect to the total order of IVIFS.

$$\omega_k = \frac{1}{s}.$$  \tag{3}

**Evaluation of relationship measures**

In this step, determined quality requirements of customer are translated into engineering measures. The goal is to translate each customer need into one or more engineering measures. To establish the relationship matrix between QRs and EMs, it is significant to set up relationships measures that exist between every QR and every EM. The evaluation of relationship measures is given by the experts who are good at product development.

Dissimilar to the acquisition of QRs, the relationship measures between QRs and EMs are evaluated by technical experts directly and represented by IVIFNs.

**Calculating the fuzzy importance of engineering measures**

As shown in Section 3.1 and Section 3.2, since the relative importance of QRs, $QR_j = \left( [r^{L}_{jk}, r^{U}_{jk}], \left[ \rho^{L}_{jk}, \rho^{U}_{jk} \right] \right)$, and relationship measures between QRs and EMs, $U_{ij} = \left( [\mu^{L}_{ij}, \mu^{U}_{ij}], [\nu^{L}_{ij}, \nu^{U}_{ij}] \right)$, are represented as IVIFNs, calculation of the importance of EMs depends on the TOPSIS-based nonlinear-programming methodology.

In this methodology, choice of reference points is a sensitive and complex problem. Similarly, the positive ideal solution represented as Interval-valued intuitionistic fuzzy numbers (IVIFPIS) and negative ideal solution represented as Interval-valued intuitionistic fuzzy numbers (IVIFNIS) denoted by $x^*$ and $x^-$, are difficult to be defined. Determining $x^*$ and $x^-$ is a pivotal problem.

The meaning of IVIFPIS and IVIFNIS needs to be explained in a completely different way, when we calculate the fuzzy importance of engineering measures in fuzzy QFD by integrating...
the TOPSIS-based nonlinear-programming methodology. \( x^* \) is, namely, the engineering measure that has a very strong relationship with all quality requirements of customer \( CR_j \) \((j = 1,2,\ldots,n)\). Therefore, membership degree and the non-membership degree of \( x^* \) on every quality requirements of customer can be briefly represented in the IVIF vector format as \([\tau_{ij}^+,\tau_{ij}^-],[\rho_{ij}^+,\rho_{ij}^-]_{j=1}^n = ([1.1],[0,0])_{i=1}^n\). In the same way, The IVIFNIS \( x^* \) is the engineering measure that has none relationship with all quality requirements of customer \( QR_j \) \((i = 1,2,\ldots,n)\). And, membership degree and the non-membership degree of \( x^* \) on every quality requirements can be briefly represented in the IVIF vector format as \([\tau_{ij}^+,\tau_{ij}^-],[\rho_{ij}^+,\rho_{ij}^-]_{j=1}^n = ([1.1],[0,0])_{i=1}^n\). It is simple to illustrate that \( x^* \) is the complement of \( x^- \).

With the target of comparing alternative EMs \( x_i \) \((i = 1,2,\ldots,m)\), distances between IVIFS can be used to measure differences between an alternative EM \( x_i \) and the IVIFPIS \( x^* \), as well as the IVIFNIS \( x^- \). Here, the weighted-Euclidean distances between \( x^* \) and \( x_i \) as well as \( x^- \) and \( x_i \) are, respectively, defined as follows:

\[
D_i^+(x_i,x^*) = \sqrt{\sum_{j=1}^{n}\left[\tau_{ij}^+(1-\mu_j^i)\right]^2 + \left[\rho_{ij}^+(1-\nu_j^i)\right]^2} \tag{4}
\]

\[
D_i^-(x_i,x^-) = \sqrt{\sum_{j=1}^{n}\left[\tau_{ij}^-(1-\mu_j^i)\right]^2 + \left[\rho_{ij}^-(1-\nu_j^i)\right]^2} \tag{5}
\]

