Structure the Nuclear Radiation Dose Function of One Dimensional

Min Zhang

Abstract

The traditional interpolation and fitting cannot restore the radiation dose curve with the radiation source characteristic. Therefore, during the reduction the field of one dimensional radiation dose, the mathematical structure method was used to restore the curve with the dose value of the radioactive source is inversely proportional to the square of the distance from the radioactive source.

Key words: inversion, construct, one dimensional radiation dose, dose dominance of adjacent point.

1. INTRODUCTION

The rectangular area of containing the radiation source is divided into grid, and the corresponding grid node data is collected. We can structure dose function to fill the missing part of the data with the limited data. Consider horizontal rectangular area with point source pollution \( \Omega = \{(x, y) | 0 \leq x \leq a, 0 \leq y \leq b\} \). In the X axis, range \([0, a]\), insert the \( n-1 \) point, \( x_1, x_2, x_3, \cdots, x_{n-1} \) and let \( x_0 = 0, x_n = a \). In the Y axis, range \([0, b]\), insert the \( m-1 \) point \( y_1, y_2, y_3, \cdots, y_{m-1} \) and let \( y_0 = 0, y_m = b \), so the search area is evenly divided.

Then \( E_{n+1} \times E_{m+1} = \{(x_i, y_j) | i = 0, 1, \cdots, n; j = 0, 1, \cdots, m\} \) is the rectangular point set, the grid node value \((x_i, y_j)\) is \( f_{i,j}, i = 0, 1, \cdots, n; j = 0, 1, \cdots, m\) (see Table 1).
Table 1. Corresponding observation values of grid nodes.

<table>
<thead>
<tr>
<th>y₀</th>
<th>y₁</th>
<th>y₂</th>
<th>...</th>
<th>yₙ</th>
</tr>
</thead>
<tbody>
<tr>
<td>x₀</td>
<td>f₀₀</td>
<td>f₀₁</td>
<td>...</td>
<td>f₀ₙ</td>
</tr>
<tr>
<td>x₁</td>
<td>f₁₀</td>
<td>f₁₁</td>
<td>...</td>
<td>f₁ₙ</td>
</tr>
<tr>
<td>x₂</td>
<td>f₂₀</td>
<td>f₂₁</td>
<td>...</td>
<td>f₂ₙ</td>
</tr>
<tr>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>xₙ</td>
<td>fₙ₀</td>
<td>fₙ₁</td>
<td>...</td>
<td>fₙₙ</td>
</tr>
</tbody>
</table>

The idea of the inversion of the nuclear radiation dose field is that the dose of any point is obtained by using the cubic spline interpolation with the data of grid node, which is based on collected grid node data. For interior grid points, we can get the value of any point by structure. Therefore, the dose of any point in the region can be obtained.

2. ONE DIMENSIONAL RADIATION DOSE DOMINANCE OF ADJACENT POINT

The source of continuous slope change structure is the continuous concentration function curve with concave function characteristic in the range of grid data.

Taking fully into account the characteristics of the radioactive source, the radioactive dose is inversely proportional to the square of the distance from the nuclear sources [1]. Based on the dose dominance of adjacent point, the radiation dose was restored and dose curve of one dimensional is obtained.

On line \( y = y_j, \ j = 1, 2, \ldots, m \), \( \forall x \in [x_{i,j}, x_{i+1,j}] \), let \( x_{0,j} \) be the maximum value in interval \( [x_{i,j}, x_{i+1,j}] \), \( d \) is the distance from \( x_{i,j} \) to \( x_{0,j} \), \( g \) is the distance from \( x \) to \( x_{0,j} \). The dose of \( x_{0,j}, x_{i,j}, x_{i+1,j} \) and \( X \) are \( f_{0,j}, f_{i,j}, f_{i+1,j} \) and \( f^*(x) \), respectively. \( \lambda, k \) are the undetermined parameters (see Fig. 2).

If \( x_{i,j} = \max\{x_{i,j}, x_{i+1,j}\} \), then let \( d^2 = (x_{0,j} - x_{i,j})^2 \), \( g^2 = (x - x_{0,j})^2 \), based on the radioactive dose is inversely proportional to the square of the distance from the nuclear sources,

\[
\frac{f^*(x)}{f_{i,j}} = \frac{\lambda}{g^2},
\]

let \( \frac{f^*(x)}{f_{i,j}} = k \frac{d^2}{g^2}, \)
then

\[ f^*(x) = \frac{kd^2}{g^2} f_{i,j}, \]

When \( x \) approaches \( x_{i,j} \), then \( d, h \) approaches 0, and \( f^* \) approaches \( f_{0,j} \), then \( k = \frac{f_{0,j}}{f_{i,j}} \), and as a result \( f^*(x) = \frac{d^2}{g^2} f_{0,j} \). Similarly, if \( x_{i+1,j} = \max\{x_{i,j}, x_{i+1,j}\} \), then \( d^2 = (x_{0,j} - x_{i+1,j})^2 \).

\[ g^2 = (x-x_{0,j})^2, \quad f^*(x) = \frac{d^2}{g^2} f_{0,j}. \]