The relative closeness coefficient of an alternative EM \( x_i \in X \) to \( x^* \) is denoted as follows:

\[
C_i(\mu_i,\nu_i,\tau_j,\rho_j) = \frac{D_i^+(x_i,x^*)}{D_i^+(x_i,x^*) + D_i^-(x_i,x^-)} \tag{6}
\]

Obviously, \(0 \leq D_i^+(x_i,x^*) \leq D_i^-(x_i,x^-) + D_i^+(x_i,x^-)\). Hence, it obviously subject to that

\[
0 \leq C_i(\mu_i,\nu_i,\tau_j,\rho_j) \leq 1 \tag{7}
\]

According to (4) and (5), \( C_i(\mu_i,\nu_i,\tau_j,\rho_j) \) could be represented as in (8):

\[
C_i(\mu_i,\nu_i,\tau_j,\rho_j) = \frac{\sqrt{\sum_{j=1}^{n}\left[\tau_{ij}^+(1-\mu_j^i)\right]^2 + \left[\rho_{ij}^+(1-\nu_j^i)\right]^2}}{\sqrt{\sum_{j=1}^{n}\left[\tau_{ij}^-(1-\mu_j^i)\right]^2 + \left[\rho_{ij}^-(1-\nu_j^i)\right]^2} + \sqrt{\sum_{j=1}^{n}\left[\tau_{ij}^+(1-\mu_j^i)\right]^2 + \left[\rho_{ij}^+(1-\nu_j^i)\right]^2}} \tag{8}
\]

Obviously, \( C_i(\mu_j,\nu_j,\tau_j,\rho_j) \) is a continuous function of \( 2(m+1)n \) variables, including \( \mu_j \in [\mu_j^L,\mu_j^U], \nu_j \in [\nu_j^L,\nu_j^U], \tau_j \in [\tau_j^L,\tau_j^U], \) and \( \rho_j \in [\rho_j^L,\rho_j^U] \) \((j = 1,\ldots,n)\).

\( C_i(\mu_j,\nu_j,\tau_j,\rho_j) \) \((j = 1,\ldots,n)\) are monotonic and nondecreasing functions of the variables \( \mu_j \in [\mu_j^L,\mu_j^U] \) \((j = 1,\ldots,n)\). Similarly, \( C_i(\mu_j,\nu_j,\tau_j,\rho_j) \) \((j = 1,\ldots,n)\) are monotonic and nonincreasing functions of the variables \( \nu_j \in [\nu_j^L,\nu_j^U] \).

\( C_i^l \) and \( C_i^u \) can be obtained by solving the nonlinear programming models that are enumerate as follows:

\[
C_i^l = \min\{C_i(\mu_j,\nu_j,\tau_j,\rho_j)\}
\]

\[
st.\left(\begin{array}{c}
\mu_j \in \Omega_j \times \Omega_j \times \Omega_j \times \Omega_j,
\nu_j \in \Omega_j \times \Omega_j \times \Omega_j \times \Omega_j
\end{array}\right)
\]

and
\[ C^u_i = \max \left\{ C_i \left( \mu_i, v_i, \tau_i, \rho_i \right) \right\} \]
\[ \text{s.t.} \left( \left( \mu_i \right)_{\text{max}}, \left( v_i \right)_{\text{max}}, \left( \tau_i \right)_{\text{max}}, \left( \rho_i \right)_{\text{max}} \right) \in \Omega_x \times \Omega_x \times \Omega_x \times \Omega_x \]

(10)

Here, \( \Omega_x = \left\{ (\mu_i)_{\text{max}} \mid \mu_i \leq \mu_{i}^{u} \right\} \) and \( \Omega_x = \left\{ (v_i)_{\text{max}} \mid v_i \leq v_{i}^{u} \right\} \) imply all possible degrees of membership and non-membership of \( \mu \) and \( v \) respectively.

Similarly, \( \Omega_{\tau} = \left\{ (\tau_j)_{\text{max}} \mid \tau_j \leq \tau_{j}^{u} \right\} \) and \( \Omega_{\rho} = \left\{ (\rho_j)_{\text{max}} \mid \rho_j \leq \rho_{j}^{u} \right\} \) imply all possible degrees of membership and non-membership of \( \tau \) and \( \rho \). The amount of variables in (11) is not a bit less than that in (10). There is no doubt that solving (11) is not a bit simpler than solving (9).

As the previous analysis to (9), \( C_i \left( \mu_i, v_i, \tau_i, \rho_i \right) \) reaches its maximum at the upper limits \( \mu_{i}^{u} \) of the intervals \( \left[ \mu_{i}^{u}, \mu_{i}^{l} \right] \) and the upper limits \( v_{i}^{u} \) of the intervals \( \left[ v_{i}^{u}, v_{i}^{l} \right] \) simultaneously. Therefore, (10) can be considerably predigest as follows:

\[ C^l_i = \min \left\{ C_i \left( \mu_i, v_i, \tau_i, \rho_i \right) \right\} \]
\[ \text{s.t.} \left( \left( \left( \tau_i \right)_{\text{max}}, \left( \rho_i \right)_{\text{max}} \right) \right) \in \Omega_x \times \Omega_x \]

(11)

Clearly, there are 2n unknown variables is a nonlinear-programming model (11), including \(\tau_j \in [\tau_{j}^{u}, \tau_{j}^{l}]\) and \(\rho_j \in [\rho_{j}^{u}, \rho_{j}^{l}]\) \((i = 1, \ldots, m; \ j = 1, \ldots, n)\). The amount of variables in (11) is not a bit less than that in (9). There is no doubt that solving (11) is not a bit simpler than solving (9).

After obtaining the IVF sets, prioritizing these EMs become the stringent problem at hand. The relative closeness coefficient of \( x_i \) to \( x^* \) is denoted by the IVF set \( \left[ C_i^l, C_i^u \right] \).

It is easily derived from (7) that \( C_i^u + (1 - C_i^l) = 1 + (C_i^l - C_i^u) \leq 1 \). Hence, \( C_i^l \) and \( C_i^u \) subject to the following conditions: \( 0 \leq C_i^l \leq 1, 0 \leq 1 - C_i^l \leq 1 \), and \( C_i^u + (1 - C_i^l) \leq 1 \). According to the definition of the AIFS, the IVF set \( \left[ C_i^l, C_i^u \right] \) may be equivalently expressed as an AIFS \( C_i = \left( C_i^l, 1 - C_i^l \right) \), which implies that the membership degrees and non-membership degrees of the \( x_i \in X \) to the IVIFPIS \( x^* \) are \( C_i^l \) and \( 1 - C_i^l \), respectively. \( \pi_{C_i} = C_i^l - C_i^u \) mirrors the uncertainty on the
The closeness degree of \( x_i \in X \) to \( x^* \). Therefore, the AIFSs \( C_i = \{ C_i^+, 1 - C_i^+ \} \) can be used to prioritize the alternative EMs \( x_i (i = 1, \ldots, m) \).

The inclusion-comparison probability of \( C_i \) and \( C_k \) can be denoted as follows:

\[
p(x_i \succ x_k) = \max \left\{ 1 - \max \left[ \frac{C_i^+ - C_k^+}{\pi_i + \pi_k}, 0 \right], 0 \right\}
\]

(13)

where \( \pi_i = C_i^+ - C_i \) and \( \pi_k = C_k^+ - C_k \) are hesitation degrees of \( x_i \) and \( x_k \), respectively.

It is obviously believed that \( 0 \leq p_a \leq 1, p_a + p_{a_i} = 1 (i, k = 1, 2, \ldots, m) \). That is to say, \( P \) is a fuzzy complementary judgment matrix. We can determine optimal degrees of membership for alternative EMs \( x_i (i = 1, 2, \ldots, m) \) as follows:

\[
\theta_i = \frac{1}{m(m-1)} \left( \sum_{i=1}^{m} p_a + \frac{m}{2} - 1 \right).
\]

(14)

Then, it is easy to rank the order of all alternative EMs \( x_i (i = 1, 2, \ldots, m) \) according to the values \( \theta_i \).