3. OPTIMIZATION STRUCTURE THE RADIATION DOSE

We can use the distance between the two endpoints of the interval \([x_{i,j}, x_{i+1,j}]\) and the maximum value of the radiation measurement to meet the inverse square of the distance between the direct calculation of the range of arbitrary points in the radiation measurement values.

\[ y = y_j, i = 1,2,\cdots,n \in \mathbb{Z}, \quad \text{Equal section} \quad [x_{i,j}, x_{i+1,j}], i = 1,2,\cdots,n - 1 \quad \text{on the X axis}, \quad \text{The dose value of any point is constructed.} \quad \forall x \in [x_{i,j}, x_{i+1,j}], \quad \text{Assuming that the length of the interval is} \quad h, \quad A \] is the maximum value of the dose near \( x_{i,j} \). The distance from \( A \) to \( x_{i,j} \) is \( d_1 \), The distance from \( x \) to \( x_{i,j} \) is \( d \), the radiation dose of \( x_{i,j}, x_{i+1,j} \) were \( f_{i,j}, f_{i+1,j} \) respectively. \( S^* \) is the dose value of \( x, \lambda, k \) are the Parameters to be determined. If \( x_{i,j} = \max\{x_{i,j}, x_{i+1,j}\} \), then

\[ d^2 = (x-x_{i,j})^2, \quad h = x_{i+1,j} - x_{i,j}, \]

and

\[ f_{i,j} = \frac{\lambda}{d_1^2}, \quad f_{i+1,j} = \frac{\lambda}{(d_1 + h)^2}, \quad f^*(x) = \frac{\lambda}{(d_1 + d)^2} \]

then

\[ \frac{f_{i+1,j}}{f_{i,j}} = \frac{d_1^2}{(d_1 + h)^2} \]

\[ f^*(x) = \frac{d_1^2 f_{i+1,j}}{[d + \frac{\sqrt{f_{i,j}} \cdot h}{\sqrt{f_{i+1,j}} - \sqrt{f_{i,j}}}]^2}. \]
When \( x \) approaches \( x_{i,j} \), \( b, h \) will approaches zero, then \( f^*(x) \) will approaches \( f_{i,j} \), so we can get \( k = 1 \) and 
\[
\frac{d_i^2 f_{i,j}}{d + \sqrt{f_{i+1,j} \cdot h}}.
\]

Similarly, if \( x_{i+1,j} = \max\{x_{i,j}, x_{i+1,j}\} \), then 
\[
d^2 = (x - x_{i+1,j})^2, \quad h = x_{i,j} - x_{i+1,j},
\]
\[
d_1 = \frac{\sqrt{f_{i,j}} \cdot h}{\sqrt{f_{i+1,j}} - \sqrt{f_{i,j}}}, \quad \text{and} \quad \forall x \in [x_{i,j}, x_{i+1,j}],
\]
\[
f^*(x) = \frac{d_i^2 f_{i+1,j}}{d + \sqrt{f_{i+1,j} \cdot h}}.
\]

Next, the adjacent point optimization construction algorithm was given.

**Algorithm Optimization structure the Algorithm of Adjacent point**

**Step 1** \( \forall x \in [x_{i,j}, x_{i+1,j}] \), input endpoint \( x_{i,j}, x_{i+1,j} \) and the radiation measurement value 
\( f_{i,j}, f_{i+1,j} \).

**Step 2** reduction radiation measurements on a grid.

**Step 2.1** \( \forall x \in [x_{i,j}, x_{i+1,j}] \), let 
\[
d^2 = (x - x_{i,j})^2 \quad \text{and} \quad h = x_{i+1,j} - x_{i,j},
\]
\[
d_1 = \frac{\sqrt{f_{i+1,j}} \cdot h}{\sqrt{f_{i,j}} - \sqrt{f_{i+1,j}}}, \quad \text{and} \quad \forall x \in [x_{i,j}, x_{i+1,j}],
\]
\[
f^*(x) = \frac{d_i^2 f_{i,j}}{d + \sqrt{f_{i+1,j} \cdot h}}.
\]

**Step 2.2** \( \forall x \in [x_{i,j}, x_{i+1,j}] \), let 
\[
d^2 = (x - x_{i+1,j})^2, \quad h = x_{i,j} - x_{i+1,j},
\]
\[
d_1 = \frac{\sqrt{f_{i,j}} \cdot h}{\sqrt{f_{i+1,j}} - \sqrt{f_{i,j}}}, \quad \text{and} \quad \forall x \in [x_{i,j}, x_{i+1,j}],
\]
\[
f^*(x) = \frac{d_i^2 f_{i+1,j}}{d + \sqrt{f_{i+1,j} \cdot h}}.
\]

**Step 3** \( \forall x \in [x_{i,j}, x_{i+1,j}] \), output radiation dose value \( f^*(x) \) of any point.
Conclusion

The traditional interpolation and fitting cannot restore the radiation dose curve with the radiation source characteristic. Therefore, during the reduction the field of one dimensional radiation dose, the mathematical structure method was used to restore the curve with the dose value of the radioactive source is inversely proportional to the square of the distance from the radioactive source. Under this basis, this paper proposes the new idea on the issue. In the future more related work will be implemented for better performance.

REFERENCE