A Numerical Example

Diesel engine is a complex power device, not only the large number of parts and complex structure. With traditional design methods, even seasoned designers often lack predictability for follow-up work, often requiring repeated modifications to complete. This example will use the new fuzzy QFD method to solve the problem of diesel engine product design. In this example, 8 major inputs (QRs) are identified for the design of the Diesel engine, which are “QR1: Sufficient power”, “QR2: Large torque”, “QR3: Safety at work”, “QR4: Long lasting”, “QR5: Small size”, “QR6: Testing convenience”, “QR7: Small vibration”, and “QR8: Start quickly”. Based on the design teams experience and expert knowledge, 5 EMs are identified corresponding to the 8 major QRs, namely, “Maximum torque” (EM1), “Maximum combustion pressure” (EM2), “Power per unit volume” (EM3), “Calibration speed” (EM4), and “Reliability” (EM5).

Then, ten surveyed customers are asked to assess the importance of eight QRs from membership and non-membership, respectively. The membership and non-membership of QRs’ importance are represented as the linguistic data at five levels, namely, extremely, very, quite, moderately and slightly.

Every customer chooses a pair of linguistic variable to evaluate the membership and non-membership of QRs’ importance respectively according to define of IVIFS, which can form an IVIFN. Utilize those IVIFNs to express their individual assessments on each QR.

According to the previous discussion, the relationships between the QRs and the EMs cannot be evaluation by linguistically judged as none, weak, moderate, strong, or very strong. In this paper, the relationships between the QRs and the EMs are IVIHFs judged by experts. Further, the fuzzy relationship matrix between the eight QRs and the five EMs can be established as shown in Table 1.
Table 1. The relationship matrix between quality requirements of customer and engineering measures.

<table>
<thead>
<tr>
<th></th>
<th>EM1</th>
<th>EM2</th>
<th>EM3</th>
<th>EM4</th>
<th>EM5</th>
</tr>
</thead>
<tbody>
<tr>
<td>QR1</td>
<td>[0.8,0.9], [0.02,0.1]</td>
<td>[0.02,0.12], [0.68,0.77]</td>
<td>[0.32,0.42], [0.32,0.52]</td>
<td>[0.45,0.54], [0.36,0.4]</td>
<td>[0.12,0.3], [0.5,0.6]</td>
</tr>
<tr>
<td>QR2</td>
<td>[0.5,0.7], [0.2,0.26]</td>
<td>[0.2,0.26], [0.36,0.39]</td>
<td>[0.89,0.92], [0.02,0.06]</td>
<td>[0.35,0.53], [0.33,0.39]</td>
<td>[0.26,0.35], [0.23,0.452]</td>
</tr>
<tr>
<td>QR3</td>
<td>[0.15,0.25], [0.36,0.39]</td>
<td>[0.3,0.36], [0.4,0.5]</td>
<td>[0.56,0.6], [0.23,0.32]</td>
<td>[0.85,0.95], [0.02,0.04]</td>
<td>[0.45,0.51], [0.35,0.45]</td>
</tr>
<tr>
<td>QR4</td>
<td>[0.39,0.45],[0.36,0.4]</td>
<td>[0.56,0.75],[0.1,0.2]</td>
<td>[0.45,0.52],[0.32,0.39]</td>
<td>[0.35,0.42],[0.33,0.33]</td>
<td>[0.89,0.92],[0.02,0.06]</td>
</tr>
<tr>
<td>QR5</td>
<td>[0.5,0.56],[0.36,0.4]</td>
<td>[0.9,1],[0]</td>
<td>[0.35,0.53],[0.23,0.32]</td>
<td>[0.15,0.3],[0.65,0.55]</td>
<td>[0.45,0.54],[0.23,0.32]</td>
</tr>
<tr>
<td>QR6</td>
<td>[0.69,0.75],[0.15,0.23]</td>
<td>[0.23,0.36],[0.4,0.5]</td>
<td>[0.18,0.28],[0.35,0.53]</td>
<td>[0.35,0.53],[0.23,0.32]</td>
<td>[0.56,0.65],[0.23,0.32]</td>
</tr>
<tr>
<td>QR7</td>
<td>[0.26,0.35],[0.36,0.45]</td>
<td>[0.45,0.54],[0.23,0.32]</td>
<td>[0.26,0.32],[0.33,0.43]</td>
<td>[0.45,0.66],[0.23,0.3]</td>
<td>[0.57,0.75],[0.12,0.22]</td>
</tr>
<tr>
<td>QR8</td>
<td>[0.36,0.45],[0.36,0.4]</td>
<td>[0.23,0.42],[0.4,0.5]</td>
<td>[0.59,0.7],[0.22,0.29]</td>
<td>[0.59,0.74],[0.12,0.2]</td>
<td>[0.39,0.5],[0.35,0.41]</td>
</tr>
</tbody>
</table>

According to the method introduced before, the fuzzy importance of engineering measures should be calculated immediately. For instance, we should construct two simplified nonlinear programming models for the alternative EM1 \((x_1)\). Using nonlinear-programming solving approach, optimal solutions of the two models are obtained as
\[ L_1 = 0.38, \quad U_1 = 0.56 \]
respectively, i.e., the relative-closeness IF set of EM1 \((x_1)\) is \(C_1 = (0.38,0.44)\).

Similarly, the relative-closeness IF sets of the alternative EMs \(x_i\) \((i = 2,3,4,5)\) are obtained as follows:
\[ C_2 = (0.32,0.45), \quad C_3 = (0.44,0.37), \quad C_4 = (0.43,0.35), \quad C_5 = (0.44,0.39) \]

The inclusion-comparison probabilities of pairwise IF sets \(C_i (i=1,2,3,4,5)\) can be obtained and expressed in the matrix format as follows:
\[
P = \begin{bmatrix}
0.5 & 0.585 & 0.324 & 0.325 & 0.342 \\
0.414 & 0.5 & 0.262 & 0.267 & 0.275 \\
0.676 & 0.738 & 0.5 & 0.488 & 0.528 \\
0.675 & 0.733 & 0.512 & 0.5 & 0.538 \\
0.657 & 0.725 & 0.472 & 0.462 & 0.5 \\
\end{bmatrix}
\]

Using (15), optimal degrees of membership for the alternative EMs \(x_i (i=1,2,3,4)\) can be calculated as follows:
\[ \theta_1 = 0.1789, \quad \theta_2 = 0.1609, \quad \theta_3 = 0.2215, \quad \theta_4 = 0.2229, \quad \theta_5 = 0.2158 \]
respectively. Therefore, the ranking order of the five alternative EMs is \(x_4 \succ x_3 \succ x_5 \succ x_1 \succ x_2\) and the best alternative is \(x_4\), the “coordinate measuring machine” (EM4).

Based on the priority of the EMs, we can determine a diesel engine product design, in which those important engineering measures should be paid more attention, and else unimportant engineering measures do not need to invest a lot of design time and cost.

**Conclusions**

The main difference between fuzzy QFD and traditional QFD is that the evaluation values are represented in INIFS instead of exact numbers. Thus, calculating and ranking the fuzzy importance of engineering measures become more difficult in fuzzy QFD. This paper proposes a novel method by integrating the new aggregation operators and the TOPSIS-based nonlinear-programming methodology to prioritize the engineering measures. The novel approach consists of four interrelated steps, namely, (1) acquisition of quality requirements of customer, (2) evaluation of relationship measures, (3) calculating the fuzzy importance of engineering measures, and (4) ranking the fuzzy importance of engineering measures. The numerical example clearly demonstrate the practicality and extensibility of the proposed fuzzy QFD.
method. With the deepening of research about fuzzy QFD, we will find that the final ordering of the EMs not only depends on the weights of QRs and relationship measures between QRs and EMs but also on the correlations among multiple engineering measures, which will be the focus of future research work in fuzzy QFD.

References